Complementary Activities, Heterogeneous Firms, and Industry Structure*

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Abstract

We present conditions that are sufficient to couple complementary organisational activities at the firm level with clear implications for the industry structure in a general-equilibrium heterogeneous-firm model. Under these conditions, we show how introduction of new activities and increased attractiveness of existing activities affect the industry structure in all dimensions by increasing the share of firms undertaking at least any given level of any activity. The results accentuate the need to incorporate complementary organisational activities into an integrated model where firms face a multi-dimensional choice set. Through examples our predictions are shown to apply to several well-known models at the intersection of organisational economics and international trade.

Keywords: Firm Heterogeneity; Organisational Decisions; Complementarities; Supermodularity; Industry Structure

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1 Introduction

A vast literature on heterogeneous firms in international trade has emerged in the wake of the seminal work by Melitz (2003). A part of this literature lets firms face decisions about more than one organisational activity.[1] These activities include: exporting, the number of products marketed, investments in advertising, technology and quality upgrading, outsourcing, and offshoring. Many of these activities are modelled to be complementary, in the sense that undertaking one of them increases the payoff from undertaking another. An immediate consequence of such complementarity is that activities cannot be considered in isolation.

The present paper takes its offset in a Melitz (2003)-type model of heterogeneous firms and then provides conditions under which complementarity between different activities at the firm level translates into strong predictions for the industry structure. When activities are complementary and certain conditions hold, we show that the introduction of a new organisational activity, or increased attractiveness of an existing activity, increases the share of firms undertaking at least any given level of any activities. The complementarity at the firm level thus implies that introducing or improving an activity advances the industry structure in all dimensions. These results suggest that in order to fully understand observed trends in the prevalence of a certain organisational activity, one should not only look at factors affecting that specific activity but also at factors affecting complementary activities.

A number of existing models such as Kasahara and Lapham (2006), Arkolakis (2010), Bustos (2011), Bernard et al. (2011), Arkolakis and Muendler (2011), and Caliendo and Rossi-Hansberg (2011) conform to the conditions we set up and they are therefore our general predictions for the industry structure applies. These examples are taken from the trade literature where the heterogeneous-firms setup we use has been advanced. However, there is nothing in our general formulation of the setup that limits the relevance of our results to trade issues. Indeed, the abstract activities we consider should be interpreted as being any complementary activities and decisions in the choice set of firms. This way, although deeply rooted in the trade literature, our study provides a framework for analysing industry-level implications of all complementary organisational activities and decisions. To this end, we

[1] See e.g. Antràs and Helpman (2004); Verhoogen (2008); Arkolakis (2010); Lileeva and Trefler (2008); Arkolakis and Muendler (2011); Bernard et al. (2011); Bustos (2011) for models where firms face a multi-dimensional choice set.

believe that the heterogeneous-firm approach has a lot to offer.

The present paper is related to Milgrom and Roberts (1990) and Milgrom and Roberts (1995) who conduct firm-level analysis of complementary organisational activities. In contrast to those studies, we analyse an industry equilibrium with monopolistic competition among heterogeneous firms and derive comparative statics on the industry structure. This is our main contribution to the literature. In obtaining our results, we draw heavily upon results on supermodularity discussed in Topkis (1978), Milgrom and Roberts (1990), Milgrom and Shannon (1994), and Milgrom and Roberts (1995).[2] By considering a supermodular profit function in a Melitz (2003)-type model, the present paper is also related to Mrazova and Neary (2011). Mrazova and Neary (2011) emphasise the role of supermodularity in the selection of firms into a single decision (market access) based on productivity and investigate the conditions under which supermodularity may arise. We take a different approach. We simply assume that firms are faced with multiple complementary activities that they have to decide upon, i.e., we assume that profits are supermodular in these activities. Given certain conditions, we then derive comparative statics on the industry structure allowing for firm heterogeneity in multiple dimensions.

The remainder of the paper is organised as follows. Section 2 describes the basic model setup, the central assumptions, and derives our general results. Along the way we provide a series of examples based on the Bustos (2011) model of exporting and technology upgrading to build intuition. Section 3 considers existing models to which our results can be applied and discusses a few of these in detail. Section 4 offers some concluding remarks.

2 The Model

After paying some fixed costs of entry, firms enter an industry characterised by a vector of fundamental parameters, β, and the endogenous demand level, A.[3] Upon entry into the industry, a firm draw a productivity level, $\theta \in$

[2]Supermodularity is the formal equivalent of complementarity.

[3]We call A the demand level throughout. You can think of A as a sufficient statistic for the market conditions faced by firms. Under an assumption of CES preferences, A would, as a minimum, depend on the endogenous price index and total expenditure. In only including one demand level, A, we implicitly assume that countries are symmetric in the case an open economy is considered.

$[\theta_0, \infty)$, and a vector of (other) firm characteristics, $\gamma \in \Gamma$, from exogenous known distributions.[4] After the realisations of θ and γ, the firm has to choose whether to start producing or to exit the industry. If it chooses to produce, the firm has to make an organisational decision $x = [x_1, \ldots, x_n]$, where x_i denotes the chosen level of activity i. The level of an activity can be either a discrete or a continuous variable.[5]

We let $x \in X$ where X is the set of all conceivable, but not necessarily admissible, organisational decisions. The choice set X is a lattice which, loosely speaking, means that undertaking a given level of any activity may not prevent undertaking a given level of another activity but it may require undertaking a given level of another activity.[6] The choice set of firms is restricted to a set of admissible choices $S \subseteq X$ with S being a sublattice of X.

In the theoretical part of her paper, Bustos (2011) presents a simple extension of the Melitz (2003) model that we will use as an example to strengthen the intuition for our approach and results.

Example 1. Take the standard Melitz (2003) model and allow firms to upgrade technology along the lines of Bustos (2011). The organisational decision faced by firms now comprises two activities: whether to export or not and whether to upgrade technology or not. Exporting gives access to a foreign market in exchange for an additional fixed cost of exporting. A technology upgrade in this context means that productivity is scaled up, also in exchange for an additional fixed cost. Let $\mathbb{1}_{\text{ex}}$ be an indicator function for exporting and $\mathbb{1}_{\text{up}}$ be an indicator function for upgrading technology. A firm's organisational decision can then be characterised by $x = (\mathbb{1}_{\text{ex}}, \mathbb{1}_{\text{up}})$ and the set of all conceivable organisational decisions, $X = \{(0,0), (0,1), (1,0), (1,1)\}$, is a lattice.

[4] The distinction between productivity, θ, and firm characteristics, γ, is made since we will impose some assumptions on θ that we will not impose on γ. That we label θ as productivity, however, should not be taken too literally. It could also be quality as in Caliendo and Rossi-Hansberg (2011), as long as θ conforms to the assumptions below.

[5] A discrete activity could be the decision of whether to export or not. A continuous activity could be the share of the market reached as in Arkolakis (2010) or the range of products sold in a given market as in Bernard et al. (2011). The term activity should not be taken too literally, as it could also refer to e.g. the number of organisational layers in the firm considered by Caliendo and Rossi-Hansberg (2011).

[6] For a good introduction to lattice theory for the purpose of the present paper, see Milgrom and Roberts (1995).

One possibility is to allow firms to choose among all elements of X, i.e., to let $S = X$. A possible restriction is to deny firms the opportunity to export. In this case of autarky, the set of admissible choices would be $S' = \{(0,0),(0,1)\}$ which is a sublattice of X. Another possible restriction is that firms need to upgrade technology in order to export. This gives the admissible choices $S'' = \{(0,0),(0,1),(1,1)\}$ which is also a sublattice of X. One case that yields a set of admissible choices that is not a sublattice of X is to deny firms access to simultaneous exporting and technology upgrading, i.e., to restrict firms to $S''' = \{(0,0),(1,0),(0,1)\}$. As previously mentioned, we do not allow admissible choice sets such as S''. ■

Firms naturally make their organisational decision, x, to maximise profits, π, under the constraint that $x \in S$ while taking θ, γ, β, and A as given. Assumption 1 describes the basic structure we impose on our setup.

Assumption 1. *Let an equilibrium exist and let the following be true:*

i) Profits, π, only depend on θ and A through $\Theta = A\theta$, i.e.,

$$\pi = \pi(x, \Theta, \gamma, \beta). \tag{1}$$

ii) For all $\gamma \in \Gamma$, the least productive firms choose not to produce.

iii) We restrict attention to equilibria where the organisational decision of the least productive active firms is constant for all $\gamma \in \Gamma$. Further, we restrict attention to changes in β that does not directly affect the profits obtained by the least productive active firms.[7]

iv) The distribution of productivities is Pareto and independent of γ, i.e., letting $F(\theta)$ be the c.d.f.,

$$F(\theta) = 1 - \left(\frac{\theta_0}{\theta}\right)^k, \tag{2}$$

where k is the shape parameter of the Pareto distribution.

At first sight Assumption 1 may seem restrictive, or even odd, but it is satisfied in many studies belonging to the heterogeneous-firm trade literature initiated by Melitz (2003).[8] Before we take a closer look at Assumption 1, let us continue with our example above.

[7] Their profits are allowed to be indirectly affected by equilibrium effects through A.

[8] See Section 3 for some of the possible references.

Example 1 (continued)**.** By a slight change of the original notation, the profit function in Bustos (2011) can be rewritten as

$$\pi(x, \Theta, \beta) = (1 + \tau^{1-\sigma})^{\mathbb{1}_{\text{ex}}} \mu^{\mathbb{1}_{\text{up}}} \Theta^{\sigma-1} - f - \mathbb{1}_{\text{ex}} f_{\text{ex}} - \mathbb{1}_{\text{up}} f_{\text{up}}, \tag{3}$$

where $x = (\mathbb{1}_{\text{ex}}, \mathbb{1}_{\text{up}})$, σ is the constant elasticity of substitution and demand, f are the minimum fixed costs of production while f_{ex} and f_{up} are the fixed costs of exporting and upgrading, respectively. As the only source of firm heterogeneity is productivity, we disregard γ in this case. The industry parameters faced by all firms, β, are made up of the inverse iceberg trade cost[9], $\tau^{-1} < 1$, and the percentage gains to variable profits from upgrading technology, $\mu > 1$, i.e., $\beta = (\tau^{-1}, \mu)$.[10]

Profits clearly only depend on productivity, θ, and the demand level, A, trough $\Theta = A\theta$. Bustos (2011) assumes that not all firms choose to produce after observing their productivity draw in order to get endogenous exit. Further, she assumes that the least productive active firms do not upgrade and do not export. This is done in order to obtain a non-degenerate sorting pattern in either dimension of the organisational decision. It is obvious that profits of the least productive active firms with $x = (0, 0)$ are not directly affected by changes in β. Finally, Bustos (2011) assumes that productivities are Pareto distributed. In short, the model of Bustos (2011) conforms to Assumption 1. ■

Assumption 1.i means that profits only depend on productivity and the demand level through their product. This is restrictive but it is satisfied due to assumptions of CES preferences in most heterogeneous-firm trade models. Θ can be thought of as demand-adjusted productivity that takes into account that a given productivity is worth more the higher is demand.

Assumption 1.*ii* and 1.*iii* basically ensure selection as touched upon in the example above. Assumption 1.*ii* extends the standard assumption of non-degenerate selection into production such that it holds for all possible firm characteristics, γ. No draw of firm characteristics is therefore sufficiently favourable to ensure that a firm will choose to produce regardless of its productivity. The first part of Assumption 1.*iii* ensures that all equilibria

[9] An iceberg trade cost at the size of τ means that τ goods must be shipped in order for one good to arrive on foreign shores as $\tau - 1$ goods melt during transit.

[10] μ is a positive monotone transformation of the factor that scales up productivity when technology is upgraded in Bustos (2011). It will soon become clear why we use τ^{-1} and not τ as a industry parameter in β.

considered in our comparative static analysis exhibit the same extent of non-degenerate selection, i.e., if all active firms undertake at least a given level of a given activity in one equilibrium, given γ, then this should also be the case in all other equilibria considered.[11] In short, we do not consider comparative statics where non-degenerate self selection, with respect to a given level of a given activity, degenerates or vice versa.[12] As earlier mentioned, this is standard in the international-trade literature as heterogeneous-firm models with self selection into specific activities often do not consider degenerate cases. An approach motivated by empirical evidence. The first part of Assumption 1.*iii* simply extends this to all $\gamma \in \Gamma$, as we allow for firm heterogeneity in multiple dimensions. The second part of Assumption 1.*iii* states that the least productive active firms are unaffected, at least directly, by changes in the fundamental industry parameters, β. This seems restrictive but it is quite standard to consider changes in parameters that have a differential impact upon firms. Often this is done in a way where the least productive active firms are not affected directly but only indirectly through the equilibrium demand level effects which we allow for.

Assumption 1.*iv* states that the distribution of productivities is Pareto. This assumption is widely used as it is analytically tractable and enjoys some empirical support.[13] For our purpose, the key property of the Pareto distribution is that the share of active firms undertaking at least a given level of an activity depends only on the relevant productivity cutoff for this level of the activity relative to the production cutoff.[14]

The role of Assumption 1 will soon become clear. Let us now introduce Assumption 2 which together with Assumption 1 gives us our first proposition.

Assumption 2. *The profit function, $\pi(x, \Theta, \gamma, \beta)$, is supermodular in x, i.e.,*

[11]While we restrict the organisational decision of the least productive active firms to be invariant across equilibria for a given γ, the identity of the least productive active firms generally changes. Through general equilibrium effects, the productivity requirement (cutoff) for producing can change.

[12]However, we do allow for scenarios where sorting into one activity is degenerate across all equilibria.

[13]The models referenced in Section 3 adopt this assumption. See e.g. Axtell (2001) for empirical support for Pareto distributed productivities.

[14]Fix γ. Let all firms with productivities above θ_{ex} export and let all firms with productivities above θ^* be active. Then the share of exporting firms only depends on the ratio of the two cutoffs, $\theta_{\text{ex}}/\theta^*$. The cutoff structure of firm sorting is discussed in Section 2.1.

for all $x, y \in X$,

$$\pi(x, \Theta, \gamma, \beta) + \pi(y, \Theta, \gamma, \beta) \leq \pi(x \vee y, \Theta, \gamma, \beta) + \pi(x \wedge y, \Theta, \gamma, \beta), \qquad (4)$$

where $x \vee y$ *is the component-wise maximum of* x *and* y *and* $x \wedge y$ *is the component-wise minimum. Moreover, the profit function,* $\pi(x, \Theta, \gamma, \beta)$*, is strictly increasing in* Θ *and has increasing differences in* (x, Θ)*, i.e., if* $x' \leq x'$*, then* $\pi(x, \Theta, \gamma, \beta) - \pi(x', \Theta, \gamma, \beta)$ *is monotone nondecreasing in* Θ*.*

Supermodularity of the profit function in the organisational decision, x, is the formal equivalent of saying that activities are complementary. That is, increasing the level of one activity increases the gains from increasing the level of other activities.[15] Increasing differences in Θ mean that, given the demand level A, it becomes more attractive to increase the level of any activity as productivity increases. These two features of the profit function are instrumental in obtaining the general cutoff structure of firm sorting discussed in Section 2.1. Since such a cutoff structure is tremendously helpful in characterising the equilibrium sorting pattern – especially with a multi-dimensional choice set – Assumption 2 is actually satisfied in many heterogeneous-firm models.[16] It is important to note that proper ordering of activity levels may be needed in order to obtain a supermodular profit function.[17]

Under the commonly-statisfied assumptions summarised in Assumption 1, complementary organisational decisions, which are reinforced by productivity as in Assumption 2, lead us directly to Proposition 1.

Proposition 1. *Let* $\lambda_i(x_i, \beta, S)$ *denote the share of active firms undertaking at least the level* x_i *of activity* i*. Under Assumption 1 and 2, if* S' *and* S

[15] To illustrate the complementarity arising from supermodularity, Milgrom and Roberts (1990) rewrites (4),

$$[\pi(x) - \pi(x \wedge y)] + [\pi(y) - \pi(x \wedge y)] \leq \pi(x \vee y) - \pi(x \wedge y),$$

where dependence on Θ, γ, and β has been suppressed for simplicity. From this equation it is obvious that increasing the level of several activities together changes profits by more than the sum of changes from increasing them separately. Hence, the activities are complementary.

[16] See Section 3 for possible references.

[17] While you could order the activites considered in Example 1 in many ways, the (partial) orderings exports $\geq$ no exports and technology upgrading $\geq$ no technology upgrading are needed in order to obtain a profit function that is supermodular in these activities.

are both sublattices of X and $S' \leq_s S$, where $\leq_s$ is the strong set order,[18] *then we have that $\lambda_i(x_i, \beta, S') \leq \lambda_i(x_i, \beta, S)$ for all levels, x_i, of all activities, $i = 1, 2, ..., n$.*

Proof. The proof is provided in Section 2.1. □

Proposition 1 states that the share of firms undertaking at least any given level of any activity is nondecreasing in an upward expansion of the set of admissible organisational decisions. Note that here we consider the overall share of firms in the industry. Thus, we do not condition on firm characteristics, γ, but aggregate across these, perhaps unobservable, characteristics. Proposition 1 reveals that the firm-level complementarity imposed by Assumption 2 asserts itself at the industry level.

We illustrate the result in Proposition 1 by continuing the example above. It turns out that Assumption 2 is also satisfied in Bustos (2011) and therefore, Proposition 1 applies.

Example 1 (continued)**.** Consider the profit function (3). It is easy to verify that this is supermodular in $x = (\mathbb{1}_{\text{ex}}, \mathbb{1}_{\text{up}})$. Thus, exporting and technology upgrading are complementary in this setup. The intuition is straight forward. Upgrading technology allows you to produce with higher productivity. Obviously, a more productive technology is worth more when applied to a larger production, and as exporting increases the production volume of the firm, it makes technology upgrading more attractive. Conversely, exporting allows you to serve another market. Obviously this is worth more when that market can be served with higher productivity and exporting is therefore more attractive when technology is already upgraded.

The profit function (3) also exhibits increasing differences in Θ. Technology upgrading is worth more when the firm is productive to start with since operating profits, which are increasing in productivity, are effectively scaled up by a fixed percentage, cf. (3). Moreover, exporting is worth more when productivity is higher.

Consequently, the Bustos (2011) framework conforms to both Assumption 1 and 2 and therefore, Proposition 1 applies. If the economies move from autarky to costly trade, this will not only imply that some firms begin to export but also that the share of active firms that choose to upgrade

[18] See e.g. Milgrom and Shannon (1994). $S' \leq_s S$ means that for all $x' \in S'$ and $x \in S$, we have that $x \wedge x' \in S'$ and $x' \vee x \in S$.

technology is nondecreasing. Conversely, the share of active firms that export is nondecreasing when technology upgrading is made available for firms. As emphasised above, including exporting or technology upgrading in the admissible choice set of firms advances the industry structure in all dimensions, in the sense that the shares of firms undertaking each activity are nondecreasing. ■

An additional assumption, this time on the dependence of profits on the industry parameters, β, will lead to a second proposition on comparative statics.

Assumption 3. *The profit function,* $\pi(x, \Theta, \gamma, \beta)$*, has increasing differences in* (x, β)*, i.e., if* $x' \leq x$*, then* $\pi(x, \Theta, \gamma, \beta) - \pi(x', \Theta, \gamma, \beta)$ *is monotone nondecreasing in* β*.*

Proposition 2. *Under Assumption 1–3, if* $\beta' \leq \beta$*, then we have* $\lambda_i(x_i, \beta', S) \leq \lambda_i(x_i, \beta, S)$ *for all levels,* x_i*, of all activities,* $i = 1, 2, ..., n$*.*

Proof. The proof is provided in Section 2.1. □

Proposition 2 implies that the share of firms undertaking at least any given level of any activity is nondecreasing in β. Basically, Assumption 3 implies that increasing β makes increasing the level of any activities weakly more attractive as profits increase by weakly more from doing so. As the activities are complementary, increasing the attractiveness of some of them implies that the share of firms that undertake at least any given level of any of them is nondecreasing. While Proposition 1 is concerned with an expansion of the firms' admissible choice set, Proposition 2 is concerned with an improvement in the attractiveness of already admissible decisions. We illustrate this distinction by returning to our example based on Bustos (2011) which also shows that care must be taken in choosing how to express the industry parameters, β, in order to obtain increasing differences.

Example 1 (continued)**.** It can easily be verified that the profit function (3) exhibits increasing differences in $\beta = (\tau^{-1}, \mu)$. Indeed, this is the reason why we use τ^{-1} and not τ as an industry parameter. The model of Bustos (2011) therefore also conforms to Assumption 3 and Proposition 2 applies.

Increases in τ^{-1}, through trade liberalisation, where the iceberg trade cost, τ, is lowered, thus imply that not only is the share of active firms who export nondecreasing, the share of active firms who upgrade technology is also nondecreasing. The reason is, that a decrease in τ makes exporters expand their production, thereby making a technology upgrade more attractive. Analogously, letting the technology upgrade have a larger impact on productivity, not only implies that the share of active firms who upgrade technology is nondecreasing, in addition, the share of active firms who export is nondecreasing.

Summing up, we note that, in the Bustos (2011) framework, introducing exporting or technology upgrading, or improving the attractiveness of either, advances the industry structure in all dimensions. ■

Proposition 1 and 2 highlight the importance of considering complementary activities in an integrated framework. It also suggests that, in explaining industry-level trends observed in the data, e.g. the rise of offshoring during the last three decades, you should not only look at factors directly related to the offshoring decision of firms but also at factors related to complementary activities. If offshoring is complementary to e.g. technology upgrading at the firm level, and the technology upgrading opportunities have been improved, then this could explain part of the rise in offshoring at the industry-level.

It is important to note that Proposition 1 and 2 are industry level predictions. As such they say little about how any given firm responds to the change in S or β. Thus, the propositions should not lead you to expect that we will only observe that firms, characterised by (θ, γ), increase their activity levels. If general equilibrium effects are negative, i.e., A decreases, we may observe that for given characteristics (γ) some firms (θ) actually decrease their level of a given activity even though we observe an increase in the overall share of firms undertaking any given level of any activity.[19]

2.1 Proof of Proposition 1 and 2

In order to see how the elements of Assumption 1 through 3 result in Proposition 1 and 2, let us run through the proof of these propositions. The role of each assumption is emphasised along the way. First, consider the optimal

[19] This is because the share of firms undertaking at least any given level of any activity increases if surviving firms reduce activity levels to a lesser extent than low-productivity firms are pushed out of production. This is somewhat related to weak versus strong effects as discussed in Section 2.2.

organisational decision, x^*, for a specific firm[20]

$$x^*(\Theta, \gamma, \beta, S) = \arg\max_{x \in S} \pi(x, \Theta, \gamma, \beta). \tag{5}$$

Now, under Assumption 2, x^* is monotone nondecreasing in Θ.[21] This means that a given equilibrium (a given A) exhibits a certain kind of cutoff structure. For given characteristics, γ, a higher demand-adjusted productivity Θ, which translates directly into higher productivity θ for given A, weakly increases the optimal level of any activity. That is, if a firm with the productivity-characteristics pair (Θ', γ) find it optimal to undertake a given level of a given activity, then $\Theta > \Theta'$ implies that firms with the pair (Θ, γ) find it optimal to undertake at least the same level of the activity in question. Formally, let $\Theta_i(x_i, \gamma, \beta, S)$ be the lowest level of demand-adjusted productivity for which it is optimal to undertake level x_i of activity i, given (γ, β, S). In effect, $\Theta_i(x_i, \gamma, \beta, S)$ is the cutoff (demand-adjusted) productivity for undertaking at least level x_i of activity i for firms with the characteristics γ. This cutoff structure, ensured by Assumption 2, is tremendously helpful for characterising the equilibrium sorting of firms into different organisational decisions.

Next, let $\Theta^*(\gamma, \beta, S)$ be the lowest level of demand-adjusted productivity for which it is profitable to produce and thus be active, i.e., $\Theta^*(\gamma, \beta, S)$ is implicitly given as

$$\pi(x^*(\Theta^*, \gamma, \beta, S), \Theta^*, \gamma, \beta) = 0. \tag{6}$$

Firms with (demand-adjusted) productivities less than $\Theta^*(\gamma, \beta, S)$ choose not to produce and shut down. Under assumption 1.*ii* we have that $\Theta^*(\gamma, \beta, S) > A\theta_0$ where θ_0 is the scale parameter of the Pareto distribution of productivities. Together with Assumption 1.*iii* this implies that for a given γ, the demand-adjusted productivity cutoff for producing, $\Theta^*(\gamma, \beta, S)$, is the same in all the equilibria considered. To see this, note that under Assumption 1.*iii*, x^* is unaffected for the least productive active firms, given γ. Further, profits under this organisational decision are not affected directly through β. Effectively, we can henceforth write $\Theta^*(\gamma, \beta, S) = \Theta^*(\gamma)$. The finding

[20] In case of non-uniqueness of the maximiser, let x^* be the component-wise maximum over the arguments maximising profits. The component-wise maximum of maximisers will itself be a maximiser (see Milgrom and Roberts, 1995).

[21] This is an immediate consequence of Theorem 5 of Milgrom and Shannon (1994).

that $\Theta^*(\gamma) > A\theta_0$ in turn ensures that the unadjusted cutoff for producing $\theta^*(\gamma, A) \equiv \Theta^*(\gamma)/A$ is able to adjust in response to changes in A to leave $\Theta^*(\gamma)$ unaffected.

Under Assumption 1.*iv*, i.e., Pareto-distributed productivities, the share of firms undertaking at least level x_i of activity i, $\lambda_i(x_i, \beta, S)$, can be written as[22]

$$\begin{aligned}\lambda_i(x_i, \beta, S) &= \frac{\int_{\gamma\in\Gamma} [1 - F(\Theta_i(x_i, \gamma, \beta, S)/A)] \,\mathrm{d}G(\gamma)}{\int_{\gamma\in\Gamma} [1 - F(\Theta^*(\gamma)/A)] \,\mathrm{d}G(\gamma)} \\ &= \frac{\int_{\gamma\in\Gamma} [\Theta_i(x_i, \gamma, \beta, S)]^{-k} \,\mathrm{d}G(\gamma)}{\int_{\gamma\in\Gamma} [\Theta^*(\gamma)]^{-k} \,\mathrm{d}G(\gamma)}, \end{aligned} \tag{7}$$

where G is the c.d.f. of the distribution of firm characteristics. Now, it is obvious that under our assumptions, the effects on $\lambda_i(x_i, \beta, S)$ of β and S are determined by the effects on the numerator in (7).

Finally, we are ready to arrive at the propositions. Under Assumption 2 we not only obtain the cutoff sorting pattern described above, we also get that x^* is nondecreasing in S.[23] Thus, $\Theta_i(x_i, \gamma, \beta, S)$ is nonincreasing in S and in turn $\lambda_i(x_i, \beta, S)$ is nondecreasing in S. Hence Proposition 1. Under Assumption 3 we have that x^* is nondecreasing in β.[24] Analogously to above, this lead to $\lambda_i(x_i, \beta, S)$ being nondecreasing in β, and hence Proposition 2.

2.2 Nondecreasing or Strictly Increasing?

So far we have only given conditions under which $\lambda_i(x_i, \beta, S)$ is nondecreasing in S and β. It is however instructive to think about conditions under which $\lambda_i(x_i, \beta, S)$ is strictly increasing. To do so, our example of Bustos (2011) will prove helpful.

Example 1. In the Bustos (2011) model as described above, let us consider opening up the economies. That is, changing S from $S' = \{(0,0), (0,1)\}$ to $S'' = \{(0,0), (0,1), (1,0), (1,1)\}$. A sufficient condition for this to result

[22] Here we have implicitly assumed that γ is continuously distributed (in all its dimensions). It could very well be discretely distributed, or some elements of γ could be continuously distributed while others could be discretely distributed. However, this does not matter for the arguments below.

[23] See Theorem 5 of Milgrom and Shannon (1994).

[24] See Theorem 5 of Milgrom and Shannon (1994).

in a strict increase in the share of upgrading firms is that $\Theta_{\text{ex}}(1, \beta, S'') < \Theta_{\text{up}}(1, \beta, S'')$ has to hold. Consequently, the cutoff for exporting has to be less than or equal to the (demand-adjusted) cutoff for upgrading technology following change in S. Thus, if the marginal upgrading firm exports following the opening of the economies then the effect on the share of upgrading firm will be strong. In other words, if the marginal upgrading firm is directly affected by the introduction of exports, i.e., it begins to export, then, through the complementarity between the two activities, this introduction affects the industry structure in the upgrading dimension.

Consider now a trade liberalisation by decreasing τ from τ' to τ''. In this case, a sufficient condition for a strict increase in the share of upgrading firms is that $\Theta_{\text{ex}}(1, \beta', S) \leq \Theta_{\text{up}}(1, \beta', S)$, $\beta' = ((\tau')^{-1}, \mu)$. The intuition is analogous to before. If the marginal upgrading firm is exporting, then it will be directly affected by a trade liberalization. In this case, the complementarity between exports and technology upgrading implies that the trade liberalisation strictly increases the share of upgrading firms.

If however, the marginal upgrading firm is not affected directly through either a change in β or S, then the share of upgrading firms is unaffected. ■

The sufficient conditions given in the above example relies on some properties of the Bustos (2011) model which are not required in Assumption 1–3. The profit function (3) is not only supermodular in $x = (\mathbb{1}_{\text{ex}}, \mathbb{1}_{\text{up}})$, it is strictly supermodular.[25] This means that the complementarity between the two activities is never weak. Exporting allways strictly increases the gains from upgrading technology, and vice versa. Further, increasing τ allways strictly increases the gains from exporting. Therefore, if the marginal technology upgrader is affected by a change in exports (availability or attractiveness) the effect on the share of upgraders is strong. Note that the marginal upgrader depends on γ in the general case. For a strong effect on the share of upgrading firms, we need only observe a strong effect on the marginal upgrader for a nonzero measure of γ and not necessarily for all γ.

2.3 A Digression on the Role of γ

The reason for introducing the firm characteristics, γ, realised simultaneously with productivity upon entry is twofold. First, it goes to show that our

[25] That is, for any unordered pair $x, x' \in X$ we have $\pi(x, \Theta, \gamma, \beta) + \pi(y, \Theta, \gamma, \beta) < \pi(x \vee y, \Theta, \gamma, \beta) + \pi(x \wedge y, \Theta, \gamma, \beta)$.

results are robust to multiple firm characteristics with hardly imposing any conditions on e.g. the distribution of these characteristics.[26] Second, it allows for more observed organisational decisions in equilibrium. When the cutoff structure of firms sorting is based solely on productivity, then the number of different organisational decisions that can be observed is greatly reduced. However, allowing γ to affect this cutoff structure through the attractiveness of the activities can greatly expand the number of organisational decisions observed in the industry, i.e., across γ's. A return to our earlier example will make this point clear.

Example 1 (continued)**.** Productivity is the only source of firm heterogeneity in Bustos (2011) and therefore you can observe at most three different organisational decisions in equilibrium although four possibilities exist.[27] To see this, suppose, as Bustos (2011) does, that the cutoff for exporting is lower than the cutoff for technology upgrading. In this case, you would observe no firms upgrading technology and not exporting. If the cutoff for exporting was larger than the cutoff for technology upgrading, you would observe no firms exporting without upgrading technology.

Let us introduce the firm characteristic γ as the fixed cost of exporting, f_{ex}. Now, it is possible that for a low draw of f_{ex}, the cutoff for exporting would be lower than that for upgrading while the opposite is true for firms with a high draw of f_{ex}. This way it is possible to observe some firms exporting, some firms upgrading, some firms doing both, and some doing neither. Allowing for this additional source of firm heterogeneity means that we can observe all four possible combinations of exporting and upgrading, which is probably what one would observe in data, even though we have the cutoff structure described above. This way we avoid having to force the model to generate a rich sorting pattern based on productivity alone.[28]

[26] We do require that the law of large numbers apply at the industry level.

[27] We have two binary activities which result in four possible combinations.

[28] This example is analogous to Kasahara and Lapham (2006), who consider a Melitz (2003)-type of model where firms are allowed to increase the variety of inputs used in production through imports. In order to observe all four possible combinations of the two binary activities (exports and imports), Kasahara and Lapham (2006) assume that firms draw both a productivity level and a fixed cost of importing upon entry.

3 Applications

The present section shows how our results can be applied in two cases. The first case is based on Helpman et al. (2004) and allows firms to serve foreign markets through horisontal FDI as an alternative to exporting. The second case considers multi-product firms based on Bernard et al. (2011) and Arkolakis and Muendler (2011).

Other models that our results can be applied to include symmetric country versions of e.g. Kasahara and Lapham (2006) who consider export and import complementarities, Arkolakis (2010) who consider endogenous market penetration, and Caliendo and Rossi-Hansberg (2011) who consider exporting and the number of hierarchical management layers in the firm.[29]

3.1 Horisontal FDI

This application will show how our results can be applied in the context of horisontal FDI. Take the Bustos (2011) model as described above and include the possibility to serve the foreign market through horisontal FDI as in Helpman et al. (2004).[30] That is, firms can establish production of their variety in the foreign country and thereby serve the foreign market from this foreign plant without incurring the iceberg trade cost, τ. This requires paying a fixed cost $f_{\mathrm{I}} > f_{\mathrm{ex}}$. As before, the firm has to decide on two activities: technology upgrading and (foreign) market access. Technology upgrading is represented by the indicator $\mathbb{1}_{\mathrm{up}}$ as above. Market access, however, is no longer binary. Let $m = 0, 1, 2$ represent a firm's choice on market access with $m = 0$ being no foreign sales, $m = 1$ being exporting, and $m = 2$ being horisontal FDI. With symmetric countries the profit function of a firm can be written as[31]

$$\pi(x, \Theta, \beta) = (1 + \rho(m))\mu^{\mathbb{1}_{\mathrm{up}}}\Theta^{\sigma-1} - f - f_m - \mathbb{1}_{\mathrm{up}} f_{\mathrm{up}}, \tag{8}$$

where $\rho(m)$ represents the degree of market access and f_m denotes the associated fixed costs. Note that we have $(\rho(0), f_0) = (0, 0)$, $(\rho(1), f_1) = (\tau^{1-\sigma}, f_{\mathrm{ex}})$,

[29] Bache and Laugesen (2012) apply our results in an analysis of vertical integration and the relation between vertical mergers and acquisitions and trade liberalisation.

[30] We treat horisontal FDI in combination with technology upgrading in order to get a multidimensional organisational decision. Including technology upgrading, as modelled by Bustos (2011), leads to a very simple extension of the Helpman et al. (2004) model.

[31] We again disregard γ as productivity is the only source of firm heterogeneity.

and $(\rho(2), f_2) = (1, f_{\mathrm{I}})$. Firms thus trade better market access off with higher fixed costs. It is easy to verify that the profit function (8) is supermodular in $x = (m, \mathbb{1}_{\mathrm{up}})$ and that it has increasing differences in (x, Θ) and (x, β) with $\beta = \mu$. Importantly, now we cannot include τ^{-1} in β since the profit function no longer has increasing differences in τ^{-1}. The reason is that the difference between exporting ($m = 1$) and horizontal FDI ($m = 2$) is decreasing in τ^{-1}, as a lower τ decreases the variable-cost savings from horisontal FDI relative to exporting. Thus, we are not able to apply our earlier results to a trade liberalisation through decreases in the iceberg trade cost, τ.

However, our results can be applied in other respects. We can for instance analyse the effects of allowing firms to undertake horisontal FDI. By Proposition 1, expanding the set of admissible choices to include horisontal FDI will imply a nondecreasing share of firms upgrading technology. Analogously, introducing technology upgrading to firms imply that both the share of firms that undertake at least exporting[32] and the share of firms that undertake horisontal FDI are nondecreasing. Using Proposition 2 we can also analyse the effect of increasing the productivity gain from technology upgrading, i.e., increasing μ. This has qualitatively the same effect on foreign market access as introducing technology upgrading in the first place.

3.2 Multi-Product Firms

Bernard et al. (2011) and Arkolakis and Muendler (2011) both consider a heterogeneous-firm model where firms are allowed to sell multiple products on each of the markets they face. In both models, the activities that firms need to decide on are the number of products to sell on each market. The basic difference between the two studies is that in Bernard et al. (2011) the number is continuous whereas in Arkolakis and Muendler (2011) it is discrete. Both models conform to our setup when countries are assumed to be symmetric. For reasons of tractability, both Bernard et al. (2011) and Arkolakis and Muendler (2011) assume that the decision about the number of products to sell on a given market is completely independent of the decisions concerning other markets. Thus, you may argue that the profit function is only supermodular in the activities in the weakest possible sense as it is simply modular. Thus, allowing firms to export or a marginal trade liberalisatione

[32]That is, the share of firms undertaking exporting or horisontal FDI. Alternatively, this could be called the share of firms serving the foreign market.

will leave the share of firms selling a given number of products domestically completely unaffected. Why then is this an interesting application of our results? We believe for two reasons.

First, because it tells us that even though individual firms scale down the number of products sold domestically in the wake of a trade liberalisation, the share of (active) firms selling a given number of products is constant. Considering a given firm, Bernard et al. (2011) notes: *"[. . .] trade liberalization reduces the range of products supplied to the domestic market."* But this should be taken as the firm-level result that it is, since the distribution of active firms over the range of products supplied to the domestic market is unaffected.

Second, maybe the decisions ought not to be independent. Although it reduces tractability of the models, it seems more reasonable a priori that the decisions are complementary. If you are already selling a given product on one market, then it should be easier to sell it on another market as well since production of the product is already up and running. If, initially, you are not selling a given product on any market, then selling it on any given market should be more costly as production of that product needs to be established. In this case, our general results of the previous section tell us that trade liberalisation actually (weakly) increases the share of firms selling a given number of products domestically. Interestingly, this result may also arise even if you let the number of domestic and export products be directly independent. To see this, introduce technology upgrading as in Bustos (2011). Liberalising trade will increase the attractiveness of upgrading technology. However, a better technology will increase the payoff from selling more products at home. Thus, despite the number of domestic and export products being seemingly unrelated, they may be made complementary to some degree through a third activity.

4 Concluding Remarks

To come. . .

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