Trade, firm selection, and innovation: the competition channel*

Giammario Impullitti†and Omar Licandro‡

Preliminary and incomplete draft: September 2009

Abstract

The availability of rich firm-level data sets has recently led researchers to uncover an interesting set of empirical findings on the effects of trade liberalization. First, trade openness forces the least productive firms to exit the market. Secondly, it induces surviving firms to increase their innovation efforts. Thirdly, together with the selection and the innovation effect, trade liberalization seems to increase the degree of product market competition. This paper presents a theoretical model aimed at providing a coherent interpretation of these empirical findings. We introduce firm heterogeneity into an innovation-driven growth model. Incumbent firms operating in oligopolistic industries perform cost-reducing innovation in order to increase their future productivity. In equilibrium more productive firms show higher investment in innovation. The oligopolistic structure implies that markups are endogenously determined by the number of firms competing in same product line. Trade liberalization leads to an higher number of firms and lower markups in each industry. This pro-competitive effect of trade forces less efficient firms out of the market and reallocates resources towards more efficient, more innovative, firms, thereby raising aggregate innovation and productivity growth. This dynamic selection effect of trade is decreasing in the level of product market competition and, as a consequence, trade liberalization has negligible effects on innovation in highly competitive economies. In a version of the model calibrated to match US aggregate and firm-level statistics we find that a 10 percent reduction in variable trade costs reduces the markups by 0.5 percent, reduces firms surviving probability by 2.3 percent and increases growth by 33.8 percent; about 80 percent of the total effect on growth can be attributed to the reallocation of market shares toward more productive firms (selection effect).

JEL Classification:
Keywords:

*We thank Jonathan Eaton, Hugo Hopenhayn, Bojan Jovanovic, Tim Kehoe, Daniel Xu, Ariel Burnstein, and seminar participants at Minneapolis FED, EUI, NYU, Penn State, IMT Lucca, EEA Barcelona 2009. Licandro acknowledges the financial support of the Spanish Ministry of Education (SEJ2007-65552).

†*IMT Lucca Institute for Advanced Studies, P. San Ponziano 6, 55100 Lucca (Italy).* Email: g.impullitti@imtlucca.it.

‡*Instituto de Analisis Economico (IAE-CSIC), Campus UAB, 08193 Bellaterra- Barcelona (Spain).* Email: omar.licandro@iae.csic.es.

1 Introduction

An interesting set of empirical regularities has recently emerged from a large numbers of studies using firm-level data. First, empirical evidence has established that large and persistent productivity differences exist among firms within the same industry (e.g. Bartelsman and Doms, 2000). The availability of micro data has also allowed researchers to assess the importance of firm heterogeneity in understanding international trade and its effect on productivity. A number of papers have shown that trade liberalization induces the least productive firms to exit the market and the most productive non-exporter firms to become exporters; this *selection effect* increases the aggregate productivity level (see e.g. Pavcnik, 2002, Topalova, 2004, and Tybout, 2003 for a survey).

A second line of research has focused on the role of firm heterogeneity in shaping the effects of trade liberalization on *innovation* activities affecting the growth rate of productivity. Bustos (2008) shows that a regional trade agreement, MERCOSUR, has selected highly productive firms into exporting and affected positively a broad set of measures of innovation (computers and software, technology transfers, R&D, and patents).[1] Bloom, Draca, and Van Reenen (2009) study the effect of Chinese import penetration on innovation in European countries. They find evidence of both the selection and the innovation effect of trade: on the one hand Chinese competition decreases employment and chances of survival of firms, and this effect is stronger for low-tech than for high-tech firms. On the other hand, surviving establishments tend to innovate more (patenting and R&D) and upgrade their technology (IT intensity). Leeiva and Trefler (2008) find that tariff cuts mandated by the Canada-US Free Trade Agreement increase productivity heavily for lower productivity plants, while productivity gains for high productivity plants are negligible. They also show that plants experiencing higher productivity gains are those investing more strongly in innovation and technology upgrading.[2]

A third piece of evidence shows that trade liberalization has *pro-competitive* effects that can potentially lead to more selection and more innovation. Bugamelli, Fabiani, and Sette (2008) using Italian firm-level manufacturing data find that import competition from China has reduced prices and markups in the period 1990-2004. Griffith, Harrison, and Simpson (2008) have studied the effects of trade integration reforms carried out under the EU Single Market

[1] Focusing on innovation has the advantage of identifying one specific channel through which improvements in productivity take place. Other studies have instead estimated productivity as a residual in the production function, with the consequence that together with technological differences, residuals captures also other differences such as market power, factor market distortion, and change in the product mix. (see i.e. Foster, Haltowanger, and Syverson, 2008, Hsieh and Klenow, 2008, and Bernard, Redding, and Schott, 2008).

[2] Several papers have investigated the related but slighlty different question of whether the exporter status implies a higher investment in innovation or technology upgrading: this has been called the *learning by exporting* mechanism. The evidence is mixed: early papers, such as Clerides, Lach, and Tybout (1998) and Bernard and Jensen (1999) do not find any evidence in favor of this mechanism. Recent studies have instead found evidence that firms improve their productitivy subsequent to entry (e.g. Delgado, Farinas, and Ruano, 2002, De Loecker, 2006, Van Biesebroek, 2005, see Lopez, 2005, for a survey). The basic difference between these studies and those discussed in the main text is that the former focus on productivity and the latter on innovation. Once exception is Criuscolo, Haskel, and Slaugther (2008) which finds that expoters and multinational firms have higher productivity because they both innovate more and learn from foreign technologies. The other difference is that Bustos (2008), Bloom et al (2008), and Llleiva and Trefler (2008) focus on trade liberalization and not on export status.

Programme (SMP) and found that these reforms have increased product market competition (measured as average markups) and stimulated innovation (R&D expenditures). Chen, Imbs and Scott (2008) using micro data on EU manufacturing for the period 1989-99, estimate the Ottaviano and Melitz (2008) model and show that trade openness reduces average prices and markups, while raising productivity through firm selection.

This paper presents a theoretical model aimed at providing a coherent interpretation of these empirical findings. More precisely, we set up a model in which trade liberalization has pro-competitive effects (reduces markups) producing firm selection and stimulating innovation. We introduce a dynamic industry model with heterogeneous firms into a model of growth with innovation by incumbents. There are two goods in the economy, an homogeneous good produced under constant returns, and a continuum of differentiated goods produced with a variable and a fixed quantity of the homogeneous good. Each variety of the differentiated good is produced by a given number of firms with the same technology, while productivity differ across varieties. Thus, as in Hopenhayn (1992) and Melitz (2003) firms are heterogeneous in their productivity. In addition to this now standard environment the model features a dynamic innovation activity performed in-house by firms and aimed at increasing productivity. The market structure for differentiated goods is oligopolistic, thus both the optimal quantity produced and the level of innovation result from the strategic interaction among firms. The oligopolistic market structure and the innovation by incumbents feature are borrowed from static trade models with endogenous market structure (e.g. Neary, 2003 and 2009) and from multi-country growth models with representative firms such as Peretto (2003) and Licandro and Navas (2008).

The open economy features two symmetric countries engaging in costly trade (iceberg type). In order to simplify the analysis, we assume that there are no entry costs into the export market, implying that all operating firms export. The fixed production costs and the heterogeneous firms structure determine the cutoff productivity level below which firms cannot profitably produce. Trade liberalization (reduction in trade costs) leads to an increase in product market competition because the number of firms producing each variety doubles. This yields a reduction in the markup and a decrease in the inefficiency of oligopolistic markets, ultimately leading to an expansion of the quantity produced by each firm. Moreover, a decline in the markup raises the productivity cutoff and forces the less productive firms out of the market. This selection effect reallocate resources from exiting firms to higher productive surviving firms, which are also those innovating at a higher pace. Thus, trade-induced firm selection increases not only the 'level' of aggregate productivity (as in Melitz, 2003) but also aggregate innovation, thus affecting the 'growth rate' of productivity as well. We call this new mechanism the *dynamic selection effect*. Secondly, the pro-competitive effect of trade have also a *direct effect* on innovation related to the increase in the quantity produced by more intense product market competition: since innovation is cost reducing, the marginal benefit from a reduction in costs is increasing with the quantity produced. Both effects are decreasing in the level of product market competition of the economy before trade liberalization: in highly competitive economies (large number of firms) trade liberalization has only negligible effects on innovation, and there exist a threshold level of competition above which trade has no effects on innovation.

All these results are obtained assuming that the number of firms in each industry is exogenous. It is plausible that trade can produce a selection effect within narrowly defined oligopolistic industries as well. We extend the baseline model to endogenize the number of firm per sector and that not only our basic results are confirmed but the effect of trade on innovation is even stronger when selection takes place both between and within industries.

Finally, we provide an evaluation of the quantitative relevance of our results by calibrating the baseline model to match salient firm-level and aggregate statistics of the US economy. The model show a sufficiently good fit of the data, and a reduction in trade costs has effects on both the first and the second moments of the productivity distribution that are quantitatively relevant. A 10 percent reduction in trade costs increases the aggregate growth rate by about 27 percent through the selecton effect and by an additional 6 percent through the direct effect. Moreover, trade liberalization increases the volatility of the firm-level growth rate and of firm sales by 3.5 and 3.2 percent respectively.

This paper is related to the emerging literature studying the joint selection and innovation effect of trade openness. A first line of research introduces a one-step technological upgrading choice into an heterogeneous firm framework. Examples are Yeaple (2005), Costantini and Melitz (2007), Bustos (2007), Navas and Sala (2007), Vannoorenberge (2008). In all these papers, with exception of Costantini and Melitz, the model economy is static, and the dynamic effects of trade on innovation cannot be analyzed. Our paper is more closely related to a second stream of research that introduces innovation as a continuous process in dynamic models of trade and productivity growth. Baldwin and Robert-Nicoud (2008) and Gustaffson and Segerstrom (2008) explore the effects of trade liberalization on innovation and growth in models of expanding variety (Romer, 1990) with heterogeneous firms. They show that the effect of trade-induced firm selection on innovation and growth depends on the form of (international) knowledge spillovers characterizing the innovation technology. Atkeson and Burnstein (2007) set up a model of process and product innovation with firm heterogeneity and show that trade has positive effects on process innovation that can be offset by negative effects on product innovation.[3]

Although differing on the type of innovation or on the specific form of innovation technology they analyze, all these papers adopt a monopolistically competitive market structure.[4] The key distinguishing feature of our model is that we study the interactions between trade, firm heterogeneity and innovation in an dynamic oligopolistic environment. In this framework the market structure is endogenous and responds to changes in trade costs, thereby representing the ideal environment to analyze the effects of trade on product market competition (the third

[3]Benedetti Fasil (2009) sets up a model featuring both product and process innovation and finds positive effects of trade liberalization on both types of innovation. Klette and Kortum (2004) and Mortensen and Lentz (2008) introduce a dynamic industry model with heterogeneous firms into a quality ladder growth model (Grossman and Helpman, 1991). They limit the analysis to the interaction between firm heterogeneity and creative destruction in closed economy, without exploring the effects of trade. Haruyama and Zhao (2008) explore the interaction between trade liberalization, selection and creative destruction in a quality ladder model of growth.

[4]One exeption is Van Long, Raff, and Stahler (2008) that features an oligopolistic market structure, but innovation is not a continuos process and the model is static.

stylized fact discussed above). Melitz and Ottaviano (2008) show that using special preferences it is possible to obtain endogenous markups in the monopolistic competitive framework. In line with our result, they find that trade liberalization produces a pro-competitive effect (lower markups). It is worth noticing that the presence of endogenous markups allows the selection effect to work through a channel different from that highlighted in Melitz (2003). In that paper, trade liberalization produces an increase in labor demand that bids up wages and forces low productivity firms to exit. In our paper, as in Melitz and Ottaviano (2008), the selection effect is produced by the reduction in markups brought about by trade liberalization. While there is evidence, as discussed above, that trade liberalization has increased product market competition, the trade-induced increase in average wages triggering firm selection in Melitz (2003) seems to be counterfactual.[5] Our model differs from that of Melitz and Ottaviano not only for the different source of endogenous markups but also because in their model there is no innovation activity aimed at improving productivity, therefore they cannot study the implications of firm heterogeneity and endogenous markups for innovation. Bernard, Jensen, Eaton, and Kortum (2003), set up a Ricardian model with Bertrand competition among firms and obtain markups responding endogenously to trade liberalization. We complement their analysis by introducing innovation and deriving endogenous markups from Cournot competition.

Summing up, to our knowledge the present paper is the first to provide a framework to interpret jointly the three stylized facts discussed above. The basic structure of the model is such that trade affects both firm selection and innovation through the *competition channel*, that is through its effect on the markup. The selection effect of trade operating through endogenous markups resulting from oligopolistic competition among firms is a novel contribution. Secondly, while the direct competition effect of trade on innovation is not new in the literature (see Peretto, 2003, and Licandro and Navas, 2007), the interaction between firm selection and innovation represents an original contribution of this paper. Finally, although our stylized economy features a rich and complex structure (oligopoly, firm heterogeneity, growth), the baseline version of the model is highly tractable and all results are obtained analytically, with no special assumption on either preferences or firms productivity distribution.

2 The model

2.1 Economic environment

The economy is populated by a continuum of identical consumers of measure L. Time is continuous and denoted by t, with initial time $t = 0$. Initial conditions are such that the economy is at a stationary equilibrium, if it exists, from the initial time.

Preferences of the representative consumer are

$$\int_0^\infty (\ln X_t + \beta \ln Y_t)\ \mathrm{e}^{-\rho t}\ \mathrm{d}t,$$

[5] For instance March CPS data show that both median and average US wages have stagnated in the last three decades, a period of progressive trade liberalization (see Acemoglu, 2002)

with discount factor $\rho > 0$. There are two types of goods, an homogeneous good, taken as the numeraire, and a differentiated good. Consumers are endowed with a unit flow of the homogeneous good. A fraction Y of it is consumed, entering utility with weight $\beta > 0$.

The differentiated good X is produced with a continuum of varieties of mass $M \in [0, 1]$ according to

$$X_t = \left(\int_0^{M_t} x_{jt}^{\alpha} \, \mathrm{d}j \right)^{\frac{1}{\alpha}}, \tag{1}$$

where x_{jt} represents variety j, and $\sigma = \frac{1}{1-\alpha}$ is the elasticity of substitution across varieties with $\alpha \in (0, 1)$. Each variety in X is produced by n identical firms by transforming the homogeneous good into this particular variety.[6] Firms face the same fixed production cost $\lambda > 0$ but may have different productivities z. A firm with productivity z has the following production technology (we omit index j)

$$c(z_t)q_t + \lambda = y_t, \tag{2}$$

where y represent inputs and q production. The labor requirement has the following form: $c(z_t) = z_t^{-\eta}$, $\eta > 0$.

Innovation activities are undertaken by incumbents according to the following technology

$$\dot{z}_t = A \; \hat{z}_t h_t, \tag{3}$$

where h represents units of the homogeneous good allocated to innovation and innovation efficiency is denoted by $A > 0$. R&D activities are also assumed to benefit from spillovers coming from direct competitors $\hat{z}_t$. Let assume, for simplicity, that all firms producing the same good have the same initial productivity $z_0 > 0$.

Irrespective of their productivity z, varieties exit the market at rate $\delta > 0$. On top of that, exit may result from productivity z being too low. This is the selection mechanism described in section 2.4 below. Exiting varieties are replaced by new varieties in order for the mass of varieties to remain constant. The productivity distribution of entrants is described in section 2.5.

2.2 Households

The representative household maximizes utility subject to its instantaneous budget constraint. The solution to this problem is straightforward and leads to the following first order conditions

[6] We also implicitly assume that preferences are such that the n identical goods enter utility symmetrically, that is, they are perfect substitutes. This is easy generalized introducing love-for-variety preferences for goods produced by the same industry. For instance, one could assume

$$x_{jt} = \left(\int_0^{n} c_{jt}^{\beta} \, \mathrm{d}j \right)^{\frac{1}{\beta}},$$

where c_{jt}^{β} is the quantity of each good within the industy j, and $1/(1-\beta)$ is the elasticity of substitution between these goods.

$$
\begin{aligned}
Y_t &= \beta E_t, &(4)\\
\frac{\dot{E}_t}{E_t} &= r_t - \rho, &(5)\\
p_{jt} &= \frac{EL}{X_t^{\alpha}} \, x_{jt}^{\alpha-1}, &(6)
\end{aligned}
$$

where r is the interest rate and p_{jt} is the price of good j. Total expenditure on the composite good X is

$$E_t = \int_0^{M_t} p_{jt} x_{jt} \, \mathrm{d}j.$$

Because of log preferences, total spending in the homogeneous good is β times total spending in the differentiated good. Equation (5) is the standard Euler equation, and (6) is the inverse demand function for variety j, $j \in [0, 1]$.

2.3 Production and Innovation

Firms producing the same good behave non-cooperatively and maximize the present value of their net cash flow.

$$V_{is} = \int_s^{\infty} \pi_{it} R_t dt,$$

where R_t is the discount factor and $\pi_{it} = (p_{it} - c(z_{it}))q_{it} - h_{it} - \lambda$ is the profit. We solve this differential game focusing on Nash Equilibrium in open loop strategies. Let $a_i = (q_{it}, h_{it})$ for $t \geq s$ be a strategy for firm i. These strategies are time paths for quantity and R&D. In the open loop equilibrium we construct firms commit to time paths strategies for quantities and R&D, which induce time paths for productivity. At time s a vector of strategies $(a_1, ..., a_i, ..., a_n)$ is an equilibrium if

$$V_{is}(a_1, ..., a_i, ..., a_n) \geq V_i(a_1, ..., a_i', ..., a_n) \geq 0$$

where $(a_1, ..., a_i', ..., a_n)$ is the vector in which only firm i deviates from the equilibrium path of quantity and R&D. The first inequality states that firm i maximize its present value of its net cash flow, and the second condition requires this to be positive.[7]

[7] We choose the open loop equilibrium because it is easier to derive in closed form solution. The drawback of focusing on the open loop equilibriun is that it does not generally have the property of subgame perfection, as firms choose their optimal time-paths strategies at the initial time and stick to them forever. In closed loop and in feedback strategies, instead, firms do not pre-commit to any path and their strategies at any time depend on the whole past history. The Nash equilibrium in this case is strongly time-consistent and therefore sub-game perfect. Unfortunately, closed loop or feedback equilibria generally do not allow a closed form solution and often they do not allow a solution at all. The literature on differential games has uncovered classes of games in which the open loop equilibrium degenerates into a closed loop and therefore is subgame perfect (e.g. Reingaum, 1982 and Fershtman,1987). A sufficient condition for the open loop Nash equilibrium to be subgame perfect is that in the first order conditions for a firm the state variable of other firms do not appear. In our model this condition is violated because of the externality in the R&D technology leading to the FOC (9) below. Although, none of the basic results of this paper depend of this externality, removing it complicates the solution of the model substantially.

The characterization of the open loop Nash equilibrium proceeds as follows: a firm producing a particular good solves at any time s the problem

$$V_s = \max_{(q_t,h_t)_{t=s}^{\infty}} \int_s^{\infty} \left[(p_t - c(z_t))q_t - h_t - \lambda\right] e^{-(\rho+\delta)(t-s)}\, dt, \qquad \text{st.} \tag{7}$$

$$\begin{aligned} p_t &= \frac{E_t L}{X_t^{\alpha}}\, x_t^{\alpha-1} \\ x_t &= \hat{x}_t + q_t \\ \dot{z}_t &= A\, \hat{z}_t\, h_t \\ z_s &> 0, \end{aligned}$$

where $\delta > 0$ is the exogenous exit rate.

In a Cournot game a firm takes as given the path of its competitors' production $\hat{x}_t$, the path of its competitors' average productivity $\hat{z}_t$, as well as the path of the aggregates E_t and X_t. The first order conditions for the problem above are

$$c(z_t) = \theta \underbrace{\frac{EL}{X_t^{\alpha}}\, x_t^{\alpha-1}}_{p_t}, \tag{8}$$

$$1 = v_t A \hat{z}_t, \tag{9}$$

$$\frac{-c'(z_t)}{v_t} q_t = \frac{-\dot{v}_t}{v_t} + \rho + \delta, \tag{10}$$

where v_t is the costate variable, and $\theta \equiv (n - 1 + \alpha)/n$ is the inverse of the markup rate. It is easy to see from (8) that firms charge a constant markup $1/\theta$ over marginal costs $c(z_t)$. As it is well known, in a Cournot-type equilibrium the markup depends not only on demand elasticity but on the number of competitors.

Note that firms producing the same good operate the same technology and face the same initial conditions, implying that at equilibrium $\hat{z}_t = z_t$ and $x_t = nq_t$. After some simple algebra, it can be shown that the symmetric equilibrium growth rate of productivity follows

$$\frac{\dot{z}_t}{z_t} = \eta A c(z_t) q_t - \rho - \delta, \tag{11}$$

$$c(z_t) q_t = \theta e_t \frac{L}{n} \left(\frac{z_t}{\bar{z}_t}\right)^{\eta \frac{\alpha}{1-\alpha}}, \tag{12}$$

where $e_t = E_t/M_t$ is expenditure per firm and average productivity is defined as

$$\bar{z}_t^{\eta \frac{\alpha}{1-\alpha}} = \frac{1}{M_t} \int_0^{M_t} z_{jt}^{\eta \frac{\alpha}{1-\alpha}}\, \mathrm{d}j.$$

To obtain the growth rate of productivity in (11), differentiate (9) and substitute the resulting $\frac{\dot{v}}{v}$ in (10), then substitute $v_t z_t$ from (9). Equation (12) derives from (8) and (1) after substituting $x_t = nq_t$.

When innovation is cost reducing and undertaken by incumbents, profitability of R&D depends on production, as shown by the right hand side in (10). The benefit of reducing production costs is larger, the larger is production. Since more efficient firms produce more, they also have more incentives to do R&D, meaning that firm's R&D activity depends positively on firm's state of technology. This is reflected in equation (11), where the growth rate of productivity depends positively on output, which depends on the normalized productivity level.

The growth rate of the average productivity $\overline{z}_t$, which we denote by g, is

$$g_t = \frac{\eta A \theta e_t L}{n} - \rho - \delta. \tag{13}$$

The term $\eta A\theta e_t L/n$ is the marginal return to R&D investment for firms with productivity $z_t = \overline{z}_t$. As time passes, firms with productivity initially smaller than the mean will grow at a smaller and smaller rate, but firms with productivity initially larger than the mean will keep growing at a growing rate.

2.4 Exit

Using results from the firm optimization problem derived above, it can be easily shown that firm z's cash flow is

$$\pi(z_t) = \left(\frac{1}{\theta} - 1\right) c(z_t)q_t - \underbrace{\left(\eta c(z_t)q_t - \frac{\rho+\delta}{A}\right)}_{h_t} - \lambda. \tag{14}$$

From (12), quantities q depend on the distance from average productivity $\tilde{z}$, defined as $\tilde{z}_t = \left(\frac{z_t}{\overline{z}_t}\right)^{\eta\frac{\alpha}{1-\alpha}}$. Consequently, we can denote $\pi = \pi\left(\tilde{z}\right)$. Let rewrite the cash flow in (14) as

$$\pi(\tilde{z}) = \left(\frac{1}{\theta} - 1 - \eta\right)\theta(L/n)e_t\tilde{z}_t + \frac{\rho+\delta}{A} - \lambda. \tag{15}$$

Since from (12) we know that $c(z_t)q_t = \theta(L/n)e_t\tilde{z}_t$, for firm profit to be increasing in productivity it suffices to assume $\eta < (1/\theta) - 1$. Let us call $\tilde{z}_t^*$ to the cutoff relative productivity below which firms exit the market. The productivity at time t of a marginal firm, i.e. a firm with relative productivity $\tilde{z}_t = \tilde{z}_t^*$, grows at a lower rate than the mean, implying that it will exit the market at $t+$ dt. For this reason, profits of the marginal firm have to be zero, that is

$$\left[1 - (1-\eta)\theta\right]\frac{L}{n}e_t\tilde{z}_t^* + \frac{\rho+\delta}{A} - \lambda = 0. \tag{16}$$

The marginal firm is making zero profits the time just before exiting.[8]

[8] In fact, firms with initial productivity smaller than the mean face an endogenous finite life, which is reached when productivity becomes equal to the cutoff level. It corresponds to an optimal control problem with horizontal terminal line. The associated terminal condition requires profits been zero at the terminal time. See section 7.4 in Chiang (1992).

2.5 Stationary Equilibrium

In order to characterize the stationary equilibrium we need to introduce the entry process of new varieties and market clearing conditions. We assume there is a mass of unit measure of potential sectors, of which $M \in [0,1]$ are operative. New varieties can enter the market at zero cost and draw a productivity from the initial productivity distribution $F(\tilde{z})$. Since firms exit at the rate δ, stationarity requires

$$(1-M)(1-F(\tilde{z}^*)) = \delta M. \tag{17}$$

This condition says that the exit flow δM equals the entry flow defined by the number of entrants $1-M$ times the probability of surviving $1-F(\tilde{z}^*)$. Consequently, the mass of operative firms M is a decreasing function of the cutoff level $\tilde{z}^*$,

$$M(\tilde{z}^*) = \frac{1-F(\tilde{z}^*)}{1+\delta-F(\tilde{z}^*)}. \tag{18}$$

The market clearing condition for the homogeneous good can be written as

$$n\int_0^M (y_j + h_j)\ \mathrm{d}j + Y = n\int_0^M (c(z_j)q_j + h_j + \lambda)\ \mathrm{d}j + \beta E = L,$$

In steady state Y and E are constant. The (measure one) total endowment of the homogeneous good is allocated to composite good production and innovation, as well as to homogeneous good consumption. The first equality comes after substitution of y from (2), and Y from (4).

Let $\mu(\tilde{z})$ be the stationary density distribution of firms defined in the $\tilde{z}$ domain.[9] The endogenous exit process related to the cutoff point $\tilde{z}^*$ implies $\mu(\tilde{z}) = 0$ for all $\tilde{z} < \tilde{z}^*$. Rewriting the labor market clearing condition after changing the integration domain from sectors $j \in [0,1]$ to productivity $\tilde{z} \in [\tilde{z}^*, \infty]$, and substituting R&D employment from (3) and (11), and $c(z)q$ from (12) we obtain

$$n\int_{\tilde{z}^*}^{\infty} \left[(1+\eta)\ \theta e\tilde{z}\frac{L}{n} + \left(\lambda - \frac{\delta+\rho}{A}\right)\right]\ \mu(\tilde{z})\ d\tilde{z} + \beta eL = \frac{L}{M}.$$

Since $\int_{\tilde{z}^*}^{\infty} \mu(\tilde{z})\, d\tilde{z} = \int_{\tilde{z}^*}^{\infty} \tilde{z}\mu(\tilde{z})\, d\tilde{z} = 1$, after integrating over all sectors we obtain

$$e = \frac{\frac{1}{M(\tilde{z}^*)} - \left(\lambda - \frac{\delta+\rho}{A}\right)\frac{n}{L}}{\beta + (1+\eta)\theta}. \tag{MC}$$

The other equilibrium condition is determined by the exit condition (16)

$$e\tilde{z}^* = \frac{n}{L}\left[\frac{\lambda - \frac{\rho+\delta}{A}}{1-(1+\eta)\theta}\right] \tag{EC}$$

Since M is decreasing in $\tilde{z}^*$, (MC) and (EC) are respectively increasing and decreasing in $(e, \tilde{z}^*)$. The following proposition establishes parameter conditions under which the solution for e and $\tilde{z}^*$ is interior.

[9] Proposition 5 in the appendix provides necessary and sufficient conditions for a stationary distribution to exists.

Assumption 1 *The following parameter restrictions hold*

$$0 < \eta < \frac{1}{\theta} - 1 \tag{a}$$

$$\frac{L}{nM} + \frac{\delta + \rho}{A} > \lambda > \frac{\rho + \delta}{A} \tag{b}$$

$$\beta < \frac{\frac{L}{nM(1)}\left[1 - (1 - \eta)\,\theta\right] - \left(\lambda - \frac{\rho+\delta}{A}\right)}{\left(\lambda - \frac{\rho+\delta}{A}\right)} \tag{c}$$

Assumption (a) makes the profit function (15) increasing in $\tilde{z}$. This assumption, together with (b), makes strictly positive the right-hand-side of both (MC) and (EC). As shown below, assumption (c) bounds the equilibrium cutoff $\tilde{z}^*$ below 1, coherently with the definition of $\tilde{z}$.

Proposition 1 *Under Assumption 1, there exists a unique interior solution* $(e, \tilde{z}^*)$ *of (MC)-(EC)*

Proof. From (MC), e is an increasing function of $\tilde{z}^*$ and from (EC), e is an decreasing function of $\tilde{z}^*$. In (EC), e goes to infinity when $\tilde{z}^*$ goes to zero, and under Assumption 1 (c), at $\tilde{z}^* = 1$ e is larger in (MC) than in (EC). Consequently, the locus $(e, \tilde{z}^*)$ in (MC) and (EC) cross once and only once for $\tilde{z}^* \in (0, 1)$. ■

Figure 1 below illustrates the proof of Proposition 1

[FIGURE 1 ABOUT HERE]

(CC) is the exit condition (EC) and (MC) is the market clearing condition (MC). Assumption (c) is sufficient for $CC(\tilde{z}^* = 1) < MC(\tilde{z}^* = 1)$, thereby (CC) always crosses (MC) for $\tilde{z}^* < 1$.

Proposition 2 *An increase in* θ *raises the productivity cutoff* $(d\tilde{z}^*/d\theta > 0)$*, reduces the number of operative varieties* $(dM(\tilde{z}^*)/d\theta < 0)$*, has an ambiguous effect on the labor resources allocated to the homogeneous sector* $(d(\beta e)/d\theta < 0)$ *and increases the growth rate* $(dg/d\theta > 0)$

Figure 2 shows the effect of an increase in the degree of competition (reduction in the markup $1/\theta$) on the equilibrium values of $\tilde{z}^*$ and e. An increase in θ shifts both the (CC) and the (MC) curves to the right, thereby increasing the equilibrium productivity cutoff $\tilde{z}^*$. Depending on the relative strengths of the shift of the two curves e can increase or decrease, but the average growth rate g always increases. Intuitively, from (13) we know that the effect of a change in θ on g is determined by its effect on θe. Multiplying the market clearing condition (MC) by θ we can obtain θe as a function of θ and $M(\tilde{z}^*)$, and since in equilibrium $M(\tilde{z}^*)$ is decreasing in θ, we can conclude that θe is increasing in θ.

[FIGURE 2 ABOUT HERE]

Two mechanisms contributes to increasing growth, a direct competition effect and a selection effect. Let describe first the *direct competition effect.* In a Cournot equilibrium, an increase in competition reduces markups and allows for an increase in produced quantities. The increase in quantities is feasible since the homogeneous good becomes relatively more expensive (i.e. the relative efficiency of the differentiate sector increases), consumers' demand moves away from it towards the composite good and resources are reallocated from the homogeneous to the composite sector. Since the payoff of cost-reducing innovation is increasing in the quantity produced, the higher static efficiency associated to lower markups brought about by competition affects positively innovation and growth. This mechanism does not depend on firm heterogeneity: it is easy to check that assuming away the dependence of M on $\tilde{z}^*$ by setting $M = 1$, the equilibrium growth rate derived from (MC) and (EC) becomes independent on the cutoff $\tilde{z}^*$, but still increasing in θ. This direct effect of competition on growth can in fact be found in representative firm models of growth with endogenous market structure (see e.g. Peretto, 2003, and Licandro and Navas, 2007).

The *selection effect* is instead specifically related to the heterogeneous firm structure of the model. The trade-induced reduction in the markup raises the productivity threshold above which firms can profitably produce, the cutoff $\tilde{z}^*$, thus forcing the least productive firms to exit the market. Resources are reallocated from exiting firms to the higher productivity surviving firms which, as shown in (11), innovate at a higher pace. Therefore this selection effect leads to higher innovation and growth. Notice that in this model the direct competition effect of trade liberalization on innovation does not hold if we eliminate the homogeneous good, because no reallocation of market shares would be possible. While the selection effect produced by the presence of firm heterogeneity would still hold because reallocation takes place within varieties of the differentiated product.

3 Open economy

Consider a world economy populated by two symmetric countries with the same technologies, preferences, and endowments as described in the previous section. We assume that trade costs are of the iceberg type: $\tau > 1$ units of goods must be shipped abroad for each unit finally consumed. Costs τ can represent transportation costs or trade barriers created by policy. For simplicity we do not assume entry costs in the export market, thus all surviving firms sell both to the domestic and foreign markets.[10]

3.1 Equilibrium characterization

Since the two countries are perfectly symmetric, we can focus on one of them. Let q_t and $\check{q}_t$ be the quantities produced for the domestic and the foreign markets, respectively. The firm solves

[10] Our main goal is to explain the interaction between trade, selection and innovation, and for this purpose having firms partitioned by their export status is not necessary.

a problem similar to that in closed economy (see appendix). The first order conditions are:

$$\begin{aligned}
c(z_t) &= \left((\alpha-1)\frac{q_t}{x_t}+1\right)p_t \\
\tau c(z_t) &= \left((\alpha-1)\frac{\check{q}_t}{x_t}+1\right)p_t \\
1 &= v_t A z_t, \\
\frac{-c'(z_t)}{v_t}\left(q_t+\tau\check{q}_t\right) &= \frac{-\dot{v}_t}{v_t}+\rho+\delta.
\end{aligned}$$

Firms face different marginal costs and set different markups for the domestic and foreign markets. In the appendix we show that the first two of the conditions above yield the following equilibrium quantities for each firm

$$c(z_t)\left(q_t+\tau\check{q}_t\right)=\theta^T e_t\left(\frac{L}{n}\right)\tilde{z}_t \tag{19}$$

where

$$\theta^T=\frac{2n-1+\alpha}{n\left(1+\tau\right)^2\left(1-\alpha\right)}\left[\tau^2\left(1-n-\alpha\right)+n\left(2\tau-1\right)+\left(1-\alpha\right)\right] \tag{20}$$

is the inverse of the average markup in the open economy. Notice that θ^T is a decreasing function of the variable trade cost parameter τ, with θ^T reaching its maximum value $\theta^T_{\max}\equiv\left(2n-1+\alpha\right)/2n$ when $\tau=1$, the polar case of no iceberg trade costs; and the autarky value $\theta=\left(n-1+\alpha\right)/n$ when $\tau=n/\left(n+\alpha-1\right)$, the alternative polar case where trade costs are prohibitive and economies do not have incentives to trade.

Using the last two first order conditions above and proceeding as in the closed economy, we find that the growth rate of productivity

$$\frac{\dot{z}_t}{z_t}=\eta A\theta^T e_t\left(\frac{L}{n}\right)\tilde{z}_t-\rho-\delta \tag{21}$$

takes the same functional form as in the closed economy. Consequently, opening to trade only affects equilibrium growth rates through changes in the markup.

As in the closed economy case, we focus on the characterization of the steady-state equilibrium. The productivity cutoff is determined solving the following equation

$$\pi(\tilde{z}^*)=\left(p-c(\tilde{z}^*)\right)q+\left(p-\tau c(\tilde{z}^*)\right)\check{q}-h-\lambda=0$$

which, as shown in the appendix, yields

$$e^T\tilde{z}^{*T}=\frac{n}{L}\left[\frac{\lambda-\frac{\rho+\delta}{A}}{1-\left(1+\eta\right)\theta^T}\right]. \tag{EC$^{\mathrm{T}}$}$$

Since firms compensate their losses in local market shares by their new shares in the foreign market, profits are only affected by the change in the markup. Consequently, the exit condition has the same functional form as in (EC) except for the θ^T.

The market clearing condition becomes

$$n\int_0^M\left(q_jc(z_j)+\tau\check{q}_jc(z_j)+\left(\left(q_j+\tau\check{q}_j\right)-\frac{\rho+\delta}{A}\right)+\lambda\right)\,\mathrm{d}j+\beta E=1,$$

which, proceeding as in the closed economy case yields

$$e^T = \frac{\frac{1}{M(\tilde{z}^{*T})} - \left(\lambda - \frac{\delta+\rho}{A}\right)\frac{n}{L}}{\beta + (1+\eta)\,\theta^T}. \tag{MC$^{\mathrm{T}}$}$$

which is equal in all aspects to (MC) except for the markup, with θ^T instead of θ. Equations (EC$^{\mathrm{T}}$) and (MC$^{\mathrm{T}}$) yield the equilibrium $(e^T, \tilde{z}^{*T})$ in the open economy. The equilibrium growth is defined by (13) with θ^T and e^T instead of θ and e.

Proposition 3 *Under Assumption 1 and for $\tau < \bar{\tau} = \frac{n}{n+\alpha-1}$ there exists a unique interior solution $\left(e^T, \tilde{z}^{*T}\right)$ of (MC^T) and (EC^T).*

Proof. At $\bar{\tau} = n/(n+\alpha-1)$ the markups under trade and autarky are equal, $\theta^T = \theta$, and the prohibitive level of trade costs is reached. Thus, for $\tau \geq \bar{\tau}$ firms do not have incentives to export, and trade does not take place. For $\tau < \bar{\tau}$ the proof of the existence is similar to that in the closed economy, and we omit it for brevity. ■

3.2 Trade liberalization

Since (MC$^{\mathrm{T}}$) and (EC$^{\mathrm{T}}$) are formally equivalent to (MC) and (EC) apart from θ, we can apply Proposition 2 to study the effects of trade liberalization. Trade openness does not affect market shares because the increase in the number of firms in the domestic market is offset by the access to the export market. The economy with costly trade is characterized by a level of product market competition higher than in autarky, $\theta^T > \theta$. A larger number of firms in the domestic market, raises product market competition, thus lowering the markup rate. From the definition of θ and the equilibrium value of θ^T we obtain

$$\theta^T - \theta = \frac{\tau\,(1-\alpha)^2 - n\,(\tau-1)^2\,(n+\alpha-1)}{n\,(1+\tau)^2\,(1-\alpha)}.$$

For $\tau < \bar{\tau}$ the markup under trade is lower, that is $\theta^T - \theta > 0$, and by differentiating the expression above it is easy to see that the distance between θ^T and θ is decreasing in τ. Hence, trade liberalization increases product market competition. When trade is completely free, $\tau = 1$, product market competition reaches its maximum level, $\theta^T_{\max} \equiv (2n-1+\alpha)/2n$. Notice that $\theta^T_{\max}$ has the same functional form as the inverse of the markup in autarky but with the number of firms doubled.

Once established that trade reduces markups, from (EC$^{\mathrm{T}}$) trade liberalization increases the productivity threshold for firms to be able to stay in the market $\tilde{z}^*$, thus leading some firms out of the market. This *selection effect* triggers a reallocation of resources from the exiting firm to the more productive firms, which are also those innovating at a faster pace. Thus, the selection effect produced by trade liberalization not only raises the level of productivity as in Melitz (2003) but also its growth rate.

As stated before, there is a another more standard channel through which trade-induced increases in competition affects growth. Trade reduces the level of oligopolistic inefficiency in the differentiated goods sector, thus raising the quantity produced of each variety. Since innovation is cost reducing, the marginal benefit from a reduction in costs is increasing with the quantity produced, therefore lower markups trigger higher investment in innovation; this is the *direct competition effect*. This channel does not rely on the presence of heterogeneous firms and, as shown by Licandro and Navas (2007), it operates also in a model with a representative firm. The selection effect instead can be obtained only in an heterogeneous firms framework.

Notice that trade liberalization has an anti-variety effect, it reduces the number of produced and consumed varieties M. This is a consequence of the assumption that there is a perfect overlap between the varieties produced by the two economies. The standard pro-variety effect of trade (e.g. Krugman 1980) could be generated by introducing asymmetry in the set of goods produced by the two countries. However, a model with asymmetric countries would complicate the algebra substantially, without adding much to the main mechanism we want to highlight (the effect of trade-induced selection on innovation and growth).

Proposition 4 *The effect of trade liberalization on selection and growth is decreasing in the number of firms*

Proof. See appendix. ■

The intuition behind this result is that for countries with high levels of product market competition, opening up the economy, or implementing a further trade liberalization, is not going to affect much the already low markup rates.

4 Discussion

The channel through which firms' selection operates in this paper is different from the one in Melitz (2003). In Melitz, selection happens through the effects of trade on the labor market: trade liberalization increases labor demand, this bids up wages and the cost of production, thus forcing the least productive firms to exit the market. In our framework, selection works through the effect of trade on product market competition: the reduction in the markup rate brought about by trade reduces profits and pushes the less productive firms out of the market. In Melitz this channel cannot operate because, under the assumption of monopolistic competition and CES preferences, a larger number of competitors does not affect the elasticity of demand. In our oligopolistic model the market structure is endogenous and trade affects the distribution of surviving firms by raising competition in the product market. The two papers are complementary in that the wage channel of firms selection can be easily introduced in our model by removing the homogeneous good and work with an economy endowed with labor.

A good question to ask is what would happen if we introduce innovation into the Melitz model and how is that different from the results of our paper. Trade induced selection would affect innovation similarly in the two models, but our oligopolistic setup yields an additional result that cannot be obtained in the monopolistic competitive framework used by Melitz. In the Melitz model firm heterogeneity does not play any role when the economy moves from autarky to free trade (zero trade costs): the effects of trade are exactly those found in the representative version of the model (i.e. Krugman, 1980). Firm selection takes place only with incremental trade liberalization (positive trade costs). In our model instead, the oligopolistic structure implies that firm selection takes place under radical trade liberalization as well. This happens because trade reduces markups which forces less productive firms to exit.

Finally, it is worth mentioning that Melitz and Ottaviano (2007) endogenize markups by assuming an ad hoc structure of preferences that makes them dependent on the number of firms (varieties), thus introducing a selection effect working through trade-induced increases in product market competition. But this transmission channel is obtained by using special preferences in the monopolistic competitive framework, while in our paper it is produced by the oligopolistic interaction among firms.

5 Quantitative analysis

In this section we explore the quantitative relevance of our mechanism. We calibrate the model's steady state to match salient aggregate and firm level statistics of the US economy, then perform a counterfactual exercise: we study the effects of a 10 percent reduction in trade costs τ on the innovation rate. Precisely, we quantitatively evaluate the effect of trade liberalization on innovation due to the *direct competition effect,* for which firm heterogeneity doesn't matter, and the *selection effect* which pushes growth through a reallocation of market shares toward more productive (more innovative) firms. We also investigate the quantitative effect of a reduction in τ on the second moments of the productivity distribution, namely of the volatility of the growth rate and of firm sales. Although the general analytical results presented above do not require assuming any particular productivity distribution, in order to perform our quantitative exercise we assume that the stationary distribution is Pareto with shape parameter κ. This is consistent with evidence on firm size distribution (see e.g. Luttmer, 2007)

5.1 Calibration

We have 12 parameters to calibrate $\alpha, \tau, \delta, \beta, \lambda, n, L, \rho, \eta, A, \kappa, z_{\min}$. We calibrate 8 parameters externally: the discount factor ρ is equal to the interest rate in steady state, thus we calibrate it to 0.03 as in the business cycle literature. For the preferences parameter α we refer to the international business cycle literature which provides estimates of the elasticity of substitution in the range $0.2 - 3.5$ (see i.e. Backus, Kehoe and Kidland, 1994, Heatcote and Perri, 2004, and Ruhl, 2008). Since the prohibitive trade cost is $n/(n + \alpha - 1)$ we have to stick to low values of α and n for the prohibitive tariff not to be too low and therefore not leaving much room for comparative statics on it. We choose a value of α yielding an elasticity of substitution in the range of the above estimates that allows a sufficiently high prohibitive trade cost: precisely we set $\alpha = 0.1$, which leads to an elasticity of substitution $\sigma = 1.1$. Anderson and Wincoop (2004) summarize the tariff and non tariff barriers using TRAINS (UNCTAD) data: for industrialized countries tariffs are on average 5% and non tariff barriers are on average 8%. We take the sum of these two costs and set $\tau = 1.13$. We use a 20 percent markup (in the range of estimates in Basu,1994) to back out the number of firms (the integer part of it) using the markup equation (20) and the calibrated values of α, and τ. Using these values we obtain $n = 3$. The choice of n and α leads to a sufficiently high prohibitive trade cost, $\bar{\tau} = 1.42$.

We set the destruction rate $\delta = 0.24$ to match the US job destruction rate of 24% per year (Davis and Haltiwanger, 1999, table I). The share of the homogeneous good β is calibrated to match a 9 percent agriculture share of private industry (BEA NIPA). We set the population $L = 1.32726800$, which is the average US civil workforce 1990-2000 Bureau of Labor Statistics (2001) rescaled to obtain the best fit of the data. We normalize the minimum value of the productivity distribution $z_{\min}$ to 0.1, this is the only free parameter in the calibration and we will provide an extensive robustness check on it.

The remaining 4 parameters $(A, \eta, \lambda, \kappa)$ are calibrated internally to match some key statistics: similar to many calibrated models of firm dynamics we target the US economy, for which many firm level moments are available (see i.e. BEJK, 2003, Luttmer, 2007, Alessandria-Choi, 2007). We use four targets, the first two are the average growth rate of productivity and the R&D ratio of GDP. We use data from Corrado, Hulten and Sichel (2009) where US national account data have been revised to introduce investment in intangible capital, including R&D. Moreover, since there is no tangible capital in the model, all statistics used in the calibration must be adapted to the model economy. Precisely, the growth rate of labor productivity and the R&D ratio to GDP, are obtained by subtracting investment in tangible capital from total income in the data. After this adjustment, Corrado et al. data report an average growth in labor productivity of 1.9% a year in the period 1973-2003. Since in the model all investment is in R&D, the targeted statistics for the R&D ratio to GDP is the investment in intangible capital share of total income; after subtracting tangible capital this leads to an average of 13.5% over the period 1973-2003. In our model the correspondent moment is

$$\frac{R\&D}{GDP} = \frac{\eta\theta^T e\frac{L}{n} - \frac{\rho+\delta}{A}}{e(1+\beta)M}$$

where from (21) we can compute the aggregate (average) equilibrium R&D $h = \eta A\theta^T e_t \, (L/n) \, \tilde{z}_t - \rho - \delta$. The growth rate in open economy can be obtained from (13) with θ^T ireplacing θ. The third targeted statistics is a standard deviation of firm growth of 0.35, which is found by Luttmer (2007) using US Census data. The standard deviation of firm growth in our model is

$$std(\tilde{z}) = \frac{\eta A\theta^T e\tilde{z}^* L}{(\kappa-1)n}\left[\frac{\kappa}{(\kappa-2)}\right]^{1/2},$$

where, since we are working with the equilibrium distribution, the bottom productivity level is $\tilde{z}^*$. Finally we use a statistics that is relevant to pin down the fixed operating costs λ, that is the average firm size of 19 workers found in Axtell (2001) for US firms in 1997.[11] In our model the average firm size is

$$y_{avg} = \theta^T e\frac{L}{n} + \lambda.$$

[11] If we set the price of the homogeneous good to be the numeraire, using labor instead of units of the homogeneous good to produce the differentiated good does not change anything in the model and in the results.

Minimizing the quadratic distance between these statistics and their theoretical counterpart we obtain the following parameters: $A = 0.19$, $\eta = 0.096$, $\kappa = 2.52$, $\lambda = 2.76$. Table 1 below shows the model fit of the data.

[Table 1 ABOUT HERE]

The calibrated model matches the targeted statistics sufficiently well. The model produces acceptable values for some key non targeted statistics as well.

5.2 Counterfactuals

In this exercise we focus on quantifying the growth effect of a 10 percent reduction in the trade cost τ, breaking it down into the two growth channels of trade liberalization that we have in the model: the direct competition effect and the selection effect.

In order to decompose the total growth effect of trade liberalization we differentiate the equation (13) as follows:

$$g_\tau = \frac{dg}{d\tau} = \underbrace{\frac{dg}{d\theta^T}\frac{d\theta^T}{d\tau} + \frac{dg}{de(\overline{\tilde{z}}^*)}\frac{de(\overline{\tilde{z}}^*)}{d\tau}}_{\texttt{Direct effect}} + \underbrace{\frac{dg}{de(\tilde{z}^*)}\frac{de(\tilde{z}^*)}{d\tilde{z}^*}\frac{d\tilde{z}^*}{d\tau}}_{\texttt{Selection effect}}$$

where the direct effect can be obtained keeping fixed the productivity threshold $\tilde{z}^*$, therefore ignoring the cutoff condition and using the market clearing condition (MC^{T}) to obtain the effect of τ on expenditure e. The resulting direct effect g_d^τ is

$$g_\tau^d = \eta A\left(\frac{L}{n}\right)e\frac{d\theta^T}{d\tau} + \eta A\theta\left(\frac{L}{n}\right)(1+\eta)\,\frac{\frac{1}{M(\tilde{z}^{*T})} - \left(\lambda - \frac{\delta+\rho}{A}\right)\frac{n}{L}}{\left[\beta + (1+\eta)\,\theta^T\right]^2}. \tag{22}$$

where $d\theta^T/d\tau$ is derived in the appendix. The selection effect is thus obtained as a residual $g_\tau^{\tilde{z}^*} = g_\tau - g_\tau^d$.

Figure 3 shows the effect of a 10 percent reduction in trade cost τ from its benchmark value of 1.13 to 1.117, on some key first moments of productivity growth.

[Figure 3 about here]

The reduction in trade cost produces a small reduction in the markup, which declines by 0.003. Interestingly, although our calibrated model does not show a large pro-competitive effect, both the productivity cutoff and innovation are fairly sensitive to changes in the markup. In fact the productivity cutoff $\tilde{z}^{*T}$ rises by 34 percent, implying a reduction of the survival probability of entering firms, $1-F(\tilde{z}^{*T})$, by 2.3 percent. The growth rate of aggregate productivity increases by 33.8 percent from 0.022 to 0.029. Comparing these results with those obtained in empirical works which studied parts of the implications of our mechanism shows that the prediction of our stylized model are fairly close to the empirical evidence. Table 3 below summarize the comparison. First we can see that the small pro-competitive effects of trade that we find are in line with empirical evidence: Chen, Imbs and Schott (2008) estimating the Melitz and Ottaviano (2008) model using European data find that a 10 percent increase in the import to production ratio lower the average markup by 0.01; Kee and Hoekman (2003) find that the a 10 percent increase in the import ratio lowers the markup by 0.014 in OECD countries. Similarly, our findings are in line with some recent estimates of the trade-induced innovation and selection effects: Bloom, Draca, and Van Reenen (2009) for instance, find that a 10 percent increase in Chinese imports is associated with a 21.4 percent increase in R&D, a 4 percent increase in IT intensity, and a 6 percent increase in patents in European countries.[12] They also find that the probability of firm survival reduces by 1.2 percent. Teshima (2009) using Mexican plant-level datasets finds that a 1 percent reduction in trade costs increases R&D by 8 percent.

[Table 3 about here]

Using (22) we find that between 18 and 20 percent of the total increase in growth can be attributed to the direct effect, while the rest is produced by the selection effect. This suggests that the main mechanism highlighted in the paper, the role of the reallocation of market shares between exiting and surviving firms in spurring growth, is quantitatively relevant. Bloom et al. (2009) instead find that the contribution of selection (between component) and of the direct effect (controlling for labor reallocation) of trade liberalization to increases in innovation is substantially similar.

In table 2 below we briefly explore the role of some key parameters. This is not meant to be a robustness analysis of our results because under the specified parameters restrictions our basic findings have been analytically in proposition 1 and 2.

[Table 2 about here]

Introduce comments..

6 Endogenous number of firms

In order to work with the simpler structure that would allow us to explain the importance of the competition channel in a firm dynamics model with innovation we have made the simplifying assumption that the number of firm producing each variety n is fixed. Under this assumption we have shown that when two symmetric countries trade, the both the size of the economy L and the number of firms simply double, thus leaving the size of the market per operating firms L/n unchanged. One implication of this assumption of constant n is that when a low productivity industry is hit by the trade shock, the firms in the industry will exit altogether. This does not allow for the possibility that only some firms in the oligopolistic industry will be forced out of business, therefore leading to a trade equilibrium in which the ratio of firms to population is lower than in autarky, because of a lower value of n. Endogenizing the number of firm would

[12] See Bloom et al (2009) table 2.

improve the explanatory power of the model to the cost of complicating it. Here we propose two ways of endogenizing the number of firms in our model.

First, we assume that the number of firms by sector is constant over time but it is endogenously determined when the sector is created, just before initial productivity is revealed. Firms face an entry cost and free entry determines the equilibrium number of firms. Secondly, we study the case in which sectors enter the economy with a maximum number of firms $n_{\max}$. Those sectors with initial productivity lower than the mean, facing decreasing profits, slowly reduce in size to allow their market value to remain positive, until they completely disappear from the economy. In this case, the economy faces an ordered sequence of cutoff productivities associated with different market sizes, with sectors below average productivity moving step by step from an $n_{\max}$-oligopoly to monopoly over time.

6.1 Endogenous but constant n

In the previous sections, it has been shown that two symmetric economies benefit from more competition when opening to trade, since the number of competitors operating in each local market doubles. This result was obtained under the assumption that the number of local firms per sector remains constant after trade liberalization. In this section, we endogenize the number of firms and show that trade openness increases the number of firms selling in each market even if the number of local firms reduces. We show it under the simplifying assumption that the mass of sectors M is constant. Since opening to trade reduces M, as shown before, the reallocation of the freed resources will allow for more firms in the surviving markets than under a constant M. In this sense, the assumption of a constant mass of sectors provides a lower bound for the pro-competitive effect of trade liberalization when the number of firms is endogenous. This simplifying assumption allows us to show how the selection effect can operate along the dimension of the number of firms active in producing each variety.

For simplicity, we focus on the closed economy and provide the intuition for the comparative statics of opening up to trade. The entry condition, as defined below, is an inequality condition since the number of firms is an integer. Let call V_n the firm's expected value in a sector with n competitors. Since profits are decreasing in the number of competitors, the expected value is decreasing too.

Definition 5 *n is said to be an equilibrium number of firms if $V_n \geq 0$, but $V_{n+1} < 0$*

When a new sector is created, firm entry decisions are assumed to be taken before the observation of initial productivity. No new firms are allowed to enter the market afterwards. Entry entails a fixed cost f, $f > 0$, measured in units of the homogeneous good. From (12) and (15), profits are linear on the relative productivity $\tilde{z}$, implying that expected profits are equal to average firm's profits, i.e. a firm with $\tilde{z} = 1$. Its profits are

$$\pi(1) = \left(\frac{1}{\theta} - 1 - \eta\right) \theta e L + \frac{\rho + \delta}{A} - \lambda.$$

Let us call $\hat{n}$ the real number that makes the average firm's value equal to the entry cost f, implying

$$[1-(1+\eta)\theta]\, eL = (\rho+\delta)\left(f-\frac{1}{A}\right)+\lambda, \tag{FE}$$

where $e=\frac{E}{nm}$. The equilibrium number of firms is the integer part of $\hat{n}$. Note that, since θ is an increasing function of n, the relation between $\hat{n}$ and eL is positive.

Proposition 6 *Under the assumption $M=1$, an interior solution for $\hat{n}$ exits and is unique*

Proof. The (FE) and (MC) conditions, under $M=1$, after substituting $\theta=\frac{n-1+\alpha}{n}$ and some simple algebra, imply

$$\left(\frac{(1+\alpha)(1+\eta)}{n}-\eta\right)L = Bn - C$$

where $B=f(\rho+\delta)(1+\beta+\eta)+\left(\lambda-\frac{\rho+\delta}{A}\right)(1+\beta)$ and $C=(1+\eta)f(\rho+\delta)(1-\alpha)$. As a function of n, the right hand side is a straight line with positive slope and the left hand side is decreasing for $n>0$. Consequently, they cross once and only once, meaning that an equilibrium for $\hat{n}$, $\hat{n}>0$, exists and is unique. ■

Let now see how an increase in the size of the economy affects competition.

Proposition 7 *$\hat{n}$ is increasing in L, but less than proportionally*

Proof. An increase in L moves the left-hand-side proportionally. Since the right-hand-side is a straight line with negative intercept, n increases less than proportionally to L. ■

In the case of two symmetric countries and a constant mass of sectors, opening to free trade is equivalent to doubling the size of the market, i.e. doubling L. Under free entry, the previous proposition shows that doubling L implies an increase in the number of firms but by a factor less than 2, that is to say that the scale of the economy (L/n) increases. Consequently, when the number of firms is endogenous moving from autarky to trade has not only a pro-competitive effect (reduces markups) but also a market-size effect, both contributing to stimulate innovation and growth. Using (FE) and (MC) it is easy to show that an increase in L will increase not only n, θ, and L/n but also the level of expenditure e. Therefore all the components of the growth equation (13) are positively affected by trade.

6.2 Decreasing number of firms

Let the number of firms producing a particular good be endogenous in the set $\mathcal{N}=\{1,2,...,n_{\max}\}$, with $n_{\max}>1$, being the number of firms of a new created sector. After a sector has been created, new firms are not allowed to enter, but firms may exit. The productivity of sectors with relative productivity smaller than one is permanently decreasing with respect to the mean, implying that the cash flow of $n_{\max}$ firms is also decreasing. When the cash flow becomes zero, the value of firms becomes zero too, and at least a firm closes down. Once the number of firms reduces to $n_{\max}-1$, the cash flow of the remaining firms becomes positive again, because the market share freed by the closing firm is reallocated to the survivors. Profits will keep decreasing until eventually they will become zero for a second time and another firm will close down, and so on and so for until the sector completely exits the market. Let us call $\tilde{z}_n^*$ the cutoff relative productivities, $n \in \mathcal{N}$, such that the number of firms is a function $n(\tilde{z})$ equal to $n_{\max}$ if $\tilde{z} \geq \tilde{z}_{n_{\max}}^*$; to n, with $n \in \{1, 2, ..., n_{\max} - 1\}$ if $\tilde{z}_n^* < \tilde{z} \geq \tilde{z}_{n+1}^*$; and to zero if $\tilde{z} < \tilde{z}_1^*$. Of course, by construction $\tilde{z}_{n_{\max}}^* < 1$. As an implication, the inverse of the markup is also a function $\theta(\tilde{z})$.

From the solution of the firm's problem in section 2.3, we can rewrite (12) as

$$c(\tilde{z})q = \theta \frac{EL}{n(\tilde{z})} \left(\frac{\theta(\tilde{z})\tilde{z}^\eta}{\overline{z}^\eta} \right)^{\frac{\alpha}{1-\alpha}}, \tag{23}$$

where $\frac{EL}{n(\tilde{z})}$ is expenditure per firm, under $M = 1$, and average productivity is defined as

$$\overline{z}^{\eta \frac{\alpha}{1-\alpha}} = \int_0^1 \left(\theta_j \tilde{z}_j^\eta \right)^{\frac{\alpha}{1-\alpha}} \mathrm{d}j.$$

In facts, $\overline{z}$ represents the price of the composite good and the numerator is the sectorial price, meaning that market shares depend on relative prices. Note that when the number of firm is the same across sectors, the θ's in the numerator and denominator cancel, as in the analysis above, implying that relative prices are equal to relative productivities.

Differently from our benchmark model with exogenous n here the we have a set of equilibrium cutoffs obtained making profits equal to zero in each industry:

$$\left(1 - (1 + \eta) \frac{n - 1 + \alpha}{n} \right) \frac{EL}{n} \tilde{z}_n^* + \frac{\rho + \delta}{A} - \lambda = 0, \tag{24}$$

for $n \in \mathcal{N}$. The market clearing condition for the homogeneous good can be written as

$$\int_0^1 n_j \left(y_j + h_j \right) \, \mathrm{d}j + Y = \int_0^1 n_j \left(c(z_j) q_j + h_j + \lambda \right) \, \mathrm{d}j + \beta E = L.$$

The total endowment of the homogeneous good is allocated to composite good production and innovation, as well as homogeneous good consumption. Let denote by $\mu(\tilde{z})$ the stationary density distribution of firms defined in the $\tilde{z}$ domain. The endogenous exit process related to the cutoff point $\tilde{z_1}^*$ implies $\mu(\tilde{z_1}) = 0$ for all $\tilde{z} < \tilde{z_1}^*$. As in the previous sections, let rewrite the labor market clearing condition after changing the integration domain from sectors $j \in [0, 1]$ to productivity $\tilde{z} \in [\tilde{z_1}^*, \infty]$,

$$(1 + \eta)\bar{\theta} eL + \left(\lambda - \frac{\delta + \rho}{A} \right) \frac{\bar{n}}{n_{\max}} + \beta eL = \frac{L}{n_{\max}}.$$

where $e = E/n_{\max}$ is the market share of the average firm, and

$$\bar{\theta} = \int_{\tilde{z_1}^*}^{\infty} \theta(\tilde{z}) \, \tilde{z} \mu(\tilde{z}) \, d\tilde{z} < \theta_{\max}$$

$$\bar{n} = \int_{\tilde{z_1}^*}^{\infty} n(\tilde{z}) \, \mu(\tilde{z}) \, d\tilde{z} < n_{\max}.$$

are the average of the inverse of the markup and the average number of firms, respectively. Both are endogenous and smaller than the corresponding values in the economy with constant number of firms. As a consequence of letting firms exit sectors, the economy is less competitive at equilibrium. The size of the market reduces, but firms increase their market shares. As an implication, firms produce more and innovate more, which is reflected in the fact that from the previous equation a reduction in $\bar{n}$ and $\bar{\theta}$ increases the market share of the average firm e.

To be completed.................................

7 Conclusion

In this paper we have built a rich but tractable model of trade with heterogeneous firms and cost-reducing innovation, in order to account for a set of findings recently emerged from the empirical analyses of trade liberalization: i) the pro-competitive effect of trade on markups, ii) the selection of the most productive firms, and iii) the positive effect on innovation at the firm level. In our framework, the competition channel is at the roots of the selection and innovation effects of trade liberalization, as all other possible channels (market-size, international technology spillovers, terms of trade) have been excluded from the analysis. The endogenous market structure derives directly from Cournot competition among firms. We have shown that trade liberalization reduces markups, thus forcing the less productive firms out of the market. This selection effect interacts with firms' innovation choice by redistributing resources towards the more productive (more innovative) firms, thereby increasing the aggregate long-run investment in innovation.

The innovation effect of trade highlighted in our model suggests the existence of a new channel of welfare gains from trade that has not been explored in the literature. To keep matters simple we have limited the analysis to the steady-state. A full understanding of the pro-competitive dynamic effects of trade requires the analysis of transitional dynamics, which we view as an interesting task for future research. Finally, studying two perfectly symmetric countries with an identical set of goods, does not allow us to obtain any pro-variety effects of trade. Introducing asymmetric countries is an important step for fully exploring the welfare effects of trade liberalization in our framework.

To be completed.............

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A Stationary Distribution

Let $\mu(\tilde{z})$ be the equilibrium productivity density, $g(\tilde{z})$ the equilibrium growth rate and $\epsilon(\tilde{z})$ the net entry rate. Finally, let define $1+\kappa(\tilde{z}) \equiv -\frac{\mu'(\tilde{z})\tilde{z}}{\mu(\tilde{z})}$. Note that κ is constant if $\mu(\tilde{z})$ is Pareto.

Proposition 8 *The density $\mu(\tilde{z})$ is a stationary solution iff $\epsilon(\tilde{z}) = -(1+\kappa(\tilde{z}))\, g(\tilde{z})$ for $\tilde{z} \geq \tilde{z}^*$, $\epsilon(\tilde{z}) + \delta \geq 0$*

Proof. The time t productivity of a variety with productivity $\tilde{z}_s$ at time $s < t$, if still operative, is

$$\tilde{z}_t = \tilde{z}_s e^{\int_s^t g(\tilde{z}_\tau)\, \mathrm{d}\tau},$$

where the exponential term cumulates productivity growth from s to t. Let define the net entry factor $\Delta_t = e^{\int_s^t \epsilon(\tilde{z}_\tau)\, \mathrm{d}\tau}$, such that

$$\mu_t(\tilde{z}_t) = \mu_s(\tilde{z}_s)\Delta_t,$$

where $\mu_t(\tilde{z}_t)$ is an equilibrium productivity density. By differentiating with respect to t,

$$\mu_t'(\tilde{z}_t)\tilde{z}_t g(\tilde{z}_t) + \frac{\partial \mu_t(.)}{\partial t} = \epsilon(\tilde{z}_t)\, \mu_t(\tilde{z}_t).$$

It's easy to see that

$$\epsilon(\tilde{z}) = \frac{\mu'(\tilde{z})\,\tilde{z}}{\mu(\tilde{z})} g(\tilde{z}) = -(1+\kappa(\tilde{z}))\, g(\tilde{z}) \tag{25}$$

is necessary and sufficient for $\mu(\tilde{z})$ to be stationary, i.e., for $\frac{\partial \mu_t(.)}{\partial t} = 0$. ■

B Firm problem in open economy

Each firm solves the following problem

$$
\begin{aligned}
V_s \;=\; & \max_{\left(q_{D,t}^D, z_{D,t}\right)_s^\infty} \int_s^\infty \left[\left(p_{D,t} - \frac{1}{z_{D,t}^\eta}\right) q_{D,t}^D + \left(p_{F,t} - \frac{\tau}{z_{D,t}^\eta}\right) q_{D,t}^F - h_{D,t} - \lambda\right] e^{-\int_s^t (r_z+\delta)\,\mathrm{d}z}\, \mathrm{d}t \\
& \text{s.t.} \\
& p_{D,t} = \frac{E_{D,t}}{X_{D,t}^\alpha}\, x_{D,t}^{\alpha-1} \qquad \text{and} \qquad p_{F,t} = \frac{E_{F,t}}{X_{F,t}^\alpha}\, x_{F,t}^{\alpha-1} \\
& x_{D,t} = \hat{x}_{D,t}^D + q_{D,t}^D + x_{F,t}^D \quad \text{and} \quad x_{F,t} = \hat{x}_{D,t}^F + q_{D,t}^F + x_{F,t}^F \\
& \dot{z}_{D,t} = A\ \hat{z}_{D,t} h_{D,t} \\
& z_{D,s} > 0,
\end{aligned}
$$

where $p_{j,t}$, $E_{j,t}$ and $X_{j,t}^\alpha$ are the domestic price, expenditure and total composite good respectively for country $j = D, F$, and q_i^j is the quantity sold from source country i to destination country j. Writing down the current value Hamiltonian and solving it yields the following first order conditions

$$
\left[(\alpha-1)\frac{q_{D,t}^D}{x_{D,t}} + 1\right] p_{D,t} = \frac{1}{z_{D,t}^\eta} \tag{26}
$$

$$
\left[(\alpha-1)\frac{q_{D,t}^F}{x_{D,t}} + 1\right] p_{F,t} = \frac{\tau}{z_{D,t}^\eta} \tag{27}
$$

$$
1 = v_{D,t} A \hat{z}_{D,t}, \tag{28}
$$

$$
\frac{\eta z_{D,t}^{-\eta-1}}{v_{D,t}}\left(q_{D,t}^D + \tau q_{D,t}^F\right) = \frac{-\dot{v}_{D,t}}{v_{D,t}} + r_t + \delta, \tag{29}
$$

Since the two countries are symmetric, $q_{D,t}^D = q_{F,t}^F \equiv q_t$, $q_{D,t}^F = q_{F,t}^D = \breve{q}_t$, $x_{D,t} = x_{F,t} \equiv x_t$, $E_{D,t} = E_{F,t}$, $X_{D,t} = X_{F,t}$, $p_{D,t} = p_{F,t}$. From (26) and (27) and using $q_t/x_t +\ \breve{q}_t/x_t = 1/n$ yields

$$
\left[(\alpha-1)\frac{q_t}{x_t} + 1\right] = \frac{2n-1+\alpha}{n(1+\tau)} \equiv \theta_D \tag{30}
$$

$$
\left[(\alpha-1)\frac{\breve{q}_t}{x_t} + 1\right] = \tau\frac{2n-1+\alpha}{n(1+\tau)} \equiv \theta_F = \tau\theta_D \tag{31}
$$

which allows us to rewrite (26) and (27) as follows

$$
\theta_D \frac{E_t}{X_t^\alpha}\, x_t^{\alpha-1} = \frac{1}{z_t^\eta} \quad \text{and} \quad \tau\theta_D \frac{E_t}{X_t^\alpha}\, x_t^{\alpha-1} = \frac{\tau}{z_t^\eta}.
$$

Multiplying the above equations by q_t and $\breve{q}_t$ and summing up we obtain

$$
\frac{q_t + \tau\breve{q}_t}{z_t^\eta} = n\left[\theta_D \frac{q_t}{x_t} + \tau\theta_D \frac{\breve{q}_t}{x_t}\right] \frac{E_t}{n}\left(\frac{x_t}{X_t}\right)^\alpha.
$$

Using $x_t = \{[1/z_t^\eta]\,(X_t^\alpha/\theta_D E_t)\}^{\frac{1}{\alpha-1}}$, it is easy to prove that $(x_t/X_t)^\alpha = \widetilde{z}_t$. From (30) and using $q_t/x_t +\ \breve{q}_t/x_t = 1/n$ we obtain

$$
\frac{q_t + \tau\breve{q}_t}{z_t^\eta} = \theta^T e_t \left(\frac{L}{n}\right) \widetilde{z}_t \tag{32}
$$

where $e_t = E_t/M$ and

$$\theta^T = \frac{2n-1+\alpha}{n(1+\tau)^2(1-\alpha)}\left[\tau^2(n+\alpha-1)+n(2\tau-1)+1-\alpha\right]$$

is the inverse of the markup in the open economy.

C Exit in open economy

The productivity cutoff is determined solving the following equation

$$\pi_t(\tilde{z}^*) = \left(p_t - \frac{1}{\tilde{z}_t^{*\eta}}\right)q_t + \left(p_t - \frac{\tau}{\tilde{z}_t^{*\eta}}\right)\breve{q}_t - h_t - \lambda = 0$$

Using $p_t = \frac{1}{\theta_D z_{D,t}^{\eta}}$ and $h_t = \eta\theta^T e_t \tilde{z}_t - (\rho+\delta)/A$ obtained from (21) yields

$$\frac{1}{\theta_D}\frac{q_t+\breve{q}_t}{\tilde{z}_t^{*\eta}} - \left(\frac{q_t+\tau\breve{q}_t}{\tilde{z}_t^{*\eta}}\right)\left(1+\frac{1}{1+b}\right) + \frac{\rho+\delta}{A} - \lambda = 0.$$

With the same procedure used to derive (32) we obtain

$$\frac{q_t+\breve{q}_t}{\tilde{z}_t^{*\eta}} = \theta_D e_t \widetilde{z}_t$$

which, together with (32), yields

$$\left[1-(1+\eta)\theta^T\right]e_t\tilde{z}_t^* + \frac{\rho+\delta}{A} - \lambda = 0.$$

This expression is similar to (EC) except for the markup $1/\theta^T$ instead of $1/\theta$.

D Non-linear effect of trade liberalization

Here we show that the competition effect of trade is decreasing in the number of firms n. This can be seen by differentiating θ^T with respect to τ

$$\frac{\partial\theta^T}{\partial\tau} = \frac{-2(2n-1+\alpha)}{n(1+\tau)^2(1-\alpha)}\left[\frac{2n\left(\tau^2+\tau-2\right) - .\left(\tau^2-1\right)(1-\alpha)}{1+\tau}\right]$$

It is easy to see that this derivative is decreasing in n.

TABLE 1

MODEL FIT

Moments	Data	Sources	Benchmark model
`Targeted`			
growth	0.019	CHS (2006)	0.022
R&D/GDP	0.135	CHS (2006)	0.12
Std. firm growth	0.35	Luttmer (2007)	0.337
avg. firm size	19	Axtell (2001)	18.8
`Non targeted`			
Std. of log sales	1.64	BJEK(2003)	0.93
Std. log productivity	0.75	BJEK(2003)	0.98(*check*)

TABLE 2

COMPARISON WITH EMPIRICAL EVIDENCE

	model	CIS	KH	BDV	Tesh*
Markups	$-.005$	$-.01$	$-.014$		
$\tilde{z}^*$	.34				
$1-F(\tilde{z}^*)$	$-.023$			$-.012$	
Growth	.338			.24($R\&D$)	.08
Direct effect	20%			50%	
Selection effect	80%			50%	

TABLE 3

GROWTH DECOMPOSITION

	benchmark	$n=4$	$L=2.6$	$\kappa=5.05$	$\lambda=5.53$	$z_{\min}$
total	0.338	0.198	0.10	0.048	0.044	
direct	0.18	0.163	0.22	0.42	0.375	
selection	0.81	0.837	0.78	0.58	0.625	

Figure 1. Steady state equilibrium

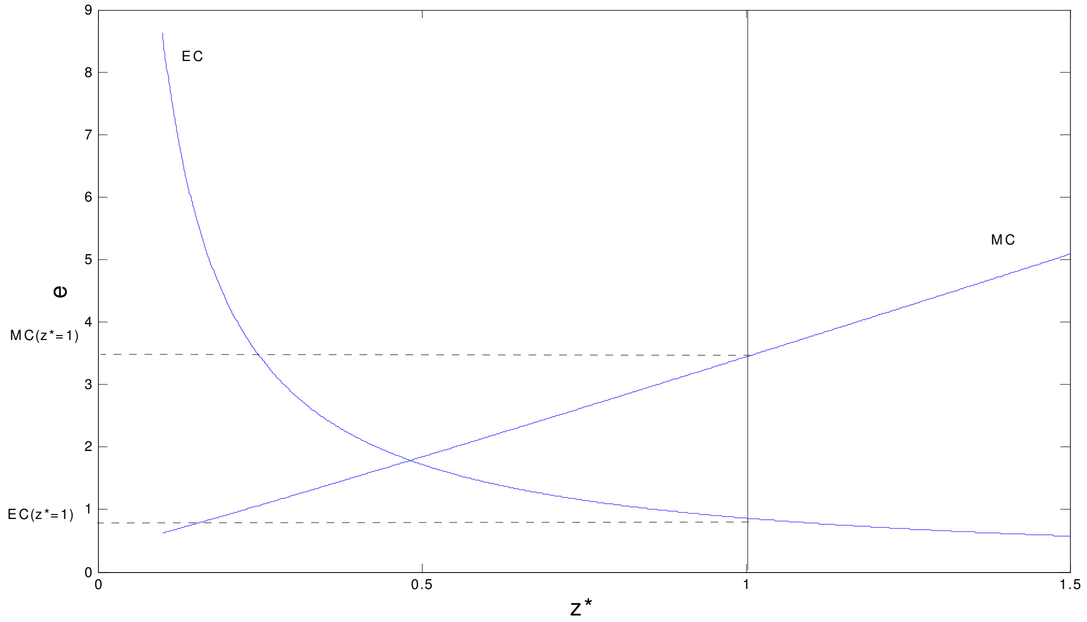

Figure 2. Increasing the number of firms n

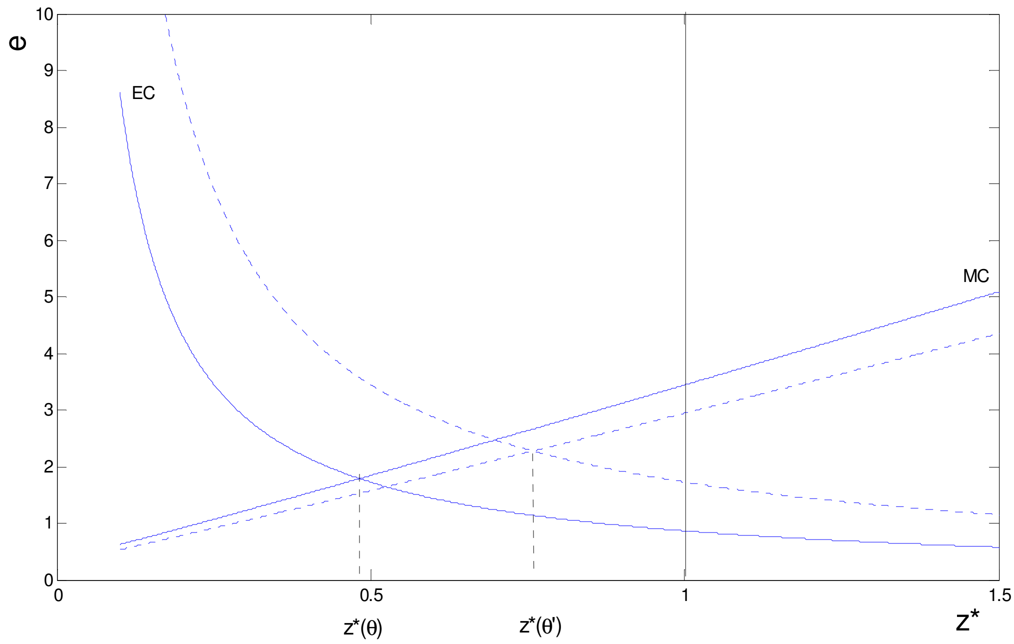

Figure 3. Trade liberalization