Skill Scarcity, Wages, and Export Intensity*

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ABSTRACT

This paper develops a trade model with skill-based product differentiation and non-iceberg transport costs that is able to account for the observed positive correlation between firms' export intensity, the price of their export, and the wages they pay to their workers. In the model, the distribution of prices and wages, as well as patterns of trade across differentiated products, are driven entirely by the distribution of workers' skill types in the economy. In equilibrium, firms that employ workers with comparatively scarcer skills export a larger fraction of their output, pay higher wages and charge higher prices. We motivate our approach with evidence from French matched employer–employee data, and firm–level custom data. We show that the observed patterns are consistent with how the model interprets the relationships between wages, output prices and export conduct.

KEY WORDS: Export Conduct, Wage Inequality, Skill-based Differentiation
JEL CLASSIFICATION: F12, F16, E24

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1 Introduction

It has been widely documented that exporting firms pay higher wages than non-exporters do.[1] Variation in export conduct across firms is typically rationalized in the literature in terms of heterogeneity in productivity across firms, either with respect to the production of horizontally differentiated varieties (along the lines of Melitz, 2003) or with respect to quality upgrading (Feenstra and Romalis, 2012); but productivity differentials cannot directly explain the observed positive correlation between wages and export performance.[2] For this correlation to arise, some additional mechanism must be present alongside productivity differentials – a positive relationship between quality choice by exporters and input quality (Bernard et al., 2003, Feenstra and Romalis, 2012), efficiency wages (Egger et al., 2013; Amiti and Davis, 2012; Davis and Harrigan, 2011), positive assortative matching between high-productivity, export-intensive firms and high-skill workers (Yeaple, 2005; Sampson, 2016), rent sharing under labor markets frictions (Felbermayr et al., 2011; Gopinath et al., 2012; Helpman et al., 2012) or sorting of workers with match specific abilities (Helpman et al., 2010).

While these various mechanisms are able to explain the relationship between export status on wages, they fail to account for differences in export conduct across exporters. In explaining the patterns of aggregate trade flows, variation across firms on the intensive margin of export performance (i.e. in terms of the share of foreign sales over total sales) is no less important than variation on the extensive margin (i.e. selection of firms into the export market). Looking at French manufacturing firms – the case we focus on in this study – variation in export levels across exporters (representing roughly 20% of all manufacturing firms) accounts for more than 90% of the average total variation of exports within sectors. This is not simply a consequence of variation in firm size: the mean standard deviation of the *export intensity* of exporters (the share of exports revenues to total revenues) within sectors is 0.25, around a mean value of 0.26; by comparison the standard deviation of exporters' total revenues is less than half the size of the corresponding mean, with the

[1]See the recent analysis by Akerman et al. (2013) for Sweden and Baumgarten (2013) for the German case. Export performance has been shown to be correlated not only with wages but also with other firm characteristics, such as size, capital intensity, prices of inputs other than labor, and export prices – the latter feature also broadly being observationally equivalent to quality. Analogous patterns are also observed within multi-product firms Manova and Zhang (2012).

[2]Models that have been developed in the literature to rationalize heterogeneous trade behavior by heterogeneous firms do not account for within-group and between-firm wage inequality. This is also the case for Helpman and Itskhoki (2010) and Felbermayr et al. (2011), in which heterogeneous firms pay the same wage in equilibrium, even tough frictions in the labor market generate unemployment.

correlation between export intensity and firm size being close to zero.[3]

This heterogeneity in behavior across exporters is also reflected in the wages they pay: we see a strong positive correlation between export conduct and wages on the intensive margin, not just on the extensive margin. In the French case, the overall mean variation of wages within product and worker categories across all producers is roughly equal to that across exporters, and both are roughly ten-fold the size of the corresponding "between-variation" between exporting and non-exporting firms.; and the mean correlation between wages and export intensity across exporters, again within product and worker categories, is 12.5%, while the corresponding mean correlation with export status (a zero/one measure) is 14.5%. A theoretical framework where export conduct is driven by factor-neutral productivity differentials across firms (such as Melitz, 2003), and where markets are separated by iceberg-type transport costs, cannot say anything about the intensive margin of trade, as it does not predict variation in export intensity across exporters.[4]

A third empirical regularity that challenges theoretical frameworks that derive variation in export conduct from factor-neutral productivity differentials comes directly from evidence on the incidence of transport costs on price in export markets. Recent studies, mainly empirical in focus (Hummels and Skiba, 2004; Gervais, 2015), show that trade costs increase less than proportionally with unit price. This could account for variation across exporters on the intensive margin: exporters that export goods with a higher unit price would face proportionally lower trade costs – the so called "Alchian-Allen effect" (Alchian and Allen, 1972). The empirical evidence on export revenues also clearly points to this: exporters that charge higher prices earn greater revenues in each export destination (Alchian and Allen, 1983; Manova and Zhang, 2012). A theoretical framework with factor-neutral productivity differentials struggles to accommodate non-proportional trade costs.[5] In order to extend the framework to accommodate non-proportional trade costs, one must posit that, for some reason, quality increases with productivity; but that is not enough: for prices to be positively related to export intensity, productivity would need to be positively correlated with marginal costs. This is a very specific structure of technology, which seems consistent with patterns that we observe in the data but which itself needs to be explained.

[3]See Bernard et al. (2012) and Fernandes et al. (2015) for a discussion of the US case.

[4]The intensive margin of exports can also be interpreted as as a sequence of extensive-margin market access decisions ordered in terms of their relative market access costs, with more productive/larger firms finding it worthwhile to export to more markets. Still, we also observe variation in export intensity across exporters within single export markets.

[5]Taken literally, it says that exporters are higher-productivity firms that face lower marginal costs and sell lower-priced goods. The burden of trade cost on price is then higher for more productive exporters. Therefore, export intensity should be higher for exporters that are comparatively *less* (rather than more) productive.

One way of rationalizing this pattern is invoking quality upgrading choices by firms that face exogenously heterogeneous quality upgrading costs and non-iceberg trade costs, as in Feenstra and Romalis (2012). However, endogenous quality upgrading cannot directly explain the positive correlation between firms' wages and their exports, unless a further mechanism is invoked that generates a skill-bias in quality upgrading. Not only do we observe that wages rise with export intensity at firm level; but they seem to contribute to price differentials in proportion to the share of labour inputs in production: if we run a simple regression of export prices on wages for French exporters, adding firm-level controls and sector-level effects (as we detail in the Section 2), the elasticity of price to wages roughly equal to the observed share of labour costs in total costs. This suggests that quality upgrading, as evidenced by a higher price, requires employing higher-wage workers, or, equivalently, that it is *determined* by the type of labour inputs employed. But since, from the firm's point of view, the supply of workers of each type is exogenously given, there is no meaningful quality upgrading choice that firms can make in response to exporting incentives.[6] To put it differently, it is the supply of worker types that determines the types of varieties that are produced and sold both domestically and in export markets, and export prices and export behavior do not correspond to a deliberate quality upgrading choice by exporting firms – no more so than individual firms are free to "choose" what to produce in a factor-proportions model.

Moreover, the patterns we see in the data concerning prices and quantities produced do not directly allow for a clear discrimination between vertical and horizontal differentiation. Again with reference to the French case, at any given price point the number of firms that sell products in export markets at that price is decreasing with the price, and the quantities produced by each exporting firm are also decreasing with price. Taken together, these patterns imply a negative relationship between a variety's price and the quantities of that variety that are produced and sold, which is compatible with an interpretation where varieties at different price points are horizontally differentiated with price differentials reflecting supply differentials, rather than vertical differentiation. In other words, as long as prices and quantities of different varieties are inversely related in demand patterns, vertical differentiation and horizontal differentiation with variation in supply levels are observationally equivalent in terms of how we can model substitution across varieties – and indeed vertical differentiation, on its own, cannot directly account for why quantities are inversely related to prices. In turn, if product differentiation is based on the type of skill employed, then comparative scarcity for products derives from comparative skill scarcity,

[6]To the extent that the supply of skills is elastic, it may respond to wage changes that are related to exporting (through market integration); but this response comes from the suppliers of the inputs, not from exporters.

which is also consistent with the observation that comparatively scarce skills being paid comparatively more, i.e. with a positively skewed wage distribution, as we actually observe it.

Starting from these empirical observations, in this paper we ask whether a simple model featuring input-based horizontal differentiation and comparative scarcity provides a sufficient representation for the patterns we see in the data in relation to export intensity, prices and wages on the extensive margin of the variation in firms' export conduct. We develop a theoretical model of trade in differentiated varieties under monopolistic competition and increasing-returns-to-scale-production technologies, along the lines of Krugman (1979). Product differentiation in the model is skill-based, and so heterogeneity in firms' choices is directly derived from comparative skill scarcity rather than from productivity differentials that are structurally orthogonal to skill levels. Firms are heterogeneous because the tasks their employees perform are different. In equilibrium wages and variety prices reflect the scarcity of skill types; hence, explaining wage dispersion even in a frictionless labor market. Exporting technologies employ the same skill type that is used in production but are less labour intensive than manufacturing technologies, giving rise to trade costs that rise less than proportionally to manufacturing costs.

The relationship between firms' productivity, export behavior, and the wages they pay arises in the model as a direct prediction of its basic structure. When product differentiation arises from input differentiation and transport costs are less than proportional to price (non-iceberg trade costs), the positive relationship between wages and export performance can be rationalized simply in terms of input scarcity: if differentiated goods are produced employing workers of different skill types, then skill scarcity translates into higher wages for scarcer skills and higher prices for the goods produced using those skills; since the incidence of transport cost on price is lower for higher-priced goods, market integration can lead to greater share of foreign sales for firms that employ higher-wage workers and produce higher-price goods. Trade will also increase the relative demand for scarcer skill-types, hence higher-wage workers, and so the model is also capable of generating predictions about the effects of trade liberalization on the distribution of wages. Such rich set of predictions is obtained from a minimal theoretical toolkit.

The approach we propose contributes to two lines of research: one explaining the relationship between export performance and prices (such as Feenstra and Romalis, 2012) and the other explaining the effect of trade on wages (such as Felbermayr et al., 2011, Helpman et al., 2012, Egger et al., 2013). Our model offers an explanation to both research questions, with the added advantages of being parsimonious and of capturing the effect of quality while circumventing the problem of measuring quality. In applications where the only observables are prices and quantities – and where predictions relating to quality or productivity differentials cannot be directly tested – the framework we propose provides a sufficient representation of the margins of choice involved. The model also generates distinctive predictions. Measured firm productivity – which the model implicitly models as being "factor embedded" – is predicted to respond endogenously to trade, even in the absence of firm exit; this implies that trade has an impact on aggregate productivity also through its intensive margin, an effect that does not end with the selection of firms out of the market. Another distinctive prediction of the model is that firms exhibiting higher export intensity and paying higher wages need not necessarily be those firms that have a comparatively larger market share in revenue terms. This is consistent with what we see in the data: while exporters are systematically larger than non-exporters – by a factor of forty on average in the French case – the correlation between export intensity and size for exporters is close to zero. Finally our analysis also contributes to the body of trade theories based on factor proportions (see Romalis, 2004 and Bombardini et al., 2012): the modelling approach we adopt is a natural extension of the factor-proportions framework, with further channels linking endowment differentials to trade-induced distributional effects – not just through trade across different categories of products, but also through trade in differentiated products.

The remainder of the paper is structured as follows. Section 2 presents evidence for French exporters. Section 3 outlines the model, characterizes an autarky equilibrium and an equilibrium in open economy, derives the main predictions on export intensity and wage concentration, and examines the relationship between skill scarcity and export intensity in the data. Section 4 discusses generalizations and extensions. Section 5 concludes.

2 Prices, wages and export intensity: evidence French firms

In this section we present evidence on the relationships between the revenues in the export market across exporters, the prices at which they sell their products and the wages they pay to their workers. The patterns of correlations that we document lend themselves to an interpretation based on input-based product differentiation. The goal of this section is indeed and only to motivate the modeling choices behind the theoretical framework, which we will present in Section 3.

Data

We use matched employer-employee administrative data that covers the universe of French firms and workers over the period 2010-2013. The dataset includes information about the categories of products sold by firms, their balance sheets (domestic and export revenues, as well as costs broken down by cost category), and, for each worker they employ, information on the worker's occupation and wages. We match this information with to customs data on firms' import and export flows, which detail for each transaction the value of the transaction, the mass of the shipment and the country of origin or destination of the products. Appendix A provides a more detailed description of the data.

The dataset contains a very large number of observations. For the purpose of visualizing correlations between prices, wages and export performance we use *binned scatterplots* that summarize a non–parametric estimate of conditional expectations. Each graph contains one hundred bins corresponding to a percentile of the horizontal-axis variable, with each bin containing a 1% mass of the total observations. The position of each bin on the horizontal axis corresponds to the mean value within the given percentile, and the vertical axis corresponds to the mean of the vertical-axis variable in the binned observations. In the exposition we will often refer to a set of controls. This is obtained with the usual partitioned regression approach only for the purpose of deriving a meaningful visual summary of the relationship between the two variables of interest, netting out the effect of the confounding factors that we refer to as controls.

Export intensity and price

Figure (1) shows the relationship between firms' export intensity (the ratio of export revenues to total sale revenues) and export price, for all exporters pooled over the full period.[7] The price is constructed as the value of the shipment divided by the mass in kilograms for

[7] In the binned scatterplot of Figure (1) we control for HS4 sector of exported product and year.

the HS6=level product in a year. In the case of multiple exported products, we compute the price of the HS6-level product which represents the largest export value for the firm in a given year. We use year dummies and sector dummies at the HS4 level (the average number of firms in a sector in a year is 40). There is a clear pattern of positive correlation between export intensity and export price.

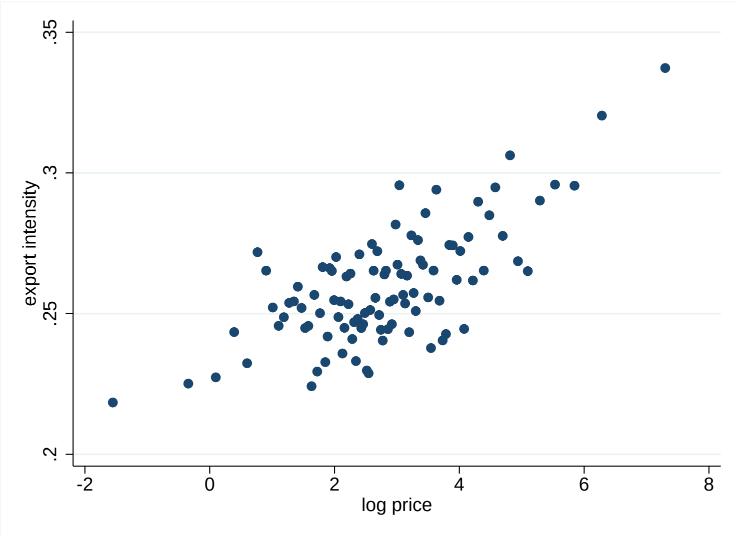


Figure 1: Export prices and export intensity

This relationship can also be examined by running an OLS regression on the following specification:

$$\ln Exp\_Intensity_{it} = \beta \ln Exp\_Price_{it} + \psi_{s(i)} + \gamma_t + \vec{\zeta}\vec{Z}_{it} + \epsilon_{it}, \quad \forall i, t, \tag{1}$$

where i identifies a firm, t is the year, $s(i)$ is the sector to which exporter i belongs, $\vec{Z}_{it}$ is a vector of firm-level controls: (i) the technological patterns of production are captured by the shares in total costs of capital costs, raw material costs, purchase of intermediate goods costs, other costs; (ii) firm size is accounted by the level of revenue in the domestic market only; (iii) the quality of input is instrumented by the price of the main product imported by the firm (which is computed analogously to the export price).

The results are shown in Table (1). When we include firm-level controls (Columns 2 and 3), the estimated value for the elasticity of export-intensity to price (β), is about 5%, and is statistically highly significant. The simplest explanation for this relationship is, as previously noted, transport costs that are less-than-proportional to price, giving rise to an Alchian-Allen effect. Building on this intuition, in Section 3, we describe a specific representation of non-price-proportional transport cost. As will be shown (in Appendix D), that representation also makes it possible to link the elasticity estimate obtained from the previous regression with literature estimates of the elasticity of transport cost to price.

Table 1: Export price and export intensity

log exp. intensity	(1) No firm controls	(2) Firm controls	(3) Price imported good
log exp. price	0.0539***	0.0518***	0.0497***
	(0.00871)	(0.00869)	(0.00871)
log capital share		0.159***	0.183***
		(0.0136)	(0.0147)
log int. good share		0.0155	0.0111
		(0.00814)	(0.00990)
log raw mat. share		0.0932***	0.0974***
		(0.00962)	(0.0105)
log other cost share		-0.0131	-0.0290
		(0.0229)	(0.0246)
log dom. revenue		-0.0495***	-0.107***
		(0.00906)	(0.00935)
log price imp.			0.0753***
			(0.00765)
Adjusted R^2	0.094	0.109	0.130
Observations	30,588	30,125	24,233

Significance: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Specification: All specifications are at the {firm, year} level and include dummies for HS4-level sector of the main exported product, HS3-level sector of the main imported product, 2-digit PCS classification of the main occupation, year; details on the data are reported in Appendix A. All specifications include a constant, standard errors are clustered at the HS4-level sector and reported in parentheses.

Prices and wages

Focusing next on input prices, previous studies have documented how exporting firms use higher-cost intermediate inputs (Manova and Zhang, 2012) and pay higher wages to similar workers (Gopinath et al., 2012). If we restrict attention to exporters, a positive relationship between input prices and export intensity is also observed. Figure (2) shows how export intensity varies with the unit price of the principal import of each firm.[8] The positive relationship in evidence in Figure (2) is reflected by the coefficient for the price of the principal export in Column 3 of Table (1).

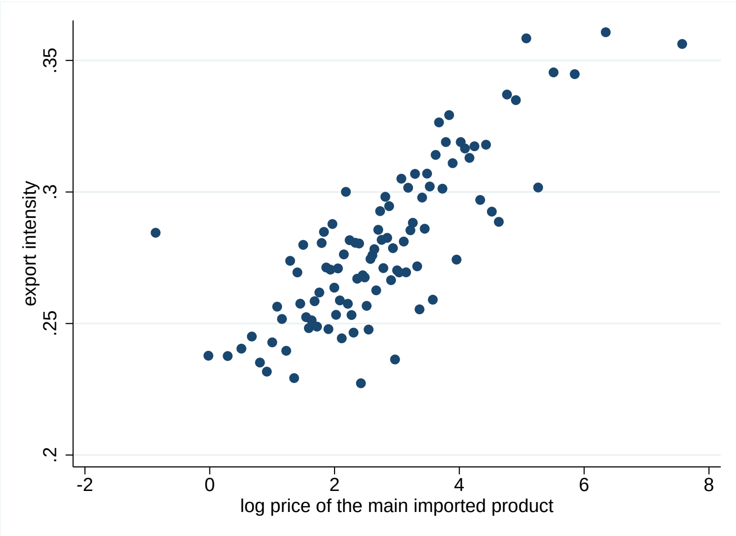


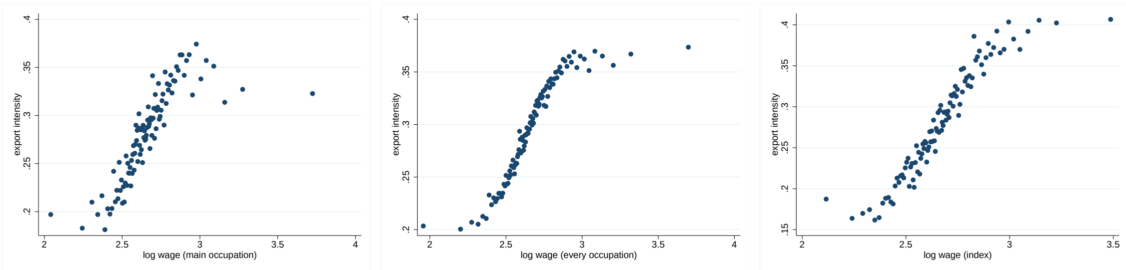
Figure 2: Price of imported intermediates and export intensity

In line with the evidence presented by Manova and Zhang (2012), more successful exporters employ factors of higher price. By exploiting the matched employer-employee structure of our dataset, we are able to show that a similar pattern holds also for firms' workers. After controlling for firms' characteristics (as evidenced by balance-sheet information), export intensity is positively correlated with the wage and the price of the main exported product is positively correlated with the wage. These relationships are shown in Figure (3) and Figure (4), respectively.[9] In panel (a) and (b) the unit of observation is the firm in a year, while in panel (c) the unit of observation is the triplet of a given occupation

[8] In the binned scatterplot of Figure (2) we control for HS4 sector of exported product, HS3 sector of the main imported product and year.

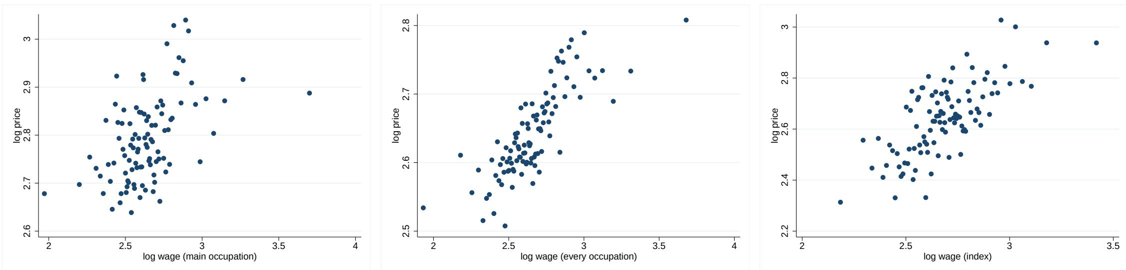
[9] In the binned scatterplot of Figure (2) and Figure (4) we control for the log of shares in total costs of capital costs, raw material costs, purchase of intermediate goods costs, other costs and for the log of revenue; in addition to a set of dummies for HS4 sector of exported product, HS3 sector of the main imported product and year.

type, in a firm, in a year. In the panels (a) and (c) we include a control for the 2-digit occupation which is more represented in the firm; in panel (b) we include dummies for all occupations in a firm, according with the 2-digit classification of occupations. In panels (a) and (b) the wage is the average hourly (net) compensation of workers in the main occupation or in each occupation respectively. In panel (c) we refer to a wage index that is constructed by weighting the average hourly wage among workers of a given occupation type with the relative use of a given occupation type.[10]



(a) (b) (c)

Figure 3: Wages and export intensity



(a) (b) (c)

Figure 4: Wages and price of the main export

Putting the two previous pieces of evidence together, it is natural to interpret the link from input prices to export intensity as flowing through output prices. We then run an

[10]For each occupation we compute the sum of wages for a given occupation in a given HS4-level sector, divided by the sum of wages in the same HS4-level sector, obtaining a weight, ω_occ_sec, for a given occupation in a given sector. For each firm we then sum the occupation and sector weights ω_occ_sec for the occupations represented in the firm. This yields the firm-level aggregate ω_firm. The ratio ω_occ_sec over ω_firm yields as many components as the number of occupations in a firm and it adds up to unity at the firm level by construction. The weights $\omega\_occ\_sec\_firm$ represent the relative use of a given occupation type in a firm, according with the patterns of the sector. The log wage index is obtained as the sum over all occupations in a given firm of $\omega\_occ\_sec\_firm$ times log $Wage_{itc}$.

OLS regression on the specification

$$\begin{aligned}\ln Exp\_Price_{it} = \quad & \alpha_L \ln Wage_{it\bar{c}(i)} + \alpha_N \ln Imp\_Price_{it} + \\ & + \psi_{s(i)} + \gamma_t + \vartheta_{\bar{c}(i)} + \vec{\zeta}\vec{Z}_{it} + \epsilon_{it\bar{c}(i)}, \qquad \forall i, t, \end{aligned} \tag{2}$$

where $Wage_{it\bar{c}(i)}$ is the mean wage paid by the firm i for the occupation $\bar{c}(i)$, and Imp_Price_{it} is the unit price of the principal import of the firm i at time t. Regression results are shown in Table (2). Columns (1)–(3) show specifications in which the unit of observation is the firm in a year. In specification (4) the unit of observation is the triplet of a given occupation type, in a firm, in a year. All specifications include sector, occupation and year controls.

After controlling for firm characteristics (Column 2), the elasticity of the export price on wage is 20% in expectation and it is highly statistically significant. Including the price of the main imported product, which controls for the quality of firm's input, (Column 3), lowers the estimate to 16% which remain highly significant. In Column 4, the unit of observation is no longer the firm, but the occupation within a firm. The estimate of the elasticity of export price on the wage is 18%, highly significant and quite similar in qualitative terms to the estimate at the firm level.

In general, a comparison between the Columns 3 and 4 shows that the pattern linking price and wage conditional on the intensity of non–labor factors and firm size looks similar whether it is investigated at the firm level looking at the main occupation or at the occupation level within a firm. This insight suggests that although on average there are 8 different types of occupation per firm (as shown by the ratio of the number of observations) a baseline framework which abstracts from the workforce composition can be sufficient to capture the salient feature of the positive relationship between price and wage.

Product differentiation and skill scarcity

We next focus on the distribution of quantities produced and sold at different price points. Figure 5, panel (a), shows the relationship between the log of export prices and the log of quantities sold by individual exporters; panel (b) shows the number of exporters at different price points.[11] The correlation between quantity and price is negative, with a slope that is greater than unity in absolute value, implying that total revenue is decreasing with price (the mean elasticity of revenue to price is approximately -17%). The number of exporting firms at a given price point falls with the export price. Taken together, these two pieces of evidence imply a negative relationship between the price of a variety and the quantity sold at that price point.

These patterns could be taken as evidence of vertical differentiation, i.e. given that consumers are willing to pay a price premium for higher quality products, the more productive

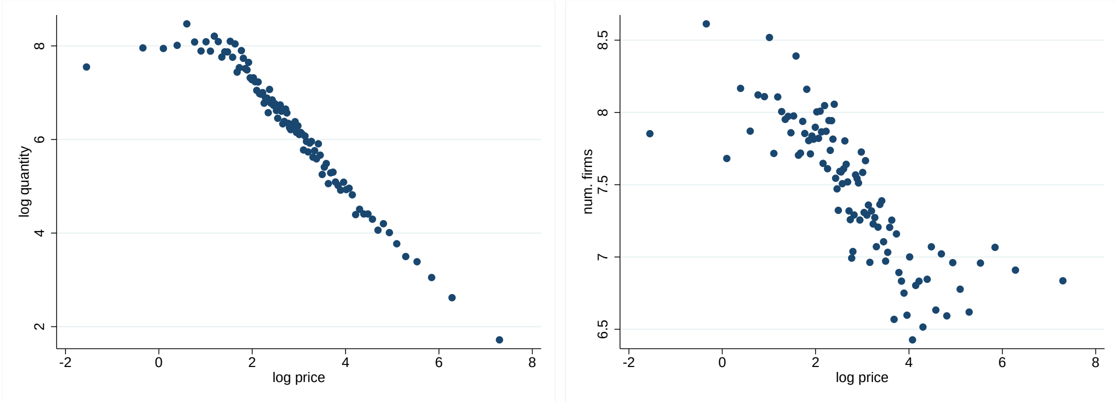
[11] In the binned scatterplot of Figure 5 we control for the HS4-level sector of exported product and year.

Table 2: Firms' wages and the price of their main export

log price	(1) Main occ.	(2) Main occ.	(3) Main occ.	(4) All occ.
log wage	0.00934	0.204***	0.163***	0.184***
	(0.0443)	(0.0433)	(0.0422)	(0.0253)
log capital share		0.0391***	0.0284*	0.0224*
		(0.0115)	(0.0113)	(0.0113)
log int. good share		-0.00193	0.00654	0.0101
		(0.00795)	(0.00771)	(0.00740)
log raw mat. share		-0.0472***	-0.0419***	-0.0476***
		(0.00903)	(0.00877)	(0.00840)
log other cost share		0.116***	0.103***	0.109***
		(0.0246)	(0.0230)	(0.0225)
log dom. revenue		-0.134***	-0.124***	-0.123***
		(0.00772)	(0.00742)	(0.00704)
log price imp.			0.164***	0.170***
			(0.00915)	(0.00903)
Adjusted R^2	0.487	0.501	0.512	0.526
Observations	25,218	24,880	24,286	197,522

Significance: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Note: Specifications (1)–(3) are at the $\{\text{firm}, \text{year}\}$ level and include dummies for HS4 sector of the main exported product, the HS3-level sector of the main imported product, the 2–digit PCS classification of the main occupation and the year. Specification (4) is at the $\{\text{firm}, \text{year}, \text{occupation}\}$ level and includes dummies for HS4-level sector of the main exported product, the 3HS sector of the main imported product, the 2-digit PCS classification of occupations for all occupations, and the year. Details on the data are reported in the appendix A. All specifications include a constant, standard errors are clustered at the HS4-level sector and reported in parentheses.



(a) Quantity of the main exported product

(b) Number of firms

Figure 5: Quantity and number of firms per level of price

(or innovative) firms choose to invest in quality upgrading as part of their profit-maximizing strategy. However, as detailed in Appendix D, for any observed decreasing relationship between price and quantity one can always find a representation of preferences consistent with symmetric horizontal differentiation. Horizontal differentiation is observationally equivalent to vertical differentiation, provided that the supply of horizontally differentiated good explains the observed price differentials. In turn, if product differentiation is based on skill differentiation, then product supply differentials – and hence price differentials across products – can be thought of as originating from differentials in the supply of different skills, and wage differentials across skills as reflecting comparative skill scarcity.

What we can see in the data is thus consistent with horizontal skill differentiation with comparative skill scarcity driving production choices and, through prices, export performance. In this interpretation, exporters do not choose to produce higher-priced goods and to employ higher-priced inputs to do so; rather, firms that produce goods that use comparatively scarcer factors end up exporting more. We formalize this idea in the next section.

3 Trade with input-based product differentiation

In this section we present a theory of trade driven by input-based product differentiation that directly links input prices (specifically wages) to output differentiation. This theory relies solely on the toolkits of neoclassical economics and Chamberlinian monopolistic competition theory and employs a minimal set of ingredients: (i) different skill types are differentially scarce; (i) manufactured goods are viewed by users of the goods as being (horizontally) differentiated according to the type of skill types with which they are produced; (ii) varieties that are produced using similar skill types are viewed by users as being closer substitutes than varieties that are produced with different skill types; (iii) trade costs involve activities that are less labour-intensive than manufacturing. We show that such a theory is able to produce a rich set of predictions on observables (values, prices and quantities) that match observed empirical patterns; specifically, it naturally predicts a positive correlation between export intensity and wages.

We first describe the model's basic structure, and then proceed to derive predictions about autarky equilibria and open-economy equilibria.

Endowments and markets

There are m identical countries, all with identical endowments.
In each country, $i \in (1, \ldots, m) \equiv M$, there is a continuum of labor skill types, $s \in [\underline{s}, \overline{s}] \equiv S$. The supply of each skill type is fixed and equal to $L(s) > 0$. Without loss of generality, let skill types be ordered so that $s'' > s' \Leftrightarrow L(s'') < L(s')$. For convenience, we assume that $L(s)$ is a differentiable function and is decreasing, i.e. $L'(s) < 0$. Additionally consumers are also endowed with non-manufactured consumables/inputs, totalling to $\overline{Y}$ in aggregate.

Preferences and market structure

Consumers in each country, i, have identical quasilinear preferences over manufactured and non-manufactured goods, represented by the utility function

$$U(i) \equiv Y + \frac{\zeta}{1+\zeta} Z(i)^{\frac{1+\zeta}{\zeta}}, \quad \zeta < -1, \tag{3}$$

where Y is consumption of non-manufactured goods, $Z(i)$ is composite consumption of manufactured goods, and ζ is the demand elasticity to price of manufactured goods. Each skill type, s, is associated with one and only one type, s, of manufactured goods, and each different good type, s, enters composite consumption of manufactured goods, M, through a constant-elasticity-of-substitution (CES) preference aggregation:

$$Z(i) \equiv \left(\int_{\underline{s}}^{\overline{s}} X(i,s)^{\frac{\sigma-1}{\sigma}} ds \right)^{\frac{\sigma}{\sigma-1}}, \tag{4}$$

where $X(i,s)$ is composite consumption of goods of type s, and $\sigma > 1$ is the elasticity of substitution across goods of different types. Notice that since every good type is produced with labor of only one skill level, one could also looks at σ as a measure of the elasticity of substitution between skill levels in consumption – and thus, indirectly, in labor demand.

For each good (and skill) type, there are $N(i,s)$ firms producing each a firm-differentiated variety of the given good type, s, in country $i \in M$. Given the total endowment, $L(s)$, of skill type s, the number of firms employing that particular skill is determined endogenously under conditions of free entry and exit, as discussed below. Under the assumption that $N(i,s)$ is large (the market for varieties of skill type s is 'thick', in Grossman-Helpman terminology), the set of firms producing varieties of each good type, s, is approximated by a continuum of firms over the set of varieties $j \in \bigcup_{i\in M}[0, N(i,s)]$, and so the market for every good type, s, is characterized by a monopolistic competitive structure. The consumption of different varieties of the same good type, s, enters the composite consumption aggregate, $X(i,s)$, of good type s in country i through to a CES preference aggregation, represented by

$$X(i,s) \equiv \left(\sum_{h\in M} \int_0^{N(h,s)} x(j,h,i,s)^{\frac{\eta-1}{\eta}} \, dj \right)^{\frac{\eta}{\eta-1}}, \quad \eta > 1, \tag{5}$$

where $x(j,h,i,s)$ is consumption of variety j of type s originating in country h by consumers in i, and η is the elasticity of substitution across different varieties of good type s.

Technology and labor markets

Each firm employs $l(j,i,s)$ units of labor and $y(j,i,s)$ units of non-manufactured goods to produce a level of output of variety (j,i,s) according to a Cobb-Douglas technology:

$$q(j,i,s) \quad \equiv \quad \left(\frac{l(j,i,s)}{\lambda}\right)^{\lambda} \left(\frac{y(j,i,s)}{1-\lambda}\right)^{1-\lambda}, \tag{6}$$

where $q(j,i,s)$ is output and $\lambda \in (0,1)$ is the share of labor in total production costs. Skill types are fully observable. Labor markets are frictionless and competitive, implying price taking behavior by firms and full employment. Since only workers endowed with skill s can be employed in firms producing goods of type s, labor markets are segmented by skill level. It follows that equilibrium wages will be skill specific, such that each skill type s in country i will be remunerated at level $w(i,s)$ and wages will be uniform within skill types.

In addition to variable costs, $v(j,i,s)$, every active firm incurs a fixed production cost consisting of $f > 0$ units of composite input. We assume a fixed cost function $w(i,s)^{\alpha} f$, with $\alpha \in (0,1)$, which implies that the fixed cost is always strictly positive and is a non-decreasing function of the wage. The total production cost function for a firm (j,i,s) equals

$$c(j,i,s) \equiv v(j,i,s) + w(i,s)^{\alpha} f. \tag{7}$$

Trade costs

Goods produced in a country and sold in the same country incur no transport costs, but there are transport costs for good that are exported. Specifically, in order to serve a certain export market, exporting firms must also combine manufactured goods with export services required to sell their goods in that market – such as developing market-specific marketing and advertising strategies, establishing and handling new distribution channels, complying with export-market specific laws, regulations, and standards. These export services are produced according to a Cobb-Douglas technology that employs non-manufactured goods and workers (of the same skill type as the good produced) as inputs, with a labor share of $\epsilon \in (0,1)$. Manufactured goods and export services are perfect complements, with the ratio of export ability per unit of exported output increasing with the level of trade barriers: one unit of exported output requires $b > 0$ units of export services.

This specification for the export technology is based on the idea that there is no substitutability between producing and exporting, which offers a simple interpretation of the measurement of export costs in the data. Taking our theory to the economic practice, we include as production those activities for which the seller is responsible in a *free–on–board* (FOB) contract. Instead the combination of production and export activities accounts for all the costs until the goods reach the destination market, that is a *cost–insurance–freight* (CIF) contract. If there is no substitutability between producing and exporting then the difference between the price of the two contracts $p_{CIF} - p_{FOB}$ identifies the export cost.

Given this export technologies, the marginal cost to an exporter of producing and exporting a unit of variety of type s is

$$w(i,s)^{\lambda} + b\, w(i,s)^{\epsilon} = \phi(i,s)\, w(i,s)^{\lambda}, \quad \phi(i,s) \equiv 1 + b\, w(i,s)^{\epsilon-\lambda}, \tag{8}$$

whereas the marginal cost of a variety sold in the domestic market is lower, $w(i,s)^{\lambda}$. Thus, $\phi(i,s) > 1$ is the factor of proportionality between the marginal cost of an exported variety over the marginal cost of the same variety sold in the domestic market. This offers a synthetic representation of the export technology as a function of the barriers to trade but also of the wage that the firm pays. An empirical assessment of export costs can be found in Hummels and Skiba (2004). Although they do not provide a micro-foundation of the export technology, their estimates of the impact of per-unit versus ad-valorem transport costs on trade flows are consistent with a cost function where export services are perfectly complementary with production. Appendix E discusses how evidence on the relationship between prices and export intensity for French exporters can be reconciled with the technology described by (8) and with Hummels and Skiba's empirical estimates.

Given the structure of transport costs, the total variable costs incurred by a firm can be expressed as

$$v(j,i,s) = w(i,s)^{\lambda}\Big(x(j,i,i,s) + \phi(i,s) \sum_{h \neq i} x(j,i,h,s)\Big). \tag{9}$$

Industry equilibrium

An equilibrium for this economy is then identified by a combination of a number, $N(i,s)$, of firms producing varieties for each good/skill type and wage levels, $w(i,s)$, for every skill type $s \in S$ in each country $i \in M$; output prices, $p(j,i,s)$, and an allocation, $\left\{q(j,i,s), l(j,i,s), y(j,i,s)\right\}$, of output and factors' employment levels, for every good variety and firm (j,i,s), $j \in [0, N(i,s)]$, $i \in M$, $s \in S$; and an allocation, $x(j,i,h,s)$, of consumption quantities for every good variety (j,i,s), $j \in [0, N(i,s)]$, $i \in M$, $s \in S$, and destination market $h \in M$; such that

(i) quantities consumed, $x(j,i,h,s)$ for every variety (j,i,s), $j \in [0, N(i,s)]$, $i \in M$, $s \in S$, maximize utility for every consumer in $h \in M$;

(ii) prices, $p(j,i,s)$, maximize profits for every firm (j,i,s), $j \in [0, N(i,s)]$, $i \in M$, $s \in S$;

(iii) revenues equal total costs for every firm (j,i,s), $j \in [0, N(i,s)]$, $i \in M$, $s \in S$;

(iv) product markets clear for every produced variety (j,i,s), $j \in [0, N(i,s)]$, $i \in M$, $s \in S$;

(v) labor markets clear for every skill type $s \in S$ in every country $i \in M$.

3.1 Autarky equilibrium

We are now in the position to characterize an equilibrium for this economy under conditions of autarky.[12] We outline the derivation of equilibrium relationships in some details for the autarky case; the corresponding derivations for the open economy case are given in the appendix. The index i is not relevant in this context and can be omitted.

Normalizing the price for non-manufactured goods to unity, the demand for manufacturing goods is:

$$Z = (P_Z)^{\zeta}, \tag{10}$$

where P_Z is the composite price index of manufactured goods. The demand for a good type composite $X(s)$ is then given by

$$X(s) = Z \left(\frac{P_Z}{P(s)}\right)^{\sigma}, \tag{11}$$

where $P(s)$ is the composite price index of a good with skill content s. The demand for variety j of good type s is then given by

$$x(j,s) = X(s) \left(\frac{P(s)}{p(j,s)}\right)^{\eta}, \tag{12}$$

[12] Autarky means that the set of markets of origin is destination-country specific and equals $M(i) \equiv \{i\}$ in this case.

where $p(j,s)$ is the price of variety j of good type s. The price index for manufacturing goods is

$$P_Z = \left(\int_{\underline{s}}^{\bar{s}} P(s)^{1-\sigma} \, ds \right)^{\frac{1}{1-\sigma}}, \tag{13}$$

such that $P_Z \, Z = \int_{\underline{s}}^{\bar{s}} P(s) \, X(s) ds$ is total expenditure in the manufacturing sector. The price index for varieties of goods with skill content s is:

$$P(s) = \left(\int_0^{N(s)} p(j,s)^{1-\eta} dj \right)^{\frac{1}{1-\eta}}, \tag{14}$$

such that $P(s) \, X(s) = \int_0^{N(s)} p(j,s) \, x(j,s) \, dj$ is the total expenditure in the good associated to the skill level s.

For given output levels, $q(j,s)$, by each firm, (j,s), the implied firm-level input demands are

$$l(j,s) = \lambda \, w(s)^{\lambda-1} \, q(j,s) + \alpha \, w(s)^{\alpha-1} f, \tag{15}$$

$$y(j,s) = (1-\lambda) \, w(s)^{\lambda} \, q(j,s) + (1-\alpha) \, w(s)^{\alpha} f. \tag{16}$$

Profit maximization by firms implies the pricing rule

$$p(j,s) = \frac{\eta}{\eta - 1} \, w(s)^{\lambda}; \tag{17}$$

From (17), and by quasi-concavity and symmetry of preference, firms producing varieties of the same good type, s, will all charge the same price $p(s)$. Thus, the composite price for good type s in equilibrium is simply

$$P(s) = N(s)^{-\frac{1}{\eta-1}} \, p(s); \tag{18}$$

and all firms producing varieties of good type s will be of identical market size and employ the same amount of labor of skill type s:

$$x(j,s) = x(s) = q(j,s) = q(s) = N(s)^{-\frac{\eta}{\eta-1}} \, X(s), \tag{19}$$

(from (5)), and

$$l(j,s) = l(s) = \lambda \, w(s)^{\lambda-1} q(s) + \alpha \, w(s)^{\alpha-1} f \tag{20}$$

(from (15)).

Defining the aggregate output as $Q(s) = N(s) \, q(s)$, and exploiting the fact that the variable cost function is linear in output to express total variable costs incurred by all firms producing varieties of good s as $V(s) = w(s)^{\lambda} \, Q(s)$, we can write the the demand of labor as a variable factor as

$$L_v^D(s) = \lambda \, w(s)^{\lambda-1} Q(s) = \lambda \, \frac{V(s)}{w(s)}, \tag{21}$$

labor demand is therefore increasing in aggregate output. Combining (19) and (18) with (10), (11) and (17), we can write

$$Q\left(s\right)=P_Z{}^{\zeta+\sigma}\left(\frac{\eta}{\eta-1}\right)^{-\sigma}N(s)^{\frac{\sigma-1}{\eta-1}}\,w(s)^{-\sigma\lambda} \tag{22}$$

which characterizes aggregate output.

The revenue of a firm (j,s) is given by $r(j,s)=\dfrac{\eta}{\eta-1}v\Big(q(s),w(s)\Big)$ for all j (from (17) and (19)). Therefore, total revenues by all firms selling varieties of good s can be expressed as $R(s)=N(s)\,r(s)=\dfrac{\eta}{\eta-1}\,V(s)=\dfrac{\eta}{\eta-1}\,w(s)\,L_v^d(s)/\lambda$.

The equilibrium number of firms producing varieties of a given good type, s is determined by the zero profit condition, which requires $R(s)=V(s)+N(s)\,w(s)^{\alpha}f$. solving for $N(s)$, we obtain the number of firms as a function of variable labor input and wage: $N(s)=\dfrac{w(s)^{1-\alpha}\,L_v^D(s)}{\lambda\,(\eta-1)f}$. Labor market clearing requires $L_v^D(s)+N(s)\,\alpha\,w(s)^{\alpha-1}\,f=L(s)$. Using this condition, the equilibrium number of firms as a function of labor endowment and wage for s can be expressed as

$$N(s)=\frac{w(s)^{1-\alpha}\,L(s)}{\left(\lambda(\eta-1)+\alpha\right)f}, \tag{23}$$

where $N(s)$ is increasing in both $w(s)$ and $L(s)$.

Substituting (23) in (22) yields total output of good s, $Q(s)$ as a function of wage and labor endowment only. We can solve the resulting equality to obtain an expression for $w(s)$ as a function of the endowment level of skill s and of a multiplicative factor that is determined endogenously in equilibrium but is independent of s:

$$w(s)=\Omega^{\frac{1}{\Theta}}P_Z{}^{\frac{\zeta+\sigma}{\Theta}}\left(\frac{\lambda(\eta-1)}{\lambda(\eta-1)+\alpha}\,\frac{L(s)}{\lambda}\right)^{-\frac{1-\theta}{\Theta}}, \tag{24}$$

where $\Omega\equiv\eta^{-\sigma}\,(\eta-1)^{\sigma-\theta}\,f^{-\theta}$ and $\Theta\equiv(1-\lambda)(1-\theta)+\lambda(\sigma-\theta)+\alpha\theta$, $\theta=(\sigma-1)/(\eta-1)$. Notice that labor productivity, defined as $N(s)\,q(s)/L_v^d(s)=(1/\lambda)\,w(s)^{1-\lambda}$, is an increasing function of the wage.

We can now characterize the relationship between skill scarcity, wages and prices:

Proposition 1 *If varieties for each good/skill types are closer substitutes than varieties across across good/skill types are ($\eta>\sigma$), then skills that are comparatively scarcer are remunerated with higher wages and firms that employ them charge higher prices for their products.*

(Proofs of propositions are given in the appendix.)

This setup thus characterizes an economy in which varieties produced with scarce skills attract a price premium. A competitive labor market then implies that a higher output price translates into a higher factor price. This requires that varieties produced with similar skill types (i.e. varieties that have a similar price) be comparatively closer substitutes than are varieties produced with different skill types (and have a different price). In what follows we restrict our discussion to scenarios in which $\eta > \sigma$, and where therefore Proposition 1 holds.

Further properties of an equilibrium allocation of output and employment can be derived:

Proposition 2 *Given that Proposition 1 holds, then:*

(i) Average labor productivity, $q(s)/l(s)$, is higher in firms employing comparatively scarcer skills.

(ii) If fixed costs include labor costs ($\alpha > 0$) then revenue per firm, $r(s)$, is higher for firms employing comparatively scarcer skills; otherwise revenue per firm is independent of the wage level (and of productivity).

(Proofs of propositions are given in the appendix.)

Thus, measured productivity per worker will be higher in firms that produce higher-priced goods and employ higher-wage workers. Higher wages (and higher productivity per worker) may also be associated with a larger firm size, but not necessarily so.

3.2 Open-economy equilibrium

We next characterize an open economy equilibrium. We use "^" to refer to equilibrium values in open economy.

The assumption of symmetric countries means that the producer price and the net-of-transport cost consumer price of varieties of goods of a given type s will be the same everywhere – and so the source and destination market indices can be omitted in the derivations that follow. However, the transport-cost-inclusive price, $\hat{p}^*(s)$, for consumers of a foreign produced variety is higher than the corresponding price, $\hat{p}(s)$, of a domestically produced variety of the same type, i.e.

$$\hat{p}^*(s) = \phi(s)\,\hat{p}(s), \tag{25}$$

where $\phi(s) = \phi(i, s),\ i \in M$ is as defined in (8), and captures the wedge in the marginal cost of an exported variety with respect to a non-exported one. Because of this wedge, varieties of imported goods and domestically produced goods will be consumed in different proportions.

Proceeding as for the autarky case, but now accounting for the effect on consumer choices of the domestic-to-imported good price wedge, and for the additional indirect demand for labor that is associated with the production of export services, we obtain expressions for the equilibrium number of firms producing varieties of type s and the wage for skills of type s:[13]

$$\hat{N}(s) = \frac{\hat{w}(s)^{1-\alpha}\, L(s)}{\Big((\eta-1)\,(\lambda+\delta(s))+\alpha\Big)\, f}; \tag{26}$$

$$\hat{w}(s) = \Omega^{\frac{1}{\Theta}}\, \hat{P}_Z^{\frac{\zeta+\sigma}{\Theta}} \Big(\Delta(s)\,\Phi(s)\Big)^{\frac{\theta}{\Theta}} \left(\frac{(\eta-1)\,(\lambda+\delta(s))}{(\eta-1)\,(\lambda+\delta(s))+\alpha}\;\frac{L(s)}{\lambda+\delta(s)}\right)^{-\frac{1-\theta}{\Theta}}, \tag{27}$$

where $\hat{P}_Z$ is a CES price aggregation for the open-economy case corresponding to (13); $\Delta(s) \equiv \left(1+(m-1)\,\phi(s)^{1-\eta}\right)/\Phi(s) > 1$ captures the gap in the variable cost due to export, $\delta(s) \equiv \dfrac{\partial\Delta(s)}{\partial\hat{w}(s)}\,\dfrac{\hat{w}(s)}{\Delta(s)}$ is the corresponding elasticity to wage; and the wedge in the marginal cost for exported varieties is also reflected in the term $\Phi(s) \equiv 1+(m-1)\phi(s)^{-\eta}$.

The equilibrium condition (27) yields an implicit function of the wage, through $phi(s)$. However, the positive relationship between factors scarcity, factor prices, and good prices that we have derived for the autarky case applies also to the open-economy case.

Lemma 1 *The results of Proposition 1 extend to the open economy equilibrium: Skills that are comparatively scarcer are remunerated with higher wages and firms that employ them charge higher prices for their products.*

(Proofs of lemmas are given in the appendix.)

The relative labour intensity of manufacturing and export services, however, is crucial in determining how wages and prices are related to export performance. To see this, we must focus on the comparison between a firms' revenues from domestic sales, $r^D(s)$, and its export revenues, $r^E(s) = (m-1)\phi(s)^{1-\eta}\, r^D(s)$. A firm's export instensity can then be measured by the ratio of export revenues to total revenues, $\rho(s) = \dfrac{\hat{r}^E(s)}{\hat{r}^D(s)+\hat{r}^E(s)}$; after simplification, we can write $\rho(s)$ simply as

$$\rho(s) = \left(1+\frac{\phi(s)^{\eta-1}}{m-1}\right)^{-1}. \tag{28}$$

Export intensity is higher the larger is the number of destinations and the lower is $\phi(s)$.

Proposition 3 *If manufacturing is more labor-intensive than the production of export services ($\lambda > \epsilon$), then firms that pay comparatively higher wages export a comparatively greater proportion of their output.*

[13]Detailed derivations are given in the appendix.

The properties stated in Proposition 2 for the autarky case also apply to the open economy case. Hence, for $\alpha > 0$, Propositions 2.ii and 3 together imply a positive relationship between firm size (in terms of revenue) and export performance:

Lemma 2 *If fixed costs include labor costs ($\alpha > 0$) then those firms that export a comparatively greater proportion of their output and pay higher wages (and exhibit higher productivity) are comparatively larger firms; otherwise export intensity is independent of firm size.*

(Proofs of lemmas are given in the appendix.)

Propositions 1–3 and the Lemmas 1–2 characterize the positive correlation between price, wage and export intensity, which we documented in Section 2. Furthermore, they also show how this framework can generate predictions on the relationship between firm productivity, firm size and export performance which are not based on exogenous idiosyncratic sources of heterogeneity. In contrast, the model explains firm heterogeneity in productivity as a consequence of how scarce is the level of skill that the firm employs. This channel alone will be sufficient to predict that scarcer factors receive higher compensation and goods produced with scarcer factors are more expensive. An export technology through which the model embeds an Alchian-Allen effect is necessary though to account for the better export performances by firms charging higher prices.

For an Alchian-Allen effect to be present, and so for higher-priced varieties to be traded comparatively more than lower-priced varieties, the incidence of export cost $b\,w(s)^{\epsilon}$ in the total consumer price in the destination market, $\phi(s)\,p(s)$, must be decreasing in the producer price, $p(s)$ – and thus in the wage, $w(s)$. Given our specification of $\phi(s)$, this is the case if and only if production is more labor intensive than the export service $\lambda > \epsilon$. In the following discussion, we shall assume that this parametric restriction is satisfied.

3.3 Skill scarcity as a predictor of trade performance

In the model there is no unemployment, and so the exogenous endowment of a given type of worker represents at the same time the aggregate demand and the aggregate supply of the corresponding skill. We characterize scarcity in the data as the proportion of a given type of occupation over the population of employed workers.[14] For each occupation level in a given year we discretize the wage support by specifying one-hundred equally spaced intervals. We then count the number of workers in each wage interval, per occupation per year. The inverse of the number of workers is our measure of scarcity for a given wage

[14]We restricted the analysis to workers who are (full time) employed over the entire 4–year period. Thus there is no selection bias on this margin.

interval, within a given occupation in a given year. We finally rank the wage intervals by their scarcity and attach to each worker (of a given occupation in a given year) the position in the ranking of the wage interval in which her compensation falls. Each worker is then assigned to a percentile of scarcity, with scarcer workers at the higher percentiles. Thus, a worker is considered scarcer than another worker if the wage bracket of the former is populated by fewer workers than the wage bracket of the latter within the same 2-digit occupation and year.

Although the wage is the pivot variable to define a skill type, we are not imposing any restriction on whether a scarcer skill level must translate into a higher wage. Moreover, firm affiliation does not play a role in the determination of worker's scarcity. Even if there was a pattern at the firm level which correlated the wage of workers within a given occupation in a given year, the number of firms is so large (in the order of thousands, per occupation per year) that this pattern would not drive the assignment of a given worker to a percentile of scarcity economy-wide.[15]

We then compute for each firm the average scarcity across its employees for a given occupation. This yields an average level of scarcity per occupation in that firm. Using these measures, we carry out OLS regressions of export intensity on scarcity using the following specification:

$$\ln Exp\_Intensity_{it} = \nu \ln Scarcity_{it\overline{c}(i)} + \psi_{s(i)} + \vartheta_{\overline{c}(i)} + \gamma_t + \vec{\zeta}\vec{Z}_{it} + \epsilon_{it}, \quad \forall i, t, \tag{29}$$

where $\overline{c}(i)$ is the occupation that is most represented within the firm. Results are reported in Table 3, and show that the scarcity measure we constructed is a good predictor of export intensity. A comparison with Table 1 reveals that scarcity can account for the variation in export intensity which was previously explained in terms of variation in the export price. The two regressions deliver substantially the same conclusion on the role played by all other determinants, with virtually identical coefficients.

Our theory predicts that a firm's export intensity is linked to its export price. In turn, the price of the variety sold by the firm is determined by the wage the firm pays for the skills that are embedded in that variety, which is ultimately determined by comparative input scarcity through the simple interplay of demand and supply. Importantly, our measure of scarcity is not driven by either firm-level or sector-level patterns, as it is constructed at the economy-wide level of workers' occupation (regardless of firm and sector). To the extent that the characteristics of the economy-wide supply of workforce can be thought

[15]Similar considerations apply to sectoral patterns. Geographical characteristics instead might be important, because the market segmentation might cluster workers regardless their occupation, sector and employer in a given year. However, constructing the scarcity percentile conditional at the province level (NUTS3 classification, the department in the French jurisdiction) does not alter the results.

Table 3: Skill scarcity as a predictor of export intensity

log exp. intensity	(1) No firm controls	(2) Firm controls	(3) Price imported good
log scarcity	0.0446*	0.0505***	0.0473***
	(0.0219)	(0.0233)	(0.0236)
log capital share		0.200***	0.194*
		(0.0148)	(0.0147)
log int. good share		0.0104	0.0117
		(0.0100)	(0.0103)
log raw mat. share		0.0884***	0.0912***
		(0.0105)	(0.0107)
log other cost share		- 0.01183	-0.0248
		(0.0245)	(0.0249)
log dom. revenue		-0.126***	-0.127***
		(0.00944)	(0.00966)
log price imp.			0.0771***
			(0.00802)
Adjusted R^2	0.114	0.144	0.148
Observations	28,279	23,230	22,666

Significance: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Note: All specifications are at the $\{\text{firm}, \text{year}\}$ level and include dummies for HS4 sector of the main exported product, the HS3 sector of the main imported product, the 2–digit PCS classification of the main occupation and the year. Details on the data are reported in the appendix, section A. All specifications include a constant, standard errors are clustered at the HS4 sector and reported in parentheses. In order to reduce the effect of possible outliers, the constructed measure of scarcity is winsorized at 1% level.

as exogenous to the decisions made by the individual firm, then input scarcity can be considered a an exoegenous determinant of firm price, wage and export performances.[16]

A clear advantage of this approach is that segmentation of the workforce by occupation types is an observable. Matched employer-employee data are then sufficient to construct an occupation- and year-specific measure of scarcity that is unrelated with firm characteristics, provided that the number of firms is sufficiently large. Based on the implications of our theoretical framework, this measure of scarcity will be informative of export success, of unobservable quality (higher priced varieties within the same product category) and also of unobservable labor productivity (as is discussed in the following section).

3.4 Trade effects on the wage distribution

The model also delivers predictions on how the wage distribution is affected by trade liberalization. These hinge on two features: (i) higher substitutability across varieties of the same good type, s, than between good types ($\eta > \sigma$), and (ii) the comparative labor content of manufacturing and export services ($\lambda > \epsilon$). To see this, consider the ratio between the (real) wage of the same skill in open economy and in autarky,

$$\frac{\hat{w}(s)}{w(s)}\left(\frac{P_Z}{\hat{P}_Z}\right)^{\frac{\zeta+\sigma}{\Theta}} = \left(1+(m-1)\phi(s)^{1-\eta}\right)\left(\frac{(\eta-1)\lambda+\alpha}{(\eta-1)(\lambda+\delta(s))+\alpha}\right)^{-\frac{1-\theta}{\Theta}}, \tag{30}$$

as a measure of gains from trade. Abstracting from the distribution of skill types, and considering the extremes of the support $\delta(s) \in (-\lambda, 1-\lambda)$ for $\phi(s) > 1$, shows that the model predicts gains from trade, for a non-trivial number of foreign markets.

Proposition 4 *The real wage is higher in open economy for both low and top income earners. The effect of trade on the real wage, however, does not have to be either positive for all skills types, or monotonically increasing or decreasing in s.*

(Proofs of propositions are given in the appendix.)

However, the effect of trade on real wage is decreasing in $\phi(s)$ and increasing in $\delta(s)$, and both are decreasing functions of the wage. This introduces the possibility of non-monotonicity discussed in Proposition 4.

[16] It might be argued that firms locate where the characteristics of the workforce are more suitable for their intrinsic nature. Still, this would only matter as long as the location, and reallocation, of firms occurs at least as often as the changes in the workforce composition. Conducting the same analysis by excluding firms that were established after 2005 (so up to five years before the sample period) does not change the results.

On these premises it is natural to investigate what can be learned from the model to the end of explaining trade–induced polarization in the wage distribution. The answer is in the following proposition.

Proposition 5 *If the endowment of the most abundant skill, s_0, is such that*

$$\phi(s_0) \leq \frac{\eta}{\eta - 1} \quad \Longleftrightarrow \quad \hat{w}(s_0) \geq \Big(b(\eta - 1)\Big)^{\frac{1}{\lambda - \epsilon}},$$

then the Alchian-Allen effect is sufficiently strong that the gap in variable cost, $\Delta(s)$, is lower the higher the wage, i.e. $\delta(s) < 0$. If this condition is met, then:

(i) if skill-based product differentiation is weak, i.e. for $\sigma \to \eta$, and the export market is sufficiently wide, i.e. m is sufficiently large, trade liberalization increases dispersion over the entire wage distribution;

(ii) if the difference of labor intensity between manufacturing and the production of export services is small, i.e. for $\epsilon \to \lambda$, and the export market is sufficiently international market is sufficiently narrow, i.e. for $m \to 1$, trade decreases dispersion over the entire wage distribution;

(iii) if the previous conditions are not met, and so $d\Big(\hat{w}(s)/w(s)\Big)/ds$ is non-monotonic in s, then trade liberalization induces more dispersion at the bottom of the wage distribution and decreases dispersion at the top of the wage distribution.

(Proofs of propositions are given in the appendix.)

It is possible to show that eliminating the Alchian-Allen effect – focusing on the limit for $\epsilon \to \lambda$ – turns off the contribution of $\phi(s)$, leading to lower real wage gains for scarcer factors. Instead, reducing the importance of skill-based product differentiation $\theta \to 1$ dampens the contribution of $\delta(s)$, leading to higher gains for scarcer factors. In the limit for $\sigma \to \eta$, the model features a single-wage equilibrium and gains from trade are unambiguous. Thus, depending on the patterns of skill based differentiation (σ versus η) and export technology (ε versus λ) of the sector for which trade exposure increases the model predicts either wage-concentration effects or wage-polarization effects from trade liberalization.

3.5 Trade effects on labour productivity

Unlike in theoretical frameworks where firm heterogeneity derives from an exogenously given distribution of factor-neutral productivity types, in this model trade liberalization changes the distribution of measured labor productivity. To characterize these changes, we can focus on the ratios $\dfrac{\hat{N}(s)\,\hat{q}(s)/\hat{l}(s)}{N(s)\,q(s)/l(s)}$. As the numbers of firms of any given type, s, change endogenously with trade liberalization, they must be included as weights (unlike the number of workers of skill type s, which is exogenous). The ratios

$$\frac{\hat{q}(s)/\hat{l}_v(s)}{q(s)/l_v(s)} = \frac{\lambda}{(\lambda + \delta(s))\Delta(s)} \left(\frac{\hat{w}(s)}{w(s)}\right)^{1-\lambda}$$

and

$$\frac{\hat{N}(s)}{N(s)} = \frac{(\eta - 1)\lambda + \alpha}{(\eta - 1)(\lambda + \delta(s)) + \alpha} \left(\frac{\hat{w}(s)}{w(s)}\right)^{1-\alpha}$$

are both proportional to the wage ratio (30), and in both cases the factor of proportionality is positive and increasing in the wage when the conditions of Proposition 5 hold – i.e. when $\delta(s) < 0$. Therefore, under those conditions, the model predicts that trade changes the shape of the productivity distribution across firms, even after controlling for changes in the wage distribution across skills. This would not be the case if transportation costs were of the iceberg type.

4 Generalization and extensions

Idiosyncratic technology

The theory can also readily accommodate exogenous idiosyncratic technology differentials, which adjust marginal cost by a firm-specific level of total factor productivity. The wage will be the same across workers endowed with the same skill but heterogeneity in productivity efficiency will translate into heterogeneity in prices, revenues and profits across firms producing varieties that use the same skill. This will imply a weakening of the relationship between input prices and output prices.[17]

Multiple occupations

Our arguments have been developed for a setup with a single category of labour inputs. In practice production combines workers with different occupations each carrying out different tasks. With multiple categories of workers the simple one-to-one mapping between input types and output types that arises in the single-input case needs be extended to allow for differentiation based on different *combinations* of input types within categories. To illustrate, suppose that there are two separate tasks, $t \in \{A, B\} \equiv T$, that need to be carried out to produce a good, each involving workers from different occupational categories; and suppose that within each occupational categories there are two differentiated

[17] A hybrid formulation of this type could be used to obtain structural estimates of the implied contribution of input-based differentiation to output price variation.

skill types, $s_t \in \{L, R\},\ t \in T$. Then, in principle, there are four possible input combinations: $(L_A, L_B),\ (L_A, R_B),\ (R_A, L_B),\ (R_A, R_B)$. If we take each of the above input combinations as translating into a distinct, horizontally differentiated variety, we can use the same modelling strategy that we have employed for the single-input case to derive predictions that link input scarcity to wages, prices and export intensity. Note, however, that this extension does not necessarily generate a prediction of positive assortative matching across inputs in terms of their price (i.e. comparatively scarcer, higher-priced inputs in a given being used in conjunction with comparatively scarcer, higher-priced inputs in a different category), but it can do so depending on substitution possibilities across input categories.[18]

Extensive-margin export choices

The main motivation behind our modelling exercise is to explain variation on intensive margin; but if we additionally posit that firms must incur a fixed cost, F_X, for exporting (as Melitz, 2003) our model also readily produces predictions concerning selection of firms into export markets. As potential export revenues are increasing with s, and since gross profits are proportional to revenues, there will be a cutoff level, $\tilde{s}$, lying between $\underline{s}$ and $\overline{s}$ above which firms will choose to export and below they will choose not to do so (as long as F_X lies between the respective levels of potential profits of firms producing $\underline{s}$ and $\overline{s}$-type varieties). With $\alpha = 0$, the variation in export revenues is unrelated to size, and so selection into export markets will be unrelated to form size (unlike in Melitz, 2003), but with $\alpha > 0$ firms with higher export revenues are also larger firms, and so selection into exporting will be positively related to size.

[18] Suppose that tasks/occupations A and B are highly substitutable in production, and that workers of skill type R within each occupational category are scarcer (and hence command a higher wage) than those of skill type L. Then in producing the variety corresponding to combination (R_A, R_B) there will be no scope for substituting lower-wage workers for higher-wage workers. However, for combination (L_A, R_B), more abundant, lower-wage L_A workers can be substituted for less abundant, higher-wage R_B workers; and for combination (L_B, R_A), more abundant, lower-wage L_B workers can be substituted for less abundant, higher-wage R_A workers. Then, if the elasticity of substitution between varieties is smaller than the elasticity of substitution between worker categories, a larger proportion of R-type workers will be matched with other R-type workers than they are with L-type workers, which will be reflected in a positive correlation between wages paid to different worker categories within firms.

5 Concluding remarks

This paper develops a model of trade that explains the positive correlation between wages, prices and share of foreign sales on total sales that has been empirically observed in cross-sectional firm level data (see Manova and Zhang (2012)). Current trade models struggle to rationalize these empirical patterns in a unifying framework, even when accounting for imperfections in the labor market or quality upgrading. These models feature factor-neutral heterogeneity in productivity across firms and price-neutral transport costs, two features that translate into predictions that are difficult to reconcile with observed wage differentials or quality differentials and with the observed relationship between trade costs and price. In a model of trade in which goods are horizontally differentiated based on the skill required to produce them output prices and input prices become naturally non-orthogonal. Among firms producing goods that employ the same skill type there exists monopolistic competition similar to Krugman (1979), with the only (but important) difference that transport costs are not of the iceberg type. In particular, by modeling the provision of export services we show that the transport cost margin in equilibrium is a decreasing function of firm wage, under fairly plausible conditions. The relative scarcity of skill types determines the wage firms pay to employ workers with a given skill type. The scarcer a skill type, the higher the wage the firm pays for it and the higher price the firm charges for goods that are produced using that skill type. The combination of non-iceberg transport technologies, skill-type based product differentiation and comparative skill scarcity is then sufficient to explain the observed empirical patterns in wage, price and export intensity.

[TBC]

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Appendix

A Data description

A.1 Firm-level data

Information on firms' characteristics comes from the FARE dataset, containing exhaustive information from firms' income statements and balance sheets, and also reports the location of firms in France and their 4-digit sector of main activity. Industries are classified according to the *Nomenclature d'activités française* (NAF), which is consistent with the NACE Rev. 2 up to the 4th digit. We restrict the analysis to manufacturing firms, from Section 10 "*Industries alimentaires*" to Section 32 "*Autres industries manufacturières*" of the NACE classification, for a total of about 37,000 firms per year (on average). As a definition of an industry we consider the 3-digit and 4-digit specifications, which respectively yield 93 and 928 groups within manufacturing.[19]

A.2 Workers and wages

The data on employees consists of administrative data reported by employers and collected by the INSEE (the statistical French authority) within the dataset DADS-postes. They covers the universe of workers who ever had a labor contract in France. We use data for four years, from 2010 to 2013. In any given year, the total number of workers is over 55 millions. We focus on the population of full-time workers who are employed for more than 240 days of the year. Workers are classified according to the *Professions et Catégories Socioprofessionnelles* (PCS), which distinguishes manufacturing workers into five broad categories: (2) *Artisans*, (3) *Cadres et professions intellectuelles supérieures*, (4) *Professions Intermédiaires*, (5) *Employés*, and (6) *Ouvriers*; each group is in turn subdivided into finer categories, up to 4 digits, yielding a detail of 465 categories.[20] In addition to information about wages and occupation, the DADS reports information on individual characteristics such as age, gender, and region of employment. Worker records include a firm identifier, making it it possible to mach employee data to employer data. The number of observations for workers employed in manufacturing firms is about 5.2 million per year on average.

A.3 Customs data

Data on import and exports by French firms are provided by the French Customs for the period 1995-2005. The information includes transaction records (detailing both transaction values and unit prices) at firm, product and market level for the universe of French firms (those located in France). Product categories are classified using Combined Nomenclature at 8 digits (CN8). We

[19]Further information can be found at https://www.insee.fr/fr/information/2406147.

[20]Further information can be found at https://www.insee.fr/fr/information/2400059.

restrict attention to manufactures – Chapters 16 to 24, and 28 to 96 in NC8, corresponding to Sections 10 to 32 in NACE.

B Proofs of propositions

In this Section we state proofs of the results presented in the text, as well as derivations of additional results that are not discussed in the main text.

Proof of Proposition 1

Autarky equilibrium

The derivation of (23) and (24) is discussed in the text. From (24) notice that $(1-\theta)/\Theta \geq 0$ if and only if $\theta \leq 1-\lambda+\lambda\sigma$; and since $\sigma > 1$, then $1-\lambda+\lambda\sigma > 1$. The condition $\eta > \sigma$ is a necessary and sufficient condition for $0 < \theta < 1$ for every $\sigma > 1$. Therefore, the wage equation (24) implies that the wage is higher for scarcer skills. Price (17) is increasing in $w(s)$. □

Proof of Lemma 1

Open-economy equilibrium

Using the fact that prices are the same for all firms producing goods of type s, the composite price for all varieties of type s consumed in a given market (domestically produced or imported) can be expressed as

$$\hat{P}(s) = \left(\hat{\Gamma}(s)\,\hat{N}(s)\right)^{-\frac{1}{\eta-1}}\,\hat{p}(s), \tag{31}$$

where $\hat{\Gamma}(s) \equiv \left(1+(m-1)\,\phi(s)^{1-\eta}\right) > 1$ is the ratio between the number of consumed varieties in open economy over the number of varieties produced by domestic firms and $\hat{p}\left(s\right) = \frac{\eta}{\eta-1}\hat{w}\left(s\right)^{\lambda}$ is the price of a domestic variety in the domestic market, which satisfies (17) at the equilibrium wage in open economy $\hat{w}\left(s\right)$. Conditions (10), (11), (17), (31) yield the consumption aggregate in a given market:

$$\hat{X}(s) = \left(\eta^{-\sigma}\left(\eta-1\right)^{\sigma}\right)\hat{P}_Z^{\zeta+\sigma}\left(\hat{\Gamma}(s)\hat{N}(s)\right)^{\frac{\sigma}{\eta-1}}\hat{w}(s)^{-\lambda\sigma}. \tag{32}$$

Domestic market demand for a domestically produced variety with skill s in open economy is $\hat{x}^D(s) = \hat{X}(s)\left(\hat{P}(s)/\hat{p}(s)\right)^{\eta}$, while export market demand is $\hat{x}^E(s) = (m-1)\phi(s)^{-\eta}\hat{x}^D(s)$. Production equals total demand from domestic and foreign consumers $\hat{q}(s) = \hat{x}^D(s) + \hat{x}^E(s)$. Let $\hat{Q}(s) = \hat{N}(s)\hat{q}(s)$ then the aggregate production of good s is given by:

$$\hat{Q}\left(\hat{N},\hat{w}\right) = \eta^{-\sigma}\left(\eta-1\right)^{\sigma}\hat{P}_Z^{\zeta+\sigma}\Phi(s)\,\hat{\Gamma}(s)^{-\frac{\eta-\sigma}{\eta-1}}\hat{N}(s)^{\theta}\hat{w}(s)^{-\lambda\sigma}, \tag{33}$$

where $\Phi(s) \equiv 1+(m-1)\phi(s)^{-\eta}$ and we substituted for (32). It follows that $\hat{q}^D(s) = \hat{q}\left(s\right)/\Phi\left(s\right)$ is the output of a type–s firm sold in the domestic market and $\hat{q}^E(s) = \hat{q}\left(s\right)\left(\Phi\left(s\right)-1\right)/\Phi\left(s\right)$

in the foreign market. The variable cost incurred to produce goods sold domestically by a firm producing a variety of good s is $\hat{v}^D(s) = \hat{w}(s)^\lambda \, \hat{q}^D(s)$. The variable cost incurred for export production is $\hat{v}^E(s) = \phi\,(s)\,\hat{w}(s)^\lambda \hat{q}^E(s)$. The variable cost function and the conditional demand for variable inputs in open economy are:

$$\begin{aligned}
\hat{v}\,(\hat{q},\hat{w}) &= \Delta\,(s)\,\hat{w}(s)^\lambda \hat{q}(s);\\
\hat{l}_v(\hat{q},\hat{w}) &= (\lambda+\delta(s))\,\Delta(s)\hat{w}(s)^{\lambda-1}\hat{q}(s);\\
\hat{y}_v(\hat{q},\hat{w}) &= (1-(\lambda+\delta(s)))\,\Delta(s)\hat{w}(s)^\lambda \hat{q}(s);
\end{aligned}$$

where $\Delta(s \equiv \left(1+(m-1)\,\phi(s)^{1-\eta}\right)/\Phi(s) > 1$ captures the gap in the variable cost due to export and $\delta(s) = \frac{\partial\Delta(s)}{\partial\hat{w}(s)}\frac{\hat{w}(s)}{\Delta(s)}$ is the corresponding elasticity to wage. A closed form expression for $\delta(s)$ is not vaery informative, since $\Delta(s)$ is a non-monotonic function of the wage. However, if both factors are employed in production then $-\lambda < \delta(s) < 1-\lambda$ and substitutability between factors $\varepsilon(s) = \frac{d(\hat{y}_v(s)/\hat{l}_v(s))}{d(\hat{w}(s)/1)}\frac{\hat{w}(s)/1}{\hat{y}_v(s)/\hat{l}_v(s)} > 0$ implies $\frac{d\delta(s)}{d\hat{w}(s)} < 0$.

The employment of factors as fixed input is:

$$\begin{aligned}
\hat{l}_f(s) &= \alpha\hat{w}(s)^{\alpha-1}f;\\
\hat{y}_f(s) &= (1-\alpha)\hat{w}(s)^{-\alpha}f;
\end{aligned}$$

which yields a fixed cost $f\hat{w}(s)^\alpha$. Monopolistic competition under free entry in the output market drives firms' profit to zero. The proportionality between revenue and variable cost implied by the demand structure and the competitive factor market allow variable cost and revenue to be computed as proportional to fix costs; then firm output and indeed factor allocations in equilibrium are obtained:

$$\begin{aligned}
\hat{v}(s) &= (\eta-1)f\hat{w}(s)^\alpha; \qquad (34)\\
\hat{r}(s) &= \eta f\hat{w}(s)^\alpha;\\
\hat{q}(s) &= (\eta-1)\Delta(s)^{-1}f\hat{w}(s)^{\alpha-\lambda};\\
\hat{l}(s) &= ((\eta-1)\,(\lambda+\delta(s))+\alpha)\,f\hat{w}(s)^{\alpha-1};\\
\hat{y}(s) &= ((\eta-1)(1-(\lambda+\delta(s)))+(1-\alpha))\,f\hat{w}(s)^\alpha.
\end{aligned}$$

Aggregate variable costs for firms of type s are $\hat{V}(s) = \Delta(s)\hat{w}(s)^\lambda\hat{Q}\,(s)$. The aggregate demand for labor as a variable input in open economy then satisfies

$$\hat{L}_v^d(s) = \Lambda(s)\hat{w}(s)^{\lambda-1}\hat{Q}(s) = \frac{\Lambda(s)}{\Delta(s)}\frac{\hat{V}(s)}{\hat{w}(s)}, \qquad (35)$$

where $\Lambda(s) \equiv (\lambda+\delta(s))\,\Delta(s)$. Total revenue of domestic firms in sector s is $\hat{R}(s) = \eta\hat{w}(s)^\alpha f\hat{N}(s)$. Total revenue is proportional to total variable costs $\hat{R}(s) = \frac{\eta}{\eta-1}\hat{V}(s)$. The zero profit condition, $\hat{V}(s)/(\eta-1) - \hat{N}(s)\,w(s)^\alpha f = 0$, identifies the equilibrium number of firms in sector s in open economy: $\hat{N}(s) = \frac{\Delta(s)\hat{w}(s)^{1-\alpha}\,\hat{L}_v^d(s)}{\Lambda(s)(\eta-1)f}$. Labor market clearing, $\hat{L}_v^d(s)+\hat{N}(s)\alpha\hat{w}(s)^{\alpha-1}f = L(s)$, yields (26).

Expenditure from domestic consumers in good s is $\hat{P}(s)\hat{X}(s)$, from (31) and (32). In an equilibrium with symmetric countries in which trade is balanced yields $\hat{R}(s) = \hat{P}(s)\hat{X}(s)$, which fixes the number of consumed varieties over domestic varieties:

$$\hat{\Gamma}(s)^{-\frac{1}{\eta-1}} = \left(\eta^{-\sigma}\left(\eta-1\right)^{\sigma-1} f^{-1}\right)^{\frac{1}{\sigma-1}} \hat{P}_Z^{\frac{\zeta+\sigma}{\sigma-1}} \hat{N}(s)^{-\frac{1-\theta}{\sigma-1}} \hat{w}\left(s\right)^{-\lambda-\frac{\alpha}{\sigma-1}}. \tag{36}$$

Conditions (33)–(36) together yield (27).

After the substitution of (36) in (33) total output reads:

$$\hat{Q}(s) = \left(\eta^{-\sigma}\left(\eta-1\right)^{\sigma}\right)^{\frac{1}{\theta}} \hat{P}_Z^{\frac{\zeta+\sigma}{\theta}} \left((\eta-1)f\right)^{1-\frac{1}{\theta}} \;\; \Phi(s)\; \hat{N}(s)^{1-\frac{\eta-\sigma}{\sigma-1}}\; w(s)^{-\lambda\eta-\alpha\frac{\eta-\sigma}{\sigma-1}}.$$

Substituting for the number of firms (26) yields:

$$\hat{Q}(s) = \left(\eta^{-\sigma}\left(\eta-1\right)^{\sigma-\theta} f^{-\theta}\right)^{\frac{1}{\theta}} \hat{P}_Z^{\frac{\zeta+\sigma}{\theta}} \;\; \Phi(s)\;\; \left(\frac{(\eta-1)\Delta(s)L(s)\hat{w}(s)}{(\eta-1)\Lambda(s)+\Delta(s)\alpha}\right)^{2-\frac{1}{\theta}} \;\; \hat{w}(s)^{-\lambda\eta-\alpha},$$

where $\Lambda(s) = \left(\lambda + \frac{\partial\Delta(s)}{\partial\hat{w}(s)}\frac{\hat{w}(s)}{\Delta(s)}\right)\Delta(s)$. Labor demand (35) and labor market clearing yield

$$\hat{Q}(s) = \frac{(\eta-1)\Delta(s)L(s)w(s)}{(\eta-1)\Lambda(s)+\Delta(s)\alpha}\frac{\hat{w}(s)^{-\lambda}}{\Delta(s)};$$

which substituted into the previous equation yields

$$\frac{\hat{w}(s)^{\lambda(\eta-1)+\alpha}}{\Delta(s)\Phi(s)} = \left(\eta^{-\sigma}\left(\eta-1\right)^{\sigma-\theta} f^{-\theta}\right)^{\frac{1}{\theta}} \hat{P}_Z^{\frac{\zeta+\sigma}{\theta}} \left(\frac{(\eta-1)L(s)\hat{w}(s)}{(\eta-1)\Lambda(s)/\Delta(s)+\alpha}\right)^{1-\frac{1}{\theta}}.$$

The two sides of the wage equation are both positive and continuous, when production is active $l(s) > 0\ \forall s$. Clearly the left hand side (l.h.s.) attains the origin zero when the wage approaches zero from above. The right and side (r.h.s.) goes to infinity for $w(s) = 0$, since $\eta > \sigma \implies \theta \in (0,1)$, as established above. The l.h.s. consists of $\frac{\hat{w}^(s)\lambda(\eta-1)+\alpha}{1+(m-1)\phi(s)^{1-\eta}}$, which is an increasing function of the wage for any $\alpha \geq 0$ and an elasticity of the export cost wedge that is bounded above

$$-\frac{d\phi(s)}{d\hat{w}(s)}\frac{\hat{w}(s)}{\phi(s)} < \lambda\ \frac{1+(m-1)\,\phi(s)^{1-\eta}}{(m-1)\,\phi(s)^{1-\eta}}.$$

Computing the elasticity from (8), e obtain

$$-\frac{d\phi(s)}{d\hat{w}(s)}\frac{\hat{w}(s)}{\phi(s)} = (\lambda-\epsilon)\frac{b\hat{w}(s)^{\epsilon-\lambda}}{1+b\hat{w}(s)^{\epsilon-\lambda}} < (\lambda-\epsilon) < \lambda,$$

which shows that the required condition is always satisfied. Regarding the r.h.s., $\frac{(\eta-1)L(s)\hat{w}(s)}{(\eta-1)\Lambda(s)/\Delta(s)+\alpha} = \Delta(s)\hat{Q}(s)\hat{w}(s)^{\lambda} = \hat{V}(s)$ is the variable cost in the industry when production is efficient and given that the labor factor is in full employment. If $\hat{w}(s)$ is the equilibrium wage, then an arbitrary positive change in the wage cannot decrease the variable cost, which implies that the r.h.s. is a decreasing function of the wage for $\theta \in (0,1)$:

$$\frac{d\delta(s)}{d\hat{w}} < 0 \implies \text{sgn}\left(\frac{d\left(\frac{\hat{w}}{\lambda+\delta(s)+\alpha/(\eta-1)}\right)}{d\hat{w}}\right) > 0$$

An intersection of the two loci does exist and it is unique. The r.h.s. tilts and shifts up for scarcer factors if and only if $\eta > \sigma$; whereas the l.h.s. does not depend on factor abundance. One concludes that scarcer factors are paid a higher wage. □

Proof of Proposition 2

2.i: Under autarky, labor productivity, $N(s)\,q(s)/L_v^d(s) = (1/\lambda)\,w(s)^{1-\lambda}$, is increasing in $w(s)$ (and thus in s, by Proposition 1). In an open-economy equilibrium, the variable cost function $\hat{v}\left(\hat{q},\hat{w}\right) = \Delta\left(s\right)\hat{w}(s)^{\lambda}\hat{q}$ is non-decreasing and concave in the wage $\hat{w}(s)$

$$\begin{aligned}\frac{\partial \hat{v}\left(\hat{q},\hat{w}\right)}{\partial \hat{w}} &= \hat{l}_v^d\left(\hat{q},\hat{w}\right) = \left(\lambda + \frac{\partial \Delta(s)}{\partial \hat{w}(s)}\frac{\hat{w}(s)}{\Delta(s)}\right)\Delta(s)\hat{w}(s)^{\lambda-1}\hat{q} \geq 0;\\ \frac{\partial^2 \hat{v}\left(\hat{q},\hat{w}\right)}{\partial \hat{w}^2} &= \frac{\partial \hat{l}_v^d\left(\hat{q},\hat{w}\right)}{\partial \hat{w}} = \frac{\partial\left(\Lambda(s)\hat{w}(s)^{\lambda-1}\right)}{\partial \hat{w}}\,\hat{q} \leq 0;\end{aligned}$$

where $\Lambda(s) \equiv \left(\lambda + \frac{\partial \Delta(s)}{\partial \hat{w}(s)}\frac{\hat{w}(s)}{\Delta(s)}\right)\Delta(s)$. These properties are sufficient to conclude that in equilibrium higher wages are associated with higher labor productivity $\hat{q}(s)/\hat{l}_v^d(s) = \left(\Lambda(s)\hat{w}(s)^{\lambda-1}\right)^{-1}$. □

2.ii: The elasticities of variable cost, revenue, output and factor demand to wage are:

$$\begin{aligned}\frac{d\hat{v}(s)}{d\hat{w}(s)}\frac{\hat{w}(s)}{\hat{v}(s)} &= \frac{d\hat{r}(s)}{d\hat{w}(s)}\frac{\hat{w}(s)}{\hat{r}(s)} = \alpha > 0;\\ \frac{d\hat{q}(s)}{d\hat{w}(s)}\frac{\hat{w}(s)}{\hat{q}(s)} &= \alpha - (\lambda + \delta(s)) \in \left(-\left(1-\alpha\right),\alpha\right);\\ \frac{d\hat{l}(s)}{d\hat{w}(s)}\frac{\hat{w}(s)}{\hat{l}(s)} &= \frac{d(\lambda+\delta(s))}{d\hat{w}(s)}\frac{\hat{w}(s)}{\lambda+\delta(s)} - (1-\alpha) < 0;\\ \frac{d\hat{y}(s)}{d\hat{w}(s)}\frac{\hat{w}(s)}{\hat{y}(s)} &= -\frac{d(\lambda+\delta(s))}{d\hat{w}(s)}\frac{\hat{w}(s)}{\lambda+\delta(s)}\frac{\lambda+\delta(s)}{1-(\lambda+\delta(s))} + \alpha > \alpha;\end{aligned}$$

where for the last result we applied the algebra of percentage variation to $\hat{w}(s)\hat{l}_v(s) = (\lambda + \delta(s))\,\hat{v}(s)$ and $\hat{y}_v(s) = \left(1 - (\lambda + \delta(s))\right)\hat{v}(s)$. When $\alpha \to 1$, then $\frac{d\hat{r}(s)}{d\hat{w}(s)}\frac{\hat{w}(s)}{\hat{r}(s)} > \frac{d\hat{q}(s)}{d\hat{w}(s)}\frac{\hat{w}(s)}{\hat{q}(s)} \geq 0$. □

Proof of Proposition 3

The claim is implied by the definition of $\rho(s)$ and by the defintion of $\phi(s)$ in (8), which is a decreasing function of the wage when $\lambda > \epsilon$, and the relationship between wage and skill scarcity in Proposition 1. □

Proof of Lemma 2

This follows directly from Propositions 2.ii and 3. □

C Distributional effects of trade liberalization

Proof of Proposition 4

Consider the wage ratio

$$\frac{\hat{w}(s)}{w(s)}\left(\frac{P_Z}{\hat{P}_Z}\right)^{\frac{\zeta+\sigma}{\Theta}} = \left(1+(m-1)\phi(s)^{1-\eta}\right)\left(\frac{(\eta-1)\lambda+\alpha}{(\eta-1)(\lambda+\delta(s))+\alpha}\right)^{-\frac{1-\theta}{\Theta}},$$

where $m>1$, $\eta>\sigma>1$, $\lambda>\epsilon \implies \frac{d\phi(s)}{d\hat{w}(s)}<0$ and $\frac{1-\theta}{\Theta}>0$.

In equilibrium:

$$\begin{aligned} \hat{l}_v(\hat{q},\hat{w})>0 &\implies \delta(s)>-\lambda; \\ \hat{y}_v(\hat{q},\hat{w})>0 &\implies \delta(s)<1-\lambda; \\ \frac{d(\hat{y}_v/\hat{l}_v)}{d(\hat{w}/1)} = -\frac{d\delta(s)/d\hat{w}}{(\lambda+\delta(s))^2}>0 &\implies \frac{d\delta(s)}{d\hat{w}}<0. \end{aligned}$$

Let $\varepsilon(s)=\frac{d(\hat{y}_v(s)/\hat{l}_v(s))}{d(\hat{w}(s)/1)}>0$ be the elasticity of substitution between homogeneous good and labor in sector s. Computing the derivative of the elasticity of substitution $\varepsilon(s)$ with respect to the wage yields:

$$\frac{d\varepsilon(s)}{d\hat{w}(s)} = \frac{2(\lambda+\delta(s))\left(d\delta(s)/d\hat{w}(s)\right)^2-(\lambda+\delta(s))^2\left(d^2\delta(s)/d\hat{w}(s)^2\right)}{(\lambda+\delta(s))^4}.$$

If it is more difficult to substitute scarcer varieties with homogeneous inputs ($\frac{d\varepsilon(s)}{ds}\leq 0$) and scarcer factors are paid more, by Proposition 2, then $\delta(s)$ is a convex function of the wage $\frac{d\varepsilon(s)}{d\hat{w}(s)}\leq 0 \implies \frac{d^2\delta(s)}{d\hat{w}(s)^2}>0$.

The non-monotonicity of $\Delta(s)$ can be understood by looking at revenue per unit of output. Notice from (34) that in open economy the ratio of revenue per unit of output deviates from the domestic price. The composition of demand in the domestic and in the foreign market yields:

$$\begin{aligned} \frac{\hat{r}(s)}{\hat{q}(s)} &= \Delta(s)p(s); \\ &= \frac{x^D(s)}{x^D(s)+x^E(s)}p(s)+\frac{x^E(s)}{x^D(s)+x^E(s)}p(s)^*; \\ &= \left(\frac{x^D(s)}{x^D(s)+x^E(s)}+\frac{\phi(s)x^E(s)}{x^D(s)+x^E(s)}\right)p(s); \\ &= \left(\frac{1/(m-1)}{1/(m-1)+\phi(s)^{-\eta}}+\frac{\phi(s)^{1-\eta}}{1/(m-1)+\phi(s)^{-\eta}}\right)p(s); \end{aligned}$$

where we substituted for the export over domestic relative demand, $\hat{x}^E(s)/\hat{x}^D(s)=(m-1)\phi(s)^{-\eta}$. If the demand in the export market is large enough relatively to the domestic demand $(m-1)\,\phi(s)^{-\eta}>\eta-1$ then the trade induced gap between revenue per unit and domestic price $\Delta(s)$ is larger $\delta(s)>0$ when the per–unit–variable cost of exported varieties relative to domestic varieties $\phi(s)$ is larger. However, the gap between revenue per unit and domestic price can actually fall with $\phi(s)$ when there is high substitutability across varieties and there are few export destinations.[21]

[21] The second component within squared brackets is decreasing in $\phi(s)$ when firms export to relatively few destinations: $(m-1)\,\phi(s)^{-\eta}<\eta-1$. The necessary and sufficient condition for $\frac{\partial\Delta(s)}{\partial\phi}<0$ is $(m-1)\,\phi(s)^{-\eta}<\eta\left(1-\phi^{-1}\right)-1$, which can be met for high substitutability across varieties η and few destinations m.

Notice that for a number of destination that is arbitrarily large then $\Delta(s) \to \phi(s)$. When this is the case then $\Delta(s)$ is a decreasing function of the wage $\delta(s) < 0$. Since $\frac{d\delta(s)}{d\hat{w}(s)} < 0$ then $\Delta(s)$ has a maximum and $\frac{d\Delta(s)}{d\hat{w}(s)} = \frac{d\Delta(s)}{d\phi(s)} \frac{d\phi(s)}{d\hat{w}(s)}$, where $\frac{d\phi(s)}{d\hat{w}(s)} < 0$ because of the A–A effect, implies that the maximum is achieved when $\frac{d\Delta(s)}{d\phi(s)} = 0$. Studying the sign

$$\text{sgn}\left(\frac{d\Delta(s)}{d\phi(s)}\right) = \phi(s)^{-\eta} - \frac{\eta}{m-1}\left(\frac{\eta-1}{\eta} - \phi^{-1}\right)$$

shows that if the elasticity of substitution across varieties of a given good type s – for η sufficiently greater than unity – a small number of markets $m \geq 2$ and low wages (so that $\phi(s) > 1$ is sufficiently high), $\Delta(s)$ and $\phi(s)$ can be negatively correlated, which results in $\Delta(s)$ being positively correlated with the wage. If the number of markets, m, is sufficiently large, or if the endowment of the most abundant skill s_0 is such that

$$\phi(s_0) \leq \frac{\eta}{\eta - 1} \iff \hat{w}(s_0) \geq (b(\eta-1))^{\frac{1}{\lambda-\epsilon}},$$

then the gap in variable costs is larger at lower wages; that is $\Delta(s)$ is negatively correlated with the wage $\delta(s) < 0$ for all skill levels.

The discussion on the wage elasticity of the variable cost motivates to conduct the analysis for $\delta(s) \in (-\lambda, 0)$ and $\frac{d\delta(s)}{d\hat{w}(s)} < 0$. For the wage of the most abundant skill $\hat{w}(s_0)$ the wedge in marginal cost is at its maximum $\phi(s_0) >> 1$ and $\delta(s_0) \to 0$. The wage ratio is:

$$\frac{\hat{w}(s_0)}{w(s_0)}\left(\frac{P_Z}{\hat{P}_Z}\right)^{\frac{\zeta+\sigma}{\Theta}} = 1 + (m-1)\phi(s_0)^{1-\eta} > 1.$$

In the limit for the most scarce skill $\hat{w}(s_1) \to \infty$ then $\phi(s_1) \to 1$ and $\lambda + \delta(s_1) \to 0$ from above. The wage ratio is:

$$\frac{\hat{w}(s_1)}{w(s_1)}\left(\frac{P_Z}{\hat{P}_Z}\right)^{\frac{\zeta+\sigma}{\Theta}} = \frac{m}{m^*} \quad , \quad m^* = \left(\frac{(\eta-1)\lambda + \alpha}{\alpha}\right)^{\frac{1-\theta}{\Theta}}.$$

Therefore, if the number of foreign markets is sufficiently large $m > m^*$ then the ratio in real wages is higher in open economy than in autarky also for top earners. Moreover, if $m > m^*(1 + (m-1)\phi(s_0)^{1-\eta})$ then top earners gain relatively more. Finally notice that if $\sigma \to \eta$ from below, then the contribution to the real wage ratio of $\delta(s)$ becomes negligible. Therefore, in the limit where the model accommodates a one wage equilibrium the real wage in open economy is higher than in closed economy for all skill levels. □

Since the mass of workers for each skill type is exogenously given and equal to $L(s)$, a higher relative change in the wage of type s'' relative to the corresponding relative change in the wage of type s', i.e. $\frac{\hat{w}(s')}{w(s')} > \frac{\hat{w}(s'')}{w(s'')}$, implies increased dispersion if $s'' > s'$ or decreased dispersion of $s'' < s'$. Focusing on this comparison, the effect of trade on the wage distribution is summarized in the following proposition:

Proof of Proposition 5

As the wage grows then $\phi(s)$ falls toward 1 and $\delta(s)$ falls toward $-\lambda$. The first effect yields a positive contribution to the wage ratio but the second effect decreases the wage ratio. Let $A(\phi(s)) = 1 + (m-1)\phi(s)^{1-\eta}$ then the contribution of the Alchian-Allen effect in rising the wage ratio is given by:

$$\frac{dA(\phi(s))}{d\phi(s)}\frac{\phi(s)}{A(\phi(s))}\frac{d\phi(s)}{d\hat{w}(s)}\frac{\hat{w}(s)}{\phi(s)} = (\lambda-\epsilon)(m-1)\mathcal{A}(s);$$

where

$$\mathcal{A}(s) = \frac{(\eta-1)(\phi(s)-1)}{1+(m-1)\phi(s)^{1-\eta}} > 0,$$

which is an increasing function of $\phi(s)$ for $\phi(s_0) \leq \frac{\eta}{\eta-1}$ – and thus a decreasing function of the wage. Let $B\left(\delta(s)\right) = \frac{(\eta-1)\lambda+\alpha}{(\eta-1)(\lambda+\delta(s))+\alpha}$ then the contribution of the skill–based product differentiation in decreasing the wage ratio is given by:

$$\frac{1-\theta}{\Theta}\frac{dB(\delta(s))}{d\delta(s)}\frac{\delta(s)}{B(\delta(s))}\frac{d\delta(s)}{d\hat{w}(s)}\frac{\hat{w}(s)}{\delta(s)} = \frac{1-\theta}{\Theta}\mathcal{B}(s),$$

where

$$\mathcal{B}(s) = -\frac{(\eta-1)\delta(s)}{(\eta-1)(\lambda+\delta(s))+\alpha}\frac{d\delta(s)}{d\hat{w}(s)}\frac{\hat{w}(s)}{\delta(s)} > 0,$$

which is a positive and decreasing function of $\delta(s)$ – accounting for $\delta(s) < 0$, $\frac{d\delta(s)}{d\hat{w}(s)} < 0$ and $\frac{d^2\delta(s)}{d\hat{w}(s)^2} > 0$ – and is thus an increasing function of the wage. The change in the wage ratio

$$\text{sgn}\left(\frac{d(\hat{w}(s)/w(s))}{d\hat{w}(s)}\right) = (\lambda-\epsilon)(m-1)\mathcal{A}(s) - \frac{1-\theta}{\Theta}\mathcal{B}(s)$$

shows that it increases with the wage $\frac{d(\hat{w}(s)/w(s))}{d\hat{w}(s)} > 0$ only if the Alchian-Allen effect is present $(\lambda > \epsilon)$, and that the effect is stronger the wider is the export market (the larger is m). The wage ratio decreases with the wage only if varieties within skill groups are better substitutes than between skill groups $\sigma < \eta \implies \theta \in (0,1)$. Both effects are present, which might result in a non-monotonic behavior of the wage ratio. However, $\mathcal{A}(s)$ is larger at lower wages (higher $\phi(s)$), whereas $\mathcal{B}(s)$ is larger at higher wages (lower $\delta(s)$). Therefore, if there is non-monotonicity then $\frac{d(\hat{w}(s)/w(s))}{d\hat{w}(s)} > 0$ at lower wages (dispersion), and $\frac{d(\hat{w}(s)/w(s))}{d\hat{w}(s)} < 0$ at higher wages (concentration). It follows that if the wage ratio at the top of the wage distribution is at least unity, then the real wage is higher in open economy than in autarky for all skill levels. □

D Generalization of horizontal differentiation to non-CES substitution possibilities

Consider a market with a given empirical distribution of unit prices for varieties within an industry, represented by a continuous and differentiable p.d.f., $f(p)$, with support $[\underline{p}, \overline{p}] \equiv P$; that is, $f(p)$

represents the proportion of varieties, measured in physical units, that are produced and sold at any given unity price, p. Suppose that this distribution has the property $f'(p) < 0$, i.e. higher priced varieties are produced and sold in smaller quantities. Starting from this observed distribution, we can introduce an identifier s for each good type and initially let $s = p$, writing prices and quantities as $p(s) = s$ and $q(s) = Q\, f(s)$, with $s \in [\underline{p}, \overline{p}] = P \equiv S$, where Q is the total number of units produced and sold.

Under these conditions, it can be shown that the observed schedules $p(s)$ and $q(s)$ can always be rationalized as resulting from utility maximizing demand choices based on homothetic preferences represented by a utility function having the following form:

$$U(q(s), s \in S) = Q \int_{\underline{s}}^{\overline{s}} v(q(s)/Q)\, ds, \quad Q \equiv \int_{\underline{s}}^{\overline{s}} q(s)\, ds,$$

with $v'(.) > 0$ and $v''(.) < 0$. We can prove this constructively by showing how the function $v(\cdot)$ can be recovered from a given $f(p)$.

The first-order conditions for budget-constrained utility maximization require

$$Q\, v'(q(s)/Q) = \lambda\, p(s), \quad s \in S,$$

where λ is the marginal utility of income. Without loss of generality, we can set $\lambda = 1$; this implies a choice of normalization of cardinal utility units, a choice that is of no consequence for the underlying utility ordering represented by $U(\cdot)$. Since $s = p$ and $q(s)/Q = f(p)$, we can re-write the above as

$$Q\, v'(f(p)) = p, \quad p \in P;$$

or

$$Q\, v'(x) = f^{-1}(x), \quad x \in [f(\overline{s}), f(\underline{s})] \equiv X.$$

This can be integrated to obtain

$$v(x) = \frac{1}{Q} \int_{f(\overline{p})}^{x} f^{-1}(z)\, dz, \quad z \in X.$$

By following the above approach, *any* pattern with $q'(s) < 0$ and $p'(s) > 0$ can be rationalized as reflecting choices based on preferences featuring symmetric (horizontal) product differentiation, i.e. preferences such that, in a counterfactual scenario with uniform prices, $p(s) = p, s \in S$, demand levels would be uniform across all good types.[22] Thus, if we observe a pattern of prices and quantities that satisfies with $q'(s) < 0$ and $p'(s) > 0$, the above formalization is a fully sufficient representation of demand choices and marginal demand responses, and produces predictions about prices and quantities that are observationally equivalent to those of a specification which makes an explicit distinction between horizontal and vertical differentiation. On the other hand, an empirical pattern with $q'(s) > 0$ and $p'(s) > 0$ or with $q'(s) < 0$ and $p'(s) < 0$, would be inconsistent with horizontal differentiation, and therefore could not be rationalized in this way.

[22] In applications where the empirical distribution is not a continuous distribution with a known form, $v(.)$ could be recovered numerically (e.g. using a monotone cubic spline approximation).

We have presented results for setup where a CES setup, but the analysis and results can be generalized to a setting that features a flexible representation, $v(q(s))$, of substitution possibilities across varieties corresponding to different skill types, while retaining a CES representation of substitution possibilities across varieties corresponding to a common skill type. The conditions we have derived for local comparison between varieties of adjacent skill types (in terms of their comparative scarcity) would then hinge on the local value of the expression $-\frac{1}{q(s)\, v''(q(s))/v'(q(s))} \equiv \sigma(s)$, which replaces the constant σ that applies to the CES case.

E Evidence on the Alchian-Allen effect

If we regress log export intensity on log price, including firm-level and sector-specific controls, we obtain an elasticity estimate of roughly 5%. We can link this to the above specification of transport costs through the following back-of-the-envelope calculation.

The expression for the elasticity of export intensity, $\rho(s)$, with respect to price, $p(s)$, we have derived in Section 3 involves the parameters η, ϵ, λ, b and m. The theoretical CIF/FOB margin is $\phi(s) - 1 = b\, w^{\epsilon}$. As domestic price equals $(\eta/(\eta-1))w^{\lambda}$, the CIF/FOB margin as a function of price can be expressed as $b\,(\eta/(\eta-1))^{1-\epsilon/\lambda} p^{\epsilon/\lambda}$. Hummels and Skiba (2004) estimate ϵ/λ (the elasticity of the CIF/FOB margin to price) to be equal to 0.6. Given a λ of 0.2, corresponding to the mean labour value share for exporters in our data, an estimate of 0.6 for ϵ/λ would translate into an ϵ equal to 0.12. Taking a value of η equal to 5 (which corresponds to middle-of-the-road literature estimates estimates of substitution elasticities across domestic and imported varieties), we are left with two free parameters m and b. If we simultaneously select these values so that, for a price normalized at unity, export intensity is 0.25 (the mean value in our data) and the elasticity of ρ to price is 0.05 (our regression estimate), we obtain $m \simeq 1.4$ and $b = 0.04$, which gives a CIF/FOB margin of 4.4%, which is within the range of literature estimates (Gelhar and McDougall, 2016). Since the expression for ρ is concave in prices, and the expression for the elasticity is convex in prices, a mean ρ of 0.25 implies a median value that is higher than 0.25, and a mean elasticity of 0.05 implies a median value that is less than 0.05, and so fitting over a distribution of prices and export intensities would produce parameters that are consistent with a mean CIF/FOB margin than is less than 4.4%.