International Trade and Technological Competition in Markets with Dynamic Increasing Returns

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Abstract

We build a simple international trade model to study the effects that the interaction between technological learning and imperfect market selection exert on the dynamics of firms, industries and export flows. The model features two countries populated by firms heterogeneous in productivity and size. Market selection in each country is driven by a *finite pairwise Pólya urn* process, which embodies dynamic increasing returns in firm size. We show that, with a static distribution of firm productivity, market selection leads either to an international or to a national monopoly depending on the entity of trade barriers. In presence of firm learning and entry-exit of firms, the firm productivity distribution changes over time and the model generates non-monopolistic market structures, whose properties depend on trade openness and market selection intensity. Finally, we show that the the model is able to jointly reproduce a wide ensemble of stylized facts concerning intra-industry trade, industry and firm dynamics as well as realistic dynamics of productivity and export leadership at the country level.

Keywords: International trade, industrial dynamics, firm dynamics, market selection.
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1 Introduction

This work proposes a simple intra-industry model of trade to study the joint effect of imperfect market selection and technological learning in the determination of the properties of firm, industry and export flows dynamics. Our work belongs to a recent stream of literature that attempts to explain international trade patterns by means of simple stochastic processes (e.g. the "balls and bins" model proposed in Armenter and Koren, 2014) and it contributes to the large theoretical and empirical literature that has stressed the pivotal role of productivity in determining firm performance in international markets (see e.g. Bernard and Bradford Jensen, 1999; Melitz, 2003). This literature has highlighted how export performance is not only influenced by productivity levels (Bernard and Bradford Jensen, 1999), but also by innovation activities and investments in R&D (Dosi et al., 2015; Grazzi et al., 2021). In addition, several works in the industrial dynamics literature have unveiled the presence of wide and persistent productivity differentials among firms (see e.g. Bartelsman and Dhrymes, 1998; Doms and Bartelsman, 2000; Foster et al., 2001; Bottazzi et al., 2010; Dosi et al., 2015). Interestingly enough, productivity differentials persist also when discriminating between exporting and non-exporting firms, as both the productivity distributions of exporters and non-exporters display right skewness. In addition, notwithstanding the presence of the so called ‘export productivity premium" (Bernard and Jensen, 1995; Bernard and Bradford Jensen, 1999; Bernard et al., 2007), the two distributions partially overlap (Bernard et al., 2003; Mayer and Ottaviano, 2008; Melitz and Trefler, 2012; Impullitti et al., 2013; Grazzi et al., 2021). This indicates that high-productivity non-exporters co-exist with low-productivity exporters, and the presence of an imperfect market selection in foreign markets. Finally, the increasing importance of international competition, witnessed by the fall of trade barriers over the last decades, has increased the awareness about the significant impact of trade flows on industry and firm dynamics. Recent studies have highlighted that market concentration (di Giovanni et al., 2011) and volatility (di Giovanni and Levchenko, 2009; Vannoorenberghe, 2012; Ćede et al., 2018) may be positively affected by trade openness.

We contribute to the above literature by building a model of intra-industry trade characterized by increasing returns in market selection. In line with traditional intra-industry trade models (see Krugman, 1980; Melitz, 2003), our model features two countries that are initially symmetric. Moreover, market selection of incumbent firms in each country is driven by a two-steps *finite pairwise*

Pólya urn process. [1] In the first step, pairs of firms are randomly drawn with a probability proportional to their size. In the second step, the selected pair of firms compete for customer demand on the basis of their productivity levels. The customer demand is then reallocated within the pair from the least productive firm to the most productive one. The micro-foundation of firm selection according to the above pairwise sampling rule captures the presence of consumers' imperfect knowledge in the product markets, similarly to the well-established models of imperfect competition with random encounters among customers (see e.g. Phelps and Winter, 1970; Bils, 1989; Rotemberg and Woodford, 1991; Greenwald and Stiglitz, 2005). Furthermore, the assumption that the probability of competing for a customer is an increasing function of market size captures the presence of *dynamic increasing returns* in market selection (see Arthur, 1989; Dosi and Kaniovski, 1994; Dosi et al., 2019), and proxies the fact that larger firms have better distribution channels and are more likely to be noticed by customers. At the same time, they are also more exposed to competition by other firms. Notice that we label the above returns as dynamic to emphasize the fact that they are an inherent feature of the competitive process of market share reallocation across firms. They do not arise instead from the presence of fixed costs as in other trade models (e.g. Melitz, 2003). Finally, competition over the productivity domain captures the fact that relatively more productive firms are able to charge a lower price and to attract a higher number of consumers, thereby increasing their market shares (Dosi et al., 1995; Melitz, 2003; Dosi et al., 2017).

The first applications of Pólya urns in economics date back to the seminal works by Herbert Simon on the firm size distribution (Simon, 1955; Ijiri and Simon, 1975, 1977). Since then, Pólya urn processes have been employed in several domains of economics, and in particular in the analysis of technical change (Arthur et al., 1987; Arthur, 1989; Dosi et al., 1994, 2019) and industry dynamics (Fu et al., 2005; Bottazzi and Secchi, 2006a; Bottazzi et al., 2007; Riccaboni et al., 2008).[2] However, to the

[1]The finite pairwise Pólya urn process differs from a standard Pólya process in three respects. First, in the former one, at each time step pairs of firms are randomly drawn, whereas in the latter only one firm is drawn. Second, market size is finite in the former, whereas it tends to infinity in the latter, as new balls are added to the urn at each time step (see also Schreiber, 2001). Third, in the finite pairwise Pólya urn process market shares are not the unique determinant of growth rates, as in standard Pólya urns processes. This last difference is the most economically relevant as it decouples the processes of market selection and learning. Thanks to these differences, the asymptotic properties of standard and finite Pólya urns are different (also see section 3.2). If standard Pólya urns converge to a fixed distribution of shares, the finite pairwise Pólya urn converges to monopoly.

[2]Another mechanism similar to our selection process is the *brochure mechanism* in the Keynes+Schumpeter (K+S) macroeconomic models proposed by Dosi et al. (see e.g. 2010, 2013, 2015). There, the consumption good firms change their supplier of intermediate capital goods when they receive a signal (a "brochure") from a capital good firm selling a more productive technology.

best of our knowledge, so far there is no application of Pólya urns to the analysis of international trade dynamics.[3]

By employing the above framework, we first prove analytically that, with a static distribution of firm productivity levels, market selection asymptotically generates either a national or an international monopoly (depending on the presence of iceberg costs). This is a result recalling the Fisher's fundamental theorem of natural selection (see Fisher, 1930), and it allows us to establish a reference point for the finite pairwise Pólya urn process that we use in this paper. We then study the model with idiosyncratic learning, where the firms' productivity distribution evolves over time, and where entry and exit of firms take place. By means of extensive Monte Carlo simulations, we show that the above extended version of the model is able to *jointly* reproduce the most important stylised facts related to international trade and industry dynamics. We also use this extended version of the model to carry out comparative dynamics exercises. In particular, we show that trade openness and the strength of the market selection process positively affect firm and industry volatility and different measures of market concentration (domestic and export concentration) by means of a winner-take-all type of dynamics. Finally, we show that the model generates persistent country-level asymmetries in export volume and aggregate productivity, as well as catching up dynamics. We also show that the odds for the laggard economy to catch up the leading one are affected by both trade openness and the strength of market selection. On the one hand, higher trade openness is associated to a higher probability of catching up in productivity, but to lower odds of catching up in export volumes. On the other hand, stronger market selection increases the likelihood of catching up both in productivity and in export volumes.

Overall, the above results indicate that our intra-industry trade model provides a fairly accurate description of how the interplay between cumulative learning and market selection shape the most interesting stylized facts concerning international trade, industry dynamics and firm dynamics. Moreover, our model also generates further predictions about the effects of trade openness and of market selection that are worth to investigate in future empirical research.

The remainder of the paper is organized as follows. In section 2 we provide a review of the empirical and theoretical literature in trade and industry dynamics related to our work. In section 3

[3]The unique exception is contained in the Appendix of the work by Dosi et al. (2015), which however employs a Pólya urn scheme different from the one we propose.

we describe the model, together with some asymptotic properties of the *finite pairwise Pólya urn process* under a static productivity distribution assumption. Section 4 presents the model results using extensive Monte Carlo simulations, and discusses their economic implications. Section 5 concludes. An appendix completes the paper providing formal proofs of the propositions presented in the paper.

2 Empirical and theoretical backgrounds

We begin by surveying the contributions to the industrial dynamics and international trade literature related to our paper, with a special focus on the interplay between firm learning and market selection, which constitute the two pillars of our model.

Market selection is an imperfect mechanism determining which firms survive in competitive environments (Foster et al., 2001; Bottazzi et al., 2010; Dosi et al., 2015) and shaping the statistical properties of firm-level productivity and size distributions. In that respect, a good deal of empirical contributions has shown that the productivity distribution of firms displays a positive skewness with a long right tail, and it is well approximated by a Log-normal law (Baily et al., 1992; Bartelsman and Dhrymes, 1998). In addition, productivity differentials between firms tend to be persistent over time. These properties of the productivity distribution are robust to different levels of aggregation and to different methods of estimation (Doms and Bartelsman, 2000; Syverson, 2011; Dosi and Grazzi, 2006; Bottazzi et al., 2008). In the context of an open economy, it is worth noticing that the presence of wide productivity differentials remains valid also when discriminating between exporting and non exporting firms. In fact, notwithstanding the presence of the so called export productivity premium (Bernard and Jensen, 1995; Bernard and Bradford Jensen, 1999; Bernard et al., 2007), the two productivity distributions of exporters and non-exporters are both right-skewed and they partially overlap (Mayer and Ottaviano, 2008; Melitz and Trefler, 2012; Grazzi et al., 2021). Thus, there exist a group of high productivity non-exporters and a group of low productivity exporters, and the first stochastically dominates the second.[4] Productivity influences export performance not only in levels.

[4]This has been rationalized by the presence of heterogeneity in some firm individual characteristic mediating the export decision (Guerini et al., 2021). The literature identified either entry costs (Bernard et al., 2011; Mayer et al., 2014), future profits uncertainty (Impullitti et al., 2013) or internal financial condition (Assenza et al., 2016) as possible mediating factors.

Firms activities related to learning and technical change (such as investment in R&D, patenting and embodied technologies) are also positively related to export (Dosi et al., 2015; Grazzi et al., 2021).

Similar properties have been uncovered also for the firm size distribution.[5] Some studies (e.g. Simon and Bonini, 1958; Axtell, 2001) indicate that the aggregate firm size distribution is well approximated by a Pareto law although the debate on the precise functional form of the size distributionis is still vivid, both because of technical concerns (Bottazzi et al., 2015) and because the properties of the distribution at the sectoral level are quite different from the aggregate ones (Bottazzi and Secchi, 2003). There is however consensus about the fact that the departure from the log-normal benchmark (and the presence of long right tails) is a systematic feature broadly recorded by all empirical studies. Other two important characteristics related to size are (i) the overlap in the two distributions of exporters and non-exporters, a fact denoting the co-existence of small exporters and large non-exporters (Grazzi et al., 2021) and (ii) a fatter and longer right tail in the size distribution of exporters with respect to the one of non-exporters (di Giovanni et al., 2011). This last feature can be explained by the distribution of the export sales by exporter firms: the vast majority of total export flows derives indeed from a small number of very large firms (Mayer and Ottaviano, 2008; Eaton et al., 2011; Freund and Pierola, 2015; Gaubert and Itskhoki, 2018). Finally, the process of market selection determines a high degree of turbulence in terms of market shares reallocation and entry and exit rates (Bartelsman et al., 2003; Bottazzi et al., 2010; Dosi et al., 2015). The turbulence affects in turn the age distribution of firms, which has been shown to be well approximated by an exponential law at different levels of aggregation (Coad, 2010b,a; Barba Navaretti et al., 2014; Coad, 2018). The process of entry and exit also affects export markets. Approximately, half of new exporters ceases to export within the first year (see Eaton et al., 2008; Amador and Opromolla, 2013; Albornoz et al., 2012). At the same time, the firm export status is very persistent as approximately 90% of firms do not change it on a yearly basis (Bernard et al., 2003; Campa, 2004; Bernard and Jensen, 2004; Bernard and Wagner, 2001).

A good deal of the literature in industry dynamics has also focused on the properties of the firm growth rate distribution. In particular, empirical works have robustly shown that the firm growth rate distribution is leptokurtic and well approximated by a Laplace distribution (see Stanley et al.,

[5] In this paper, with size we refer to firm sales.

1996; Bottazzi and Secchi, 2003, 2006a). Moreover, firm growth volatility is negatively correlated with size (see Hymer and Pashigian, 1962; Stanley et al., 1996; Sutton, 2002; Calvino et al., 2018). These two features are inconsistent with the Gibrat's Law of Proportionate Effects (Gibrat, 1931). [6]. Interestingly, the fat-tails property of growth rate distributions is robust to aggregation levels (Fu et al., 2005). Industry growth rates are also well approximated by a Laplace density (Castaldi and Sapio, 2008), as well as aggregate output growth rates (see Fagiolo et al., 2008; Castaldi and Dosi, 2009).

Fat-tails in the growth rate distributions at various aggregation levels imply that growth events that are larger in magnitude (either positive or negative) are statistically more relevant than what a Gaussian model would predict. This is very much related to the issue of growth rates volatility, which is a topic of primary concern in the policy debate (see e.g. Rodrik, 1998; Easterly et al., 2001). In this respect, recent contributions have studied the possible channels of transmission from micro to macro volatility (see for example Gabaix, 2011; Acemoglu et al., 2012; di Giovanni et al., 2014). Others have instead focused on the impact of international trade on volatility. In particular, some of the latter studies have found that exporters sales growth rate volatility is on average greater than non-exporters (Ćede et al., 2018; Vannoorenberghe, 2012). Furthermore the reallocation of market shares between foreign and domestic incumbents impacts the odds to grow or to shrink also at higher levels of aggregation (i.e. at industry and country levels, see di Giovanni and Levchenko, 2009, 2012; di Giovanni et al., 2019). This is because bilateral trade, firm intensive and extensive export margins are all positively associated to different degrees with trade openness (Bernard et al., 2011; Mayer and Ottaviano, 2008). In that respect, bilateral trade takes also place at both the country and the sector level (see the intra-industry trade findings in Balassa, 1966; Grubel and Lloyd, 1975; Fontagné and Freudenberg, 1997).

Table 1 summarizes the different stylized facts on international trade and industry and firm dynamics that we have discussed so far. The above statistical properties have had implications for the theoretical analysis of the mechanisms of market selection. However, except for the works in the evolutionary tradition (Nelson and Winter, 1982) the empirical insights on firm heterogeneity were

[6]Empirical estimates of the Gibrat's model suggest that the firm size coefficient is close to unity, as predicted by the law, but with strong deviations in the statistical properties of the residuals with respect to the Gibrat's law assumptions, especially when small firms are considered (Santarelli et al., 2006; Dosi and Nelson, 2010).

Stylized Facts of Industry Dynamics	
SF 1	The volatility of industry sales is positively associated with trade openness
SF 2	The distribution of industry growth rates is tent-shaped and leptokurtic
Stylized Facts of Intra-Industry Trade and Firms' Margins	
SF 3	Bilateral trade, firm intensive and extensive export margins are positively associated with trade openness
SF 4	The firms' trade status is persistent
SF 5	A large share of new exporters ceases exporting within the year
Stylized Facts of Firm Dynamics	
SF 6	The size distribution is more right skewed than a log-Normal law
SF 7	The growth rates distribution is tent-shaped and leptokurtic
SF 8	The volatility of firms' growth rates is negatively associated with size
SF 9	The age distribution follows an exponential law
SF 10	The productivity distribution follows a log-Normal law
Stylized Facts of Firm Dynamics in International Trade	
SF 11	The productivity distributions of exporters and non-exporters partially overlap
SF 12	The productivity of exporters is on average higher than the one of non-exporters
SF 13	The size distributions of exporters and non-exporters partially overlap
SF 14	The size volatility of exporters is on average higher that the one of non-exporters
SF 15	Firm level exports are more right skewed than a log-Normal law
SF 16	The distribution of size for exporters looms larger with higher trade openness
SF 17	Exporters growth rate is on average more volatile than non-exporters growth rate
SF 18	Innovation activity is positively related to export performance

Table 1: Summary of the stylised facts of industrial dynamics and international trade at different levels of aggregation.

mostly analysed until the 2000s by assuming perfectly competitive markets with fixed costs of entry and production (Jovanovic, 1982; Hopenhayn, 1992). The first contribution that models heterogeneous agents in a monopolistic competition equilibrium is the work of Melitz (2003), which provides an intra-industry trade theory consistent with the evidence on exporters self-selection (Bernard and Jensen, 1995; Bernard and Bradford Jensen, 1999) and on the reallocation effects of trade liberalization (Pavcnik, 2002; Trefler, 2004). In Melitz' model, firms take decisions on the basis of expected profits (as in Hopenhayn, 1992). More precisely, the firms serve each market that grants them non-negative profits conditional on productivities and on the respective fixed costs. Productivities are heterogeneous, fixed in time and drawn from an exogenous general cumulative distribution function. As productivities are fixed, firms exit takes place exogenously with a constant probability. Trade liberalization induces a selection effect by generating two productivity thresholds in equilibrium, one for the domestic and one for the foreign market, under which the firm cannot cover the respective fixed costs. This leads to the two main results of the model. First, more productive firms become exporters, thereby generating an export productivity premium. Second, trade liberalization generates a selection effect. The productivity thresholds become higher and selection becomes tougher, implying the exit of firms with productivity lower than the new thresholds. In this context, new gains from trade arise in terms of higher aggregate productivity, that reduces price level and increases real wages by reallocating shares to more productive firms.

The research stream triggered by the work of Melitz (2003) is still very active and proved to be flexible enough able to explain additional stylised facts of trade and industry dynamics and to introduce new technical improvements. For instance, the Melitz' model generates a Pareto distribution of size by assuming a Pareto distribution of firm productivity levels (see Baldwin, 2005; di Giovanni and Levchenko, 2012; Bernard et al., 2018).[7]

Differently from intra-industry trade models linked to Melitz' work, ours does not feature rational firms and it does not explain trade dynamics as an equilibrium outcome. Rather, we put emphasis on the interaction between simple (i.e., reduced-form) out-of-equilibrium processes of firm learning and imperfect market selection dynamics. We show that this interaction is able to generate jointly and endogenously the main stylised facts of industry, firm and trade dynamics. Notice that this also relates to the heterogeneity in firms' characteristics such as size and productivity, which are characterised by the skewed and/or fat-tailed distributions generally found in empirical studies. In addition, we show that the above interaction can also generate a realistic trade dynamics at the country level, encompassing processes of catch-up and falling behind. In these respects, our contribution is strongly rooted in the literature of stochastic models of firm dynamics (Ijiri and Simon, 1975) and in the evolutionary tradition (see Nelson and Winter, 1982). In particular, the evolutionary industrial dynamics models of Dosi et al. (1995, 2017) are close to ours in terms of the main economic mechanisms employed, but differs from it in two main aspects. First, they only focus on a closed economy. Our model encompasses instead intra-industry trade flows, which may affect both learning (via imitation of foreign exporters) and selection (via entry and competition of foreign exporters) at the country-level. Furthermore, these models employ the quasi-replicator equation for

[7]The assumption of Pareto productivity, albeit at odds with empirical evidence (see Doms and Bartelsman, 2000), is necessary to provide a Pareto distributed size in equilibrium for two reasons. First, the Pareto distribution is scale invariant (i.e. truncated Pareto distribution remains Pareto distributed). This property is exploited by the works using the Melitz' framework because the equilibrium distribution of productivity is the assumed firm productivity distribution truncated by the cut-off productivity threshold above which it is optimal to produce. Second, size is proportional to productivity in monopolistic competition models, thus a Pareto size can only emerge from a Pareto productivity. Furthermore, the works of Atkeson and Burstein (2010) Burstein and Melitz (2011) and Impullitti et al. (2013) endogenously generates the Pareto productivity distribution by introducing a stochastic mechanisms of firms learning based on Luttmer (2007).

firms' selection and a firm entry mechanism that encompasses entrants automatically substituting bankrupt firms. Conversely, in our model selection is an explicit and micro-founded mechanism based on random firm-to-firm market interactions, which is mediated by both firm's size and productivity. This difference is key, as it allows us to generate endogenous stochastic entry barriers for foreign exporters.

Finally, our approach is also close to the one of the 'balls and bins" model of Armenter and Koren (2014). However, differently from this work, we do not study how shipments are allocated to products and destinations, thereby trying to replicate stylised facts related to sparsity of trade flows data. We put indeed special emphasis on the interplay between market selection and firm learning, which allow us to focus on intra-industry trade stylised facts related to firms' heterogeneity.

3 A model of intra-industry trade

We now describe our dynamic model of intra-industry trade. In line with the tradition of intra-industry trade models (see Krugman, 1980; Melitz, 2003), the model features two countries that are initially symmetric and it thus encompasses trade flows between similar countries (see the evidence in Grubel and Lloyd, 1975). Firms are heterogeneous in terms of productivity, which determines firm competitiveness. Firm productivity may change over time as a result of stochastic learning. In addition, each firm can compete in the domestic market and/or in the foreign one. The main mechanisms driving the model dynamics are market selection, firm learning and entry/exit processes. In the baseline version of the model that we present in the next section we abstract from firm idiosyncratic learning and from entry-exit. This abstraction allows us to obtain a full analytical description of the properties of the finite pairwise Pólya urn process governing market selection. We first present the analytical properties of this baseline version of the model under autarchy. We then discuss the implications of the process in a two-countries setting with trade. Next, in Section 3.3 we develop the extended model including also firm learning and entry-exit of firms.

3.1 Market Selection as a finite Pólya Urn Process: the baseline model

Consider a country with one industry populated by N firms. Firms compete in a market characterized by a finite and time-constant number M of customers. Each customer has a demand of unitary size. Therefore, the total number of customers corresponds to the total market size. In each period $t = 1, \ldots, T$, a firm $i = 1 \ldots, N$ is characterised by three state variables: size $S_{i,t} \in [0, M]$; market share $s_{i,t} \in [0, 1]$, defined as the ratio between firm size and total market size; and productivity level $a_{i,t}$. In this baseline version of the model we assume that productivity is heterogeneous across firms and constant over time, so that $a_{i,t} = a_i > 0 \;\; \forall t$. Productivities are randomly drawn from a distribution $G(a_i)$ with positive support. The size of a firm i is measured as the number of customers allocated to that firm at every period t. The sum of firm sizes is therefore constant over time and equal to total market size:

$$M = \sum_{i=1}^{N} S_{i,t} \quad \forall t$$

In every period t, firm size $S_{i,t}$ is determined as the outcome of a two-steps Pólya urn process, which assigns K customers to firms. More precisely, $K > 0$ pairs of firms are sequentially drawn *without replacement* with probability equal to their market shares and compete for the allocation of the customer $k \in \{1, ..., K\}$. The most productive firm in the pair gains the customer k thereby "stealing" its demand from the least productive firm (which then decreases its size). Notice that the parameter K also identifies the number of customers that at each time step look around for a better firm. Therefore, it captures the intensity of market competition. Next, let us suppose that firms have homogeneous size at time 0, and let us denote initial size with $s_{i,0}$. The law of motion of firm size resulting from the above competition process can be written as:

$$S_{i,t+1} = S_{i,t} + \sum_{k=1}^{K} \xi_{i,k,t} \tag{1}$$

where $\xi_{i,k,t}$ represents the shock to sales that a firm i experiences at period t when competing for the kth customer. The shock $\xi_{i,k,t}$ is defined by:

$$\xi_{i,k,t} = \begin{cases} z_{ij,t} & p_{ij,k,t} \ \forall j \neq i \\ 0 & 1 - \sum_{\forall j \neq i} p_{ij,k,t} \end{cases} \tag{2}$$

The probability $p_{ij,k,t}$ measures the odds that firm i is drawn to compete with firm j at the kth assignment and is proportional to the size of both firms. With probability $1 - \sum_{\forall j \neq i} p_{ij,k,t}$ a firm i is not drawn in any pair and it does not change size at the kth assignment. The shock $z_{ij,t}$ is equal to 1 when i is more productive than j, and to -1 vice versa.[8] Moreover, the above process of market reallocation generates a symmetric shock ($z_{ij,t} = -z_{ji,t}$) to the size of the two firms in the pair. The most productive firm in the pair gets a positive shock of unitary size, whereas the least productive firm gets a negative shock, also of unitary size.

The probability $p_{ij,k,t}$ is increasing in the size of the two firms and it is defined by:

$$p_{ij,k,t} = s_{i,k,t} \cdot s_{j|i,k,t} + s_{j,k,t} \cdot s_{i|j,k,t} \tag{3}$$

where $s_{i,k,t}$ is the market share of firm i at the kth assignment (i.e., probability of drawing firm i) and $s_{j|i,k,t}$ the share of firm j at the kth assignment computed on the market net of $s_{i,k,t}$ (i.e., the probability of selecting firm j conditional on drawing the first firm without replacement).[9] The latter probabilities are equal to firm market shares, and they read as:

$$\begin{cases} s_{i,k,t} & = \dfrac{S_{i,k,t}}{M} \\ s_{j|i,k,t} & = \dfrac{S_{j,k,t}}{M - S_{i,k,t}}. \end{cases} \tag{4}$$

Where:

$$S_{i,k,t} = S_{i,t-1} + \sum_{k=1}^{k} \xi_{i,k,t} \tag{5}$$

[8]The vector of firm productivities a_t enters the model only as an ordinal measure by determining whether the firm i wins the pairwise competition against j. It does not determine the size of the shock experienced by a firm, but only its sign.

[9]Notice that this definition of the matching probabilities excludes the possibility that the same firm i is matched with herself.

The assumption that the probability of selecting a firm is increasing in firm market shares introduces *dynamic increasing returns* in market selection. Bigger firms have a higher probability of being selected in order to compete with other firms. Moreover the resulting increase (or decrease) in market share immediately maps into a higher value of the probability $p_{ij,k,t}$, determining the increase (or decrease) in the odds to compete again.[10] Notice that the law of motion in (1) has two fixed points (0 and M). The fixed point in 0 corresponds to the case in which a firm loses all its market share, the one in M to industry monopoly.

Finally, by combining equations (2), (3) and (4) with equation (1) we obtain that firm size in expected terms follows a multiplicative process:

$$E(S_{i,k,t}|a_t) = S_{i,k-1,t} \cdot \left(1 + \frac{f(s_{i,k-1,t}, a_{i,t})}{M}\right) \tag{6}$$

where a_t indicates the vector of firms' productivities. The growth rate factor $f_i(s_{k,t}, a_t)$ in the last equation is equal to:

$$f(s_{i,k,t}, a_{i,t}) = \sum_{\forall j \neq i} s_{j|i,k,t} \cdot z_{i,j,t} \cdot \eta_{i,j,t} \tag{7}$$

where $s_{j|i,k,t}$ is the share of firm j at the kth assignment relative to the market size net of firm i, $z_{i,j,t}$ is a shock taking positive (negative) value when firm i is more (less) productive than firm j, and $\eta_{i,j,t} = 1 + \frac{M-S_{i,k,t}}{M-S_{j,k,t}}$ is a firm interaction term, which is decreasing in firm size $S_{i,k,t}$.[11] Accordingly, firm growth rates in our model depend on firms' relative productivities via $z_{i,j,t}$ and on firm size via the interaction term $\eta_{i,j,t}$.[12]

Notice that Equation (7) implies that the firm growth process in our model similar to a Gibrat-type dynamics (see Gibrat, 1931; Ijiri and Simon, 1975) where growth is proportional to current size. At the same time, and similar to other recent works in the industrial dynamics literature (e.g. Bottazzi and Secchi, 2006a,b), our firm growth process also departs in some important way from a standard Gibrat one. This is because the firm size $S_{i,k,t}$ influences the firm growth rate by determining the

[10] Moreover, such an allocation process breaks the i.i.d. characterization of the shocks that we would have if $p_{ij,k,t}$ was not updated, implying that the growth rate distribution cannot be generated by the Central Limit Theorem.

[11] The presence of $s_{j|i,k,t}$ is due to the sampling without replacement.

[12] Firm growth rate in our model can also be considered as weighted average of possible firm-to-firm encounters (captured by $\eta_{i,j,t}$). Indeed, the components $s_{j|i,k,t}$ can be thought of as weights: $\sum_{\forall j \neq i} s_{j|i,k,t} = \sum_{\forall j \neq i} \frac{S_{j,k,t}}{M - S_{i,k,t}} = 1$

probability that a firm is hit by a growth rate shock $z_{i,j,t}$. Moreover firm size also determines the magnitude of the growth rate shock (captured by the term $\eta_{i,j,t}$).

3.2 Asymptotic properties of market selection

The market selection process described in the previous section implies that the firm with maximal productivity always experiences non-negative growth shocks as its customers cannot be stolen by any other firms. Intuitively, such a firm will asymptotically become the industry monopolist. This result is more rigorously stated by proposition 1. Notice that the asymptotic results of the pairwise Pólya urn in proposition 1 are similar to other well-known results in the economic literature (i.e., the convergence of replicator equations to monopoly, see Metcalfe, 2007). They also recall Fisher's fundamental theorem of natural selection (see Fisher, 1930) in evolutionary biology. However, they are preparatory for next sections, as they allow to interpret the results from the trade model with learning, entry and exit developed in section 3.3.

Proposition 1. *Consider a closed economy with a finite pairwise Pólya urn market allocation process and with a time-constant distribution of firm productivity levels $G(a_i)$. The market structure of this economy converges to monopoly as $K \to \infty$.*

Proof. *See Appendix A* □

The proof of the foregoing proposition is easy to understand in a 2-firms setting. If the same pair of firms is repeatedly drawn, the most productive firm will keep on stealing customers from the least productive one, that will eventually reach a zero size. This argument can easily be generalized to a N-firms setting. In such a framework the least productive firms will continuously lose market shares every time they are selected, eventually reaching a null size and thus a zero probability of being extracted. The firm that will converge faster to zero size is, on average, the least productive one. Once this happens, then the industry has only $N - 1$ firms with positive size. The second least productive firm becomes now the least productive firm with positive market share, and it will lose market shares when selected. The process is reiterated until only one firm, i.e. the most productive firm, controls the whole market.[13]

[13] An equivalent result has been obtained in the finance literature by Blume and Easley (1992); Bottazzi and Giachini (2019) and in quasi-replicator dynamics (Metcalfe, 2007), which have studied a stochastic process similar to the one analyzed here.

Let us now introduce trade in the baseline model by considering a symmetric two-country setting with trade wherein N domestic and N foreign firms have the same initial size in every market as in Krugman (1980). The number of active firms in both markets is thus equal to $2 \cdot N$. Furthermore, we assume that exporters' competitiveness in the foreign market is affected by an iceberg costs $0 \leq c \leq 1$ that decreases a firm productivity in the foreign market:

$$\tilde{a}_{i,t} = a_{i,t}(1 - c) \tag{8}$$

where $\tilde{a}_{i,t}$ denotes the foreign productivity of firm i. Iceberg costs c in our model are a proxy for the inverse of the degree of trade openness. They encompass both the traditional interpretation of geographical distance as well as the level of tariffs which can be applied to exported products, or unfavourable exchange rates. One shall also notice that productivity $a_{i,t}$ determines firm price competitiveness in our model. Indeed, if we assume like in Krugman (1980) and Melitz (2003) that wages are given as *numeraire* and that mark-ups are homogeneous across firms, then firm's i price reads as:[14]

$$p_{i,t} = (1 + \mu)\frac{w}{a_{i,t}} \tag{9}$$

The next proposition shows that, depending on the level of iceberg costs, the market structure in this two countries model can converge either to a domestic monopoly (where one firm absorbs all customers in the domestic market) or to an international monopoly (i.e. a situation where a single firm dominates markets in both countries).

Proposition 2. *Consider a two-country economy with trade where market allocation is determined by a finite pairwise Pólya urn allocation process. Assume the two countries have identical firm productivity level distributions $G(a_{i,t})$. Let $c \in [0, 1]$ be the level of iceberg cost determining a firm foreign productivity according to Equation (8). The following asymptotic results hold as $K \to \infty$:*

1. *if $c = 0$, then the market structure of the two countries converges to an international monopoly*
2. *if $c \in (0, 1)$, then the market structure of the two countries converges either to a domestic or to an international monopoly*

[14] This equation implies that modelling competition trough productivity or prices is equivalent.

3. *if $c = 1$, then the market structure of two countries converges to a domestic monopoly*

Proof. *See Appendix A*

□

The case with zero iceberg cost in the above proposition ($c = 0$) corresponds to a situation where the two economies are perfectly integrated, and where they thus behave as a single market. In this case, as shown by Proposition 1 above, one firm will eventually absorb all customers. When the iceberg cost is positive ($0 < c < 1$), the international monopoly may arise or not and the probability of observing it depends on the level of the iceberg cost as well as on the moments of the productivity distribution (see also the proof of the proposition). Finally, when $c = 1$ then a firm does not have any productive advantage when trading in the foreign market. In such a case the two markets are completely separated and the finite Pólya urn market allocation process will generate local monopolies in both countries.

3.3 Learning, entry-exit and selection: the extended model

The asymptotic results presented in the previous section state that market selection, based on the finite pairwise Pólya process, will generate either a local or an international monopoly depending on the level of iceberg cost. These results were however obtained assuming a static distribution of firm productivity levels. We now extend the two-country model to account for more realistic features. In particular, we introduce a process of firm technological learning that impacts on firm productivity, and we allow for entry and exit of firms. We shall show that monopoly is not a limit market structure in this more general framework and that a more competitive industry emerges.

We model the learning process by following Dosi et al. (1995, 2017). Each incumbent firm increases its productivity according to a geometric random walk with non-negative productivity shocks. The law of motion of firm productivity is:

$$a_{i,t} = a_{i,t-1}\left(1 + max\left\{0, \theta_{i,t}\right\}\right) \tag{10}$$

where $\theta_{i,t}$ is a i.i.d. random shock that proxies for new available technological opportunities and that is drawn from a Beta distribution – i.e. $\theta \sim \mathcal{B}(\beta_1, \beta_2, \beta_{min}, \beta_{max})$.

The learning process described by Equation (10) implies cumulativeness in learning (see Dosi, 1988) as firms improve upon their past productivity level $a_{i,t}$.[15] Moreover, this process has two straightforward consequences for the dynamics of our model. First, it shifts to the right (and rescales) the whole distribution of firm productivity as time goes by. Second, it reshuffles the firm productivity rankings in each period, thus affecting firms odds to grow or shrink according to equation 2. These changes are sufficient to invalidate the results of Propositions 1 and 2, which depend on a time-constant productivity ranking of firms.[16]

Another important difference with respect to the baseline model presented in the previous sections, is the presence of a variable number of firms in each market as a result of entry and exit. We assume that exiting firms are firms whose market share in period t is lower than a strictly positive threshold δ.[17] Exit from one market does not automatically imply the failure of a firm, as the same firm could still be active in the market of the other country. We assume that a firm fails whenever its market share is lower than δ in both markets.

Furthermore, in each period t firms that are not yet active in a market try to enter it by competing with incumbents according to the pairwise Pólya competition illustrated in the previous section. The pool of potential entrants is composed by foreign firms that are not yet present in the market as well as by domestic firms that exited previously. In addition, entry in the model occurs also to replace failed firms. When a firm from country A goes bankrupt, a newborn firm belonging to the same country tries to enter by competing with incumbents in the domestic market. Similarly to Dosi et al. (2010, 2017), this newborn firm has an initial productivity level that is "copied" from the one of a randomly selected domestic incumbent. [18]

Each potential entrant competes with an incumbent that is randomly selected with a probability proportional to its market share. Let s_{jt} denote the size of the selected incumbent at time t. If the potential entrant has a higher productivity level than the incumbent then it actually enters in the

[15]In addition, productivity shocks in Equation (10) are non-negative as we assume that a firm is sufficiently rational not to transform the production process if the outcome of learning implies the adoption of a technique that is inferior to the old one.

[16]It is also worth noticing that, notwithstanding the absence of strictly negative productivity shocks in equation 10, a positive productivity shock, but smaller than average, might have a negative impact on firm competitiveness.

[17]This is also consistent with the fixed cost mechanism of Melitz (2003) if one implicitly assumes that firms with $s_{i,t} < \delta$ are also unable to pay for these costs.

[18]The copied incumbent can be either a domestic or foreign exporters. In the latter case the copied productivity level is discounted by a factor τ that captures the extent of differences in country-level technological structure and is therefore a proxy of a country absorptive capacity.

market by stealing η customers from the incumbent, where η is an integer drawn from a truncated Poisson distribution $P(\delta \cdot M)$ such that $1 \leq \eta \leq s_{jt}$.[19]

In each period, the following order of events takes place simultaneously in the domestic and the foreign markets:

1. incumbent firms update their productivity levels according to learning process described in Equation 10;
2. entry of new firms occurs;
3. firm market shares are reallocated;
4. exit of firms occurs.

4 Simulation results

We have fully characterized the dynamics of the baseline model in a closed form. This is not feasible for the extended model with firm learning and firm entry-exit that we described in the previous section. We thus resort to Monte Carlo simulations for its analysis.[20]

Numerical simulations aim to show that the extended model is able to *jointly* reproduce the empirical evidence on industrial dynamics and international trade. We thus present our results in relation to the list of stylised facts described in section 2. We investigate the emergence of these stylized facts in relation to the parameters of the model governing the market selection intensity and the level of trade openness. The first is captured by the number of pairwise interactions K, while the second is inversely related to the level of iceberg costs c. In the next sections, and unless stated otherwise, we shall refer to these two parameters jointly as "the competition regime" Table 2 summarizes the different competition regimes we use in our Monte Carlo exercises, while Table 3 reports the value of the other parameters (and that are kept fixed in the various simulations exercises). Notice that our model features a relatively small number of parameters with respect to

[19]The foregoing competitive process for market entry avoids that new firms always enter with a size that is close to the exit threshold, something that would inflate firm exits and market turbulence in the model. Also notice that the mean of the Poisson distribution of entrants' customers is equal to the exit threshold multiplied by the market size. This allows us to match the empirical evidence that entrants have heterogeneous size, but are on average smaller than incumbents (Dunne et al., 1988; Geroski, 1995; Bartelsman et al., 2003).

[20]In a Monte Carlo analysis the target statistics are computed as averages in multiple instances of the same model, with same parameters, but with different pseudo random draws. For a comparison between different scenarios instead, parameters are varied while the pseudo random draws are kept constant.

the large number of stylised facts that it aims to replicate. In addition, notice that our model starts from a complete homogeneity condition, at the firm and country level (see also Table 3). Firms and countries then become heterogeneous as the dynamics of the model unfolds. Accordingly firm heterogeneity (in size, productivity and export) and country heterogeneity (in productivity and export volume) are therefore a fully endogenous property of the model. Finally, as far as the results at the firm level, statistical tests of equality in means do not reject the assumption of equality between the results for the two countries. Thus, in what follows we report only the results for one economy.

Parameters		**Competition regimes**	
K	c	*Selection Intensity*	*Trade Openness*
500	0.5		Low
500	0.25	Low	Medium
500	0		High
750	0.5		Low
750	0.25	High	Medium
750	0		High

Table 2: Competition regimes used in the Monte Carlo simulations.

Parameters	**Value**	**Description**
N	250	Number of firms in each country
T	400	Number of time steps
δ	0.001	Threshold market share
s_0	100	Size at time 0
a_0	1	Productivity at time 0
β_1	5	Shape parameter of the learning shock
β_2	5	Scale parameter of the learning shock
β_{min}	-0.25	Minimal learning shock
β_{max}	0.25	Maximal learning shock

Table 3: Model parametrization used in the Monte Carlo simulations.

4.1 Competition and industry dynamics

Competition regime		**Business and Industry Statistics**				
Selection Intensity	*Trade Openness*	*Exit Rate*	*Turbulence Index*	*HHI*	*Domestic Firms Share*	*Foreign Firms Share*
Low	Low	0.075 (0.006)	0.081 (0.005)	0.042 (0.023)	0.893 (0.017)	0.107 (0.035)
	Medium	0.113 (0.007)	0.125 (0.005)	0.05 (0.028)	0.844 (0.021)	0.339 (0.055)
	High	0.141 (0.009)	0.167 (0.016)	0.065 (0.042)	0.659 (0.018)	0.658 (0.021)
High	Low	0.091 (0.007)	0.1 (0.005)	0.052 (0.027)	0.858 (0.017)	0.068 (0.033)
	Medium	0.133 (0.006)	0.145 (0.005)	0.056 (0.025)	0.817 (0.022)	0.274 (0.049)
	High	0.165 (0.008)	0.202 (0.018)	0.06 (0.031)	0.627 (0.017)	0.621 (0.019)

Table 4: *Exit Rate* is the share of exiting firms on the total incumbents, *Turbulence Index* is the total sum of absolute change in market shares, *HHI* is the Hirschman-Herfindahl Index at the industry level, *Domestic Firms Share* and *Foreign Firms Share* are respectively the share of domestic and foreign incumbents with respect to the maximum number of firms in the market (N). Standard errors in parentheses.

We begin by looking at the effects of competition regimes on variables related to market turbulence (see Table 4). These effects are well established in the trade and industry dynamics literature (see also Section 2) and their replication represents a first test of the empirical performance of our model. We observe that an increase in either selection intensity or trade openness increases exit rates as well as the turbulence index (measured as the total sum of the absolute changes in market shares). A key property of our model is that pairwise competition produces a reallocation of customers towards more productive firms. Increasing selection intensity or lowering iceberg costs clearly boosts the foregoing reallocation process. The result is that a higher share of low productivity firms is driven out of the market and that more productive firms become bigger. The latter process also results in higher levels of market concentration, measured by the Herfindahl-Hirschman index (cf. the third column Table 4). Accordingly, while one might expect that opening a closed economy would generate a more competitive market structure, our results suggest that this is not always the case (in line with the predictions of evolutionary models, see e.g. Dosi et al., 1995).

Next, we look at the effects of competition regimes on industry output volatility. The work of

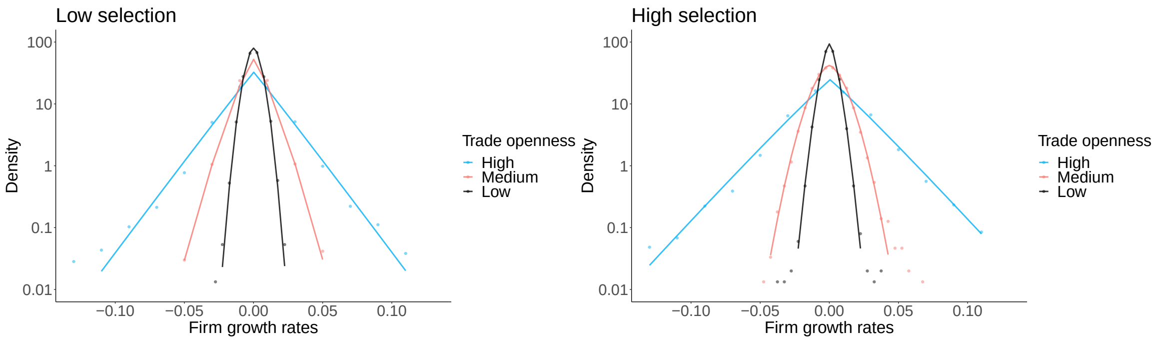

Figure 1: Pooled distribution of industry growth rate (points) and exponential power fit (line). The y-axis is in log-scale.

di Giovanni and Levchenko (2009) suggests that trade openness is positively correlated with industry growth rate since it increases the export opportunities for all the firms belonging to the industry (**SF1** in Table 1). The results in the first column of Table 5 corroborate this evidence. We also fit the pooled sample of industry growth rates in a given scenario with an Exponential-Power distribution. Such a distribution is very flexible as it encompasses both the Normal distribution as well as the Laplace distribution characterized by fat tails (see Bottazzi and Secchi, 2006a; Fagiolo et al., 2008; Castaldi and Dosi, 2009). [21]. The second column of Table 5 indicates that the shape parameter ("b") of the fitted Exponential-Power distributions is always below 2. Thus, regardless of the competition regime, our model generates industry growth rate distributions that deviate from the Normal benchmark and display fat-tails, in line with **SF2** (see Table 1). Moreover, the shape parameter falls when trade openness increases and this effect is magnified by higher selection intensity. The value of the scale parameter of the distribution - which is related to industry growth volatility - follows an opposite trend, as it increases with trade openness and selection intensity. These results are explained by the fact that a decrease in trade barriers removes the productivity gaps between domestic and foreign firms. This boosts the process of reallocation of market shares among firms, thus generating more volatile and more leptokurtic industry growth rate distributions.

[21] More precisely, the Exponential-Power is a class of distributions with three parameters (mean, scale (a) and shape (b)). The shape parameter b is a proxy of thickness of the tails of the distribution, as it determines how fast the probability function approaches its extremes. The Normal and Laplace distributions, for example, are particular cases of the Exponential Power with $b = 2$and $b = 1$ respectively. Thus we can discriminate between the goodness of their relative approximations by fitting an EP distribution.

Competition regime			**Exponential Power Fit**	
Selection Intensity	*Trade Openness*	*Average Volatility*	b	a
Low	Low	0.005 (0.001)	1.806	0.005
	Medium	0.01 (0.002)	1.268	0.010
	High	0.02 (0.011)	1.022	0.021
High	Low	0.005 (0.001)	1.526	0.005
	Medium	0.01 (0.002)	1.691	0.010
	High	0.026 (0.009)	1.050	0.027

Table 5: Monte Carlo average of the growth rate volatility of industry output And fitted Exponential-Power parameters. b and a are respectively the shape and scale parameters.

4.2 Competition and intra-industry trade

Competition regimes also impact on the characteristics of the population of active firms in a market. The last two columns of Table 4 indicate that, when trade openness is low, almost all the incumbents are domestic firms. When openness is high, instead, incumbents are evenly spread between domestic and foreign firms. It follows that our model produces a monotonic increasing (decreasing) relation between trade openness and the presence of foreign (domestic) incumbents in a market.

Table 6 takes a deeper look at the trade dynamics emerging in our model by reporting statistics on export margins, intra-industry trade (Balassa, 1966; Grubel and Lloyd, 1975) and trade flows in different competition regimes. In line with empirical evidence (cf. **SF3** in Table 1), the extensive margin (firm participation in foreign markets), the intensive margin (the average firm-level export flow) as well as the measures of bilateral intra-industry trade flows are all positively related to trade openness data. When barriers to trade decrease, more firms export, and they export more (see Table 6). Accordingly, the bilateral trade flows increase on average and they become more volatile (cf. the last two columns of the Table). This is coherent with the stream of empirical literature which has shown the existence of a inverse relation between country-level distance (which reflects entry barriers in the foreign markets) and foreign market participation or intra-industry trade flows

Competition regime		Export Margins		Intra-Industry Trade		Export Flows	
Selection Intensity	*Trade Openness*	*Extensive*	*Intensive*	*Bilateral Trade*	*Grubel-Lloyd Index*	*Mean*	*St. Deviation*
Low	Low	0.1 (0.036)	28.818 (1.818)	0.031 (0.008)	0.674 (0.112)	0.03 (0.013)	0.013 (0.009)
	Medium	0.333 (0.055)	40.777 (13.058)	0.137 (0.018)	0.73 (0.11)	0.136 (0.038)	0.04 (0.026)
	High	0.659 (0.024)	75.29 (12.776)	0.48 (0.01)	0.84 (0.056)	0.488 (0.06)	0.076 (0.029)
High	Low	0.067 (0.027)	27.99 (1.694)	0.019 (0.008)	0.641 (0.112)	0.019 (0.009)	0.01 (0.006)
	Medium	0.274 (0.051)	37.821 (5.133)	0.106 (0.015)	0.708 (0.108)	0.106 (0.03)	0.037 (0.017)
	High	0.62 (0.022)	75.609 (11.119)	0.476 (0.011)	0.825 (0.054)	0.464 (0.059)	0.084 (0.025)

Table 6: Margins, bilateral and intra-industry trade for different competition regimes. Standard errors in parentheses. The extensive margin is defined as the share of exporters, the intensive margin in terms of average exported sales per firm. The bilateral trade is the total bilateral export divided the world total production. Export flows are defined as the export share on the total country production.

(Bernard et al., 2011, 2012). However, notice the share of exporting firms is barely 60% in regimes with high openness, which indicates that more than 1/3 of firms are excluded from participating to the export market. The latter firms are low productivity firms that are unable to enter foreign markets as they are outperformed by bigger and more productive domestic and foreign incumbents.

We also investigate how competition regimes affect the export status and the probability of survival in foreign markets. Table 7 reveals that our model generates a very persistent export status (see **SF4**). Moreover, export persistence decreases with openness, hinting at a higher turbulence in presence of more trade flows. Table 7 also indicates that the share of exporting firms is pretty high and invariant across competition regimes. In contrast, less than half of new exporters survive after the first year of entry in the foreign market (in line with **SF5)** and the probability of survival marginally decreases with stronger selection. The latter two results are inherent to the strong competitive pressure a firm is subject to when it enters a foreign market. Indeed, the pairwise selection process we introduced in Section 3.3 implies that foreign entrants will typically compete with bigger and thus more productive firms, at the time of their entry in the market but also afterwards. It follows that only a small fraction of these potential entrants, i.e. those with high productivity, will be able to outperform incumbents and acquire a stable presence in the foreign market.

Competition Regime		**Export Status**		
Selection Intensity	*Trade Openness*	*Export Status Persistence*	*New Exporters Survival*	*All Exporters Survival*
Low	Low	0.946 (0.02)	0.427 (0.037)	0.878 (0.022)
	Medium	0.861 (0.024)	0.433 (0.022)	0.852 (0.012)
	High	0.773 (0.013)	0.455 (0.014)	0.867 (0.008)
High	Low	0.952 (0.022)	0.386 (0.043)	0.897 (0.035)
	Medium	0.848 (0.025)	0.404 (0.018)	0.834 (0.012)
	High	0.741 (0.012)	0.439 (0.015)	0.851 (0.007)

Table 7: Model-generated statistics on export persistence and survival. *Export Status Persistence* is the Monte Carlo average of the share of incumbents whose export status does not change from one period to the other. *New Exporters Survival* is the share of newly exporting firms that survive after the first year. *All Exporters Survival* is the share of exporters that survives from one period to the other.

Competition Regime		**Export Margins**		
Selection Intensity	*Trade Openness*	*T%*	*Ratio*	*%Ratio>1*
Low	Low	0.072 (0.02)	0.118 (0.021)	0
	Medium	0.208 (0.047)	0.361 (0.131)	1
	High	0.563 (0.044)	1.261 (0.182)	95
High	Low	0.072 (0.017)	0.114 (0.02)	0
	Mild	0.203 (0.057)	0.353 (0.124)	0
	High	0.602 (0.047)	1.426 (0.204)	100

Table 8: The decomposition of the total export in the contribution from the extensive and the intensive margin. $T\%$ is the percentage of time step in which the intensive was greater than the extensive margin, $Ratio$ is the average ratio of intensive to extensive margin, $\%Ratio > 1$ is the percentage of times in which the ratio is greater than 1 across simulations.

Finally, the work of Bernard et al. (2009) decomposes the rate of change of export activity and finds that the intensive margin plays a more relevant role over the extensive one. Our simulations data allows us to verify whether this claim holds true in our model. The results in Table 8 confirm the findings of Bernard et al. (2009). In particular, our model predicts that intensive margin is more important than the extensive one only when trade openness is maximal (i.e. when iceberg costs are null). With a positive iceberg cost instead (i.e. in the "Low" and "Medium" openness scenarios), the extensive margin effect dominates the intensive margin one. Such results are also coherent with the work of Eaton et al. (2011), which finds that the intensive margin dominates the extensive one in presence of a fall in trade barriers.

4.3 Statistical properties of firm size, growth and age distributions

The results discussed in the previous section document how our model predicts an increase in exit rates and in market turbulence when competitive pressures are higher. We now turn to analyse how the model fares in reproducing the main statistical properties concerning the firm size, growth and age distributions. The empirical evidence discussed in Section 2 indicates that firms not only display wide asymmetries in productivity levels. They also remarkably differ in their sizes at every level of aggregation. In particular, several studies have pointed out that the size distribution of firms is characterized by a leptokurtic right tail, fatter than the one produced by a log-Normal distribution (cf. **SF6** in Table 1). Figure 2 shows that our model reproduces this property. The pooled distributions of firm sizes generated from our model deviate from the Log-normal in all competition regimes we consider.

Dynamic increasing returns in firm selection explain the foregoing shape of the size distribution. This is because the Pólya urn selection mechanism implies that bigger and more productive firms are more likely to attract customers and to grow. We also estimate the tail exponent of the distribution and we find that the hypothesis that the firm size distribution is a Pareto is not rejected in any scenario analysed (see Table 9). [22] In addition, most fitted coefficients in Table 9 are smaller than 3. This indicates that relative differences in firm size are so wide that only the first and the second moments of the distribution exist.

[22] The Pareto fit is tested statistically by means of the test of Clauset et al. (2009), that builds upon a Kolmogorov-Smirnov test.

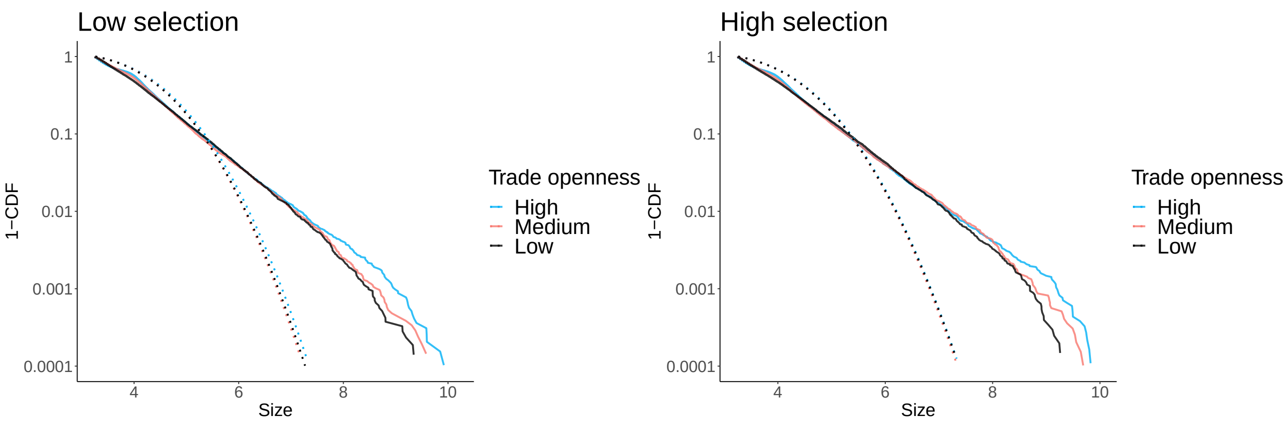

Figure 2: Log-centred size distribution (lines) and fitted log-Normal distribution (dots) across trade regimes. Size is the sum of domestic and foreign sales. Results are pooled over Monte Carlo simulations.

Competition Regime		**Pareto Estimates**				
Selection Intensity	*Trade Openness*	*Aggregate*	e^5	e^6	e^7	e^8
	Low	2.249	2.272	2.349	2.627	3.052
Low	Medium	2.349	2.273	2.271	2.426	2.685
	High	2.244	2.240	2.205	2.273	2.531
	Low	2.203	2.237	2.322	2.460	2.925
High	Medium	2.227	2.187	2.148	2.342	2.978
	High	2.244	2.217	2.175	2.215	2.326

Table 9: The fitted coefficient α of the Pareto distribution $P(x) = x^{-\alpha}$ for the aggregate empirical distribution of firm size and for different cut-off points of the empirical distribution ($\{e^5, e^6, e^7, e^8\}$ corresponding to $\{5, 6, 7, 8\}$ in Figure 2). The Pareto distributions were fitted by using the procedure described in Clauset et al. (2009). It is not possible to reject a Pareto fit at the 5% confidence level for all Pareto estimates.

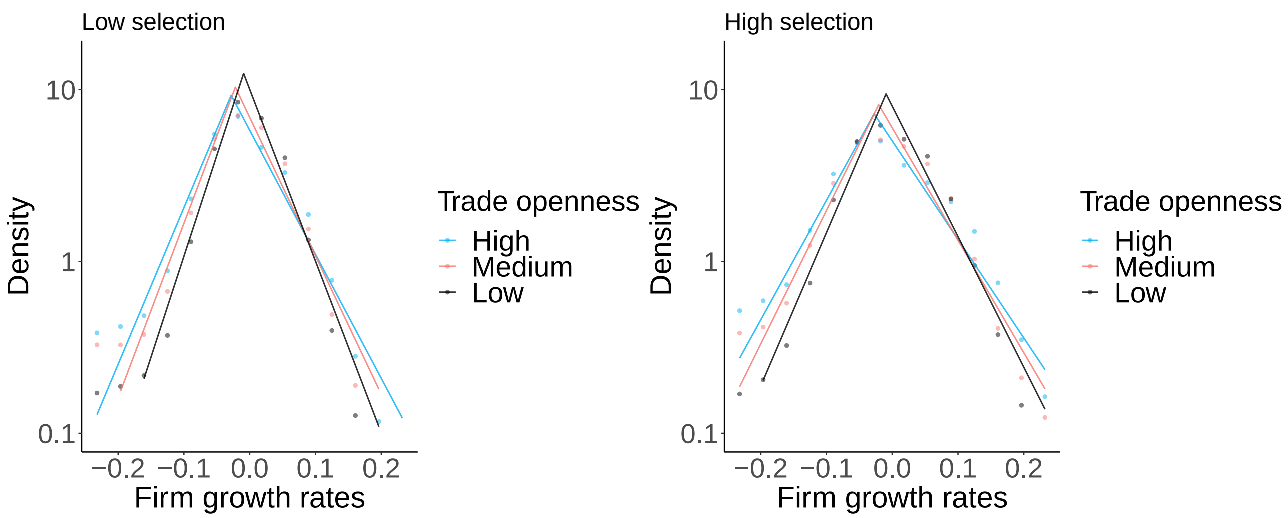

Figure 3: Pooled growth rates distribution with Laplace distribution fit across competition regimes. The y-axis is in log-scale.

Increasing returns also generate Laplace-like firm growth rate distributions (see **SF6** of Table 1) in our model. See Figure 3. As firms do not have the same probability to compete for customers, shocks to sales are not identically distributed across firms. Since what is gained by a firm is lost by another in the model, shocks are not independently distributed as well. These two properties entails a violation of the i.i.d. distribution of shocks that would instead generate a Normal distribution of firm growth rates because of the Central Limit Theorem (see also Fu et al., 2005; Bottazzi and Secchi, 2003, 2006a, for an explanation of Laplace distributed firm growth rates along the same lines). Furthermore, Figure 3 also shows that an increase in trade openness increases firm volatility. The Laplace distributions of firm growth rates are more dispersed when openness is higher and this results holds irrespectively of the selection intensity. Thus, the effect of removing trade barriers on volatility, that we already spotted at the industry level in Section 4.1, is also replicated at the firm level. This effect is explained by the increase in the pool of customers that the higher trade openness allows to compete for. When trade barriers are lower firms can enter more easily in foreign markets and thus attract not only domestic but also foreign customers. This increases their growth potential (both positively and negatively).

Firm growth volatility is also inversely related to firm size (see **SF8** in Table 1). This implies that smaller firms are more likely to experiment very high or very low growth rates. Our model is able to replicate this stylised fact, which is depicted in Figure 4). The linear fit in the log-log space approximates fairly well the relations in the different competition regimes and do not change when considering the regression on pooled data or the Monte Carlo average coefficient of the regression (Table 10). The relation shifts upward and the slope increases when the market selection strengthens and when trade openness gets higher.[23]

Finally, our model is also able to reproduce an exponential distribution of firm age (as highlighted by Calvino et al., 2021, cf. **SF9**). See Figure 5. The exponent of the distribution, estimated with a log-log regression relating numerosity to age, is negative and it decreases with the level of competition

[23]The above documented inverse relation between firm growth rates and firm size is often considered as a violation of the Gibrat's Law of Proportionate Effects, which predicts the independence of firm growth rates from firm size (see Dosi, 2007). The violation of the Gibrat's law in our model should not surprise because, as we already mentioned, growth shocks are by construction not independent in our model. The gain of a customer by one firm directly corresponds to a loss for another one. As a further robustness check on the violation of the Gibrat's law we have estimated several independent Gibrat regressions in all competition regimes we consider. The null hypothesis that the regression coefficient of firm size is 1 is rejected in most cases.

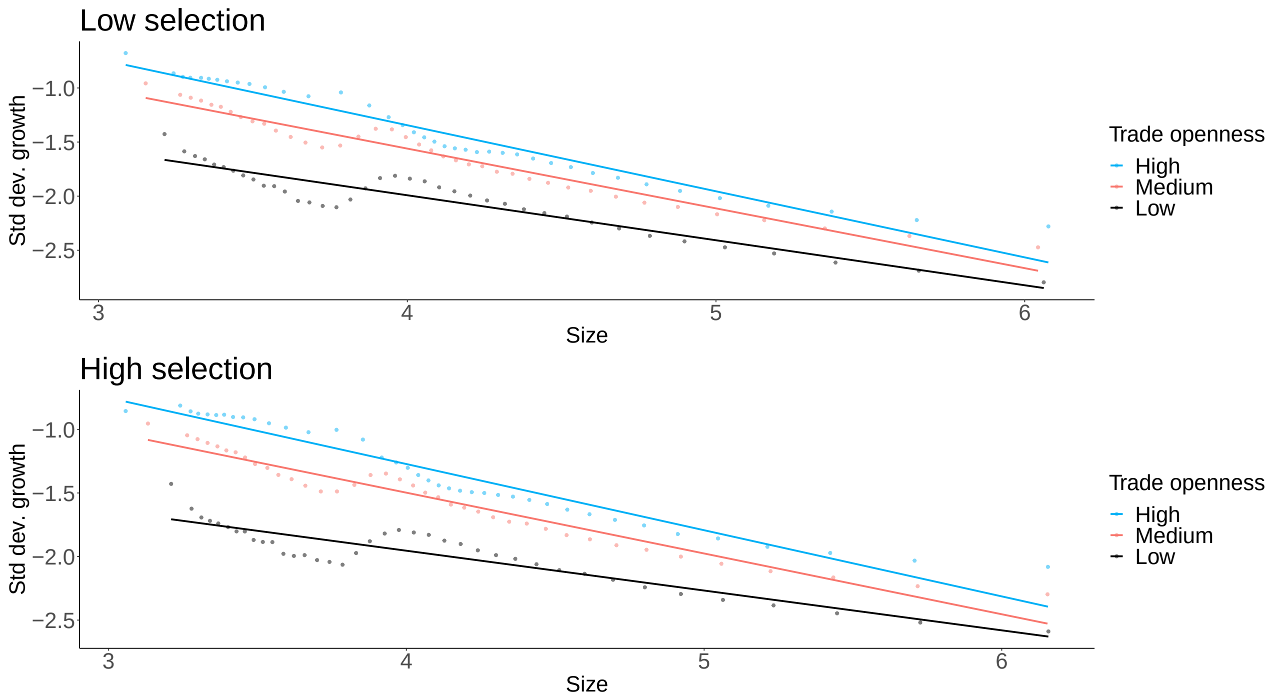

Figure 4: Scaling growth variance relation with fitted binned linear regression across trade regimes. Data points have been computed on equally sized bins of firm growth and size. Both the x and the y-axis are in log-scale.

Competition Regime		**Pooled OLS**		**Monte Carlo OLS**		
Selection Intensity	*Trade Openness*	$\hat{\beta}$	R^2	$\bar{\hat{\beta}}$	$\bar{R}^2$	*% rej.* H_0
Low	Low	-0.416 (0.025)	0.876	-0.386 (0.092)	0.664	1
	Medium	-0.553 (0.02)	0.955	-0.533 (0.066)	0.896	1
	High	-0.611 (0.022)	0.954	-0.583 (0.036)	0.911	1
High	Low	-0.313 (0.024)	0.822	-0.279 (0.104)	0.544	0.96
	Medium	-0.478 (0.018)	0.949	-0.462 (0.049)	0.891	1
	High	-0.521 (0.021)	0.941	-0.503 (0.032)	0.896	1

The * indicates significance at 0.1% confidence level. $H_0 : \hat{\beta} = 0$.

Table 10: Estimated coefficient and R^2 for the linear regression fitted to the binned points of the scaling growth variance relation of Figure 4 (columns 1 and 2) and Monte Carlo averages of the coefficient and of the R^2 from the regression fit at the end of every simulation (columns 3 and 4). The null hypothesis of independence between size and growth rate standard deviation (null fitted coefficient) is always refused (column 5).

(see Table 11). Higher competition also exerts a positive influence on the rate of exit (cfr. Table 4) and thus affects the slope of the age distribution.

Competition Regime		**Firm Age Regression**	
Selection Intensity	*Trade Openness*	β	R^2
	Low	-0.05	0.97
Low	Medium	-0.068	0.987
	High	-0.083	0.996
	Low	-0.066	0.97
High	Medium	-0.083	0.984
	High	-0.105	0.99

All estimates are significance at the 0.1% confidence level.

Table 11: Estimated coefficient and R^2 for the log-log linear model relating the numerosity of each firm age profile to firm age values. These numbers represent the slope and the quality of the dashed lines fitted in Figure 5.

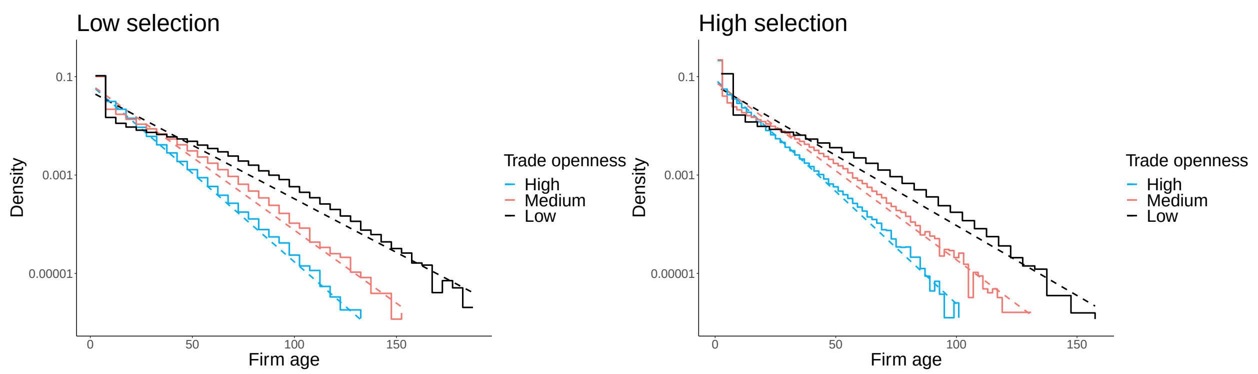

Figure 5: Pooled distribution of firm-level age across different regimes with fitted exponential distributions (the dashed lines). The y-axis is in log scale.

4.4 Firms in international trade

We now focus on the ability of the model to reproduce the stylized facts relating to the characteristics of exporting and non-exporting firms. Figure 6 shows the kernel density of firm productivity levels (rescaled by the industry mean) in the six different competition regimes we consider. In line with empirical evidence (see **SF10** in Table 1) our model endogenously generates a productivity distribution which is Log-Normal and thus characterised by right-skewness. This is an indication of the presence of wide productivity differentials between the firms. Moreover, as in Melitz (2003), we observe a clear selection effect following an increases in trade openness. High trade barriers (low trade openness) generate more dispersed productivity distributions. They also generate fatter right tails (see Figure 6 and Table 12).

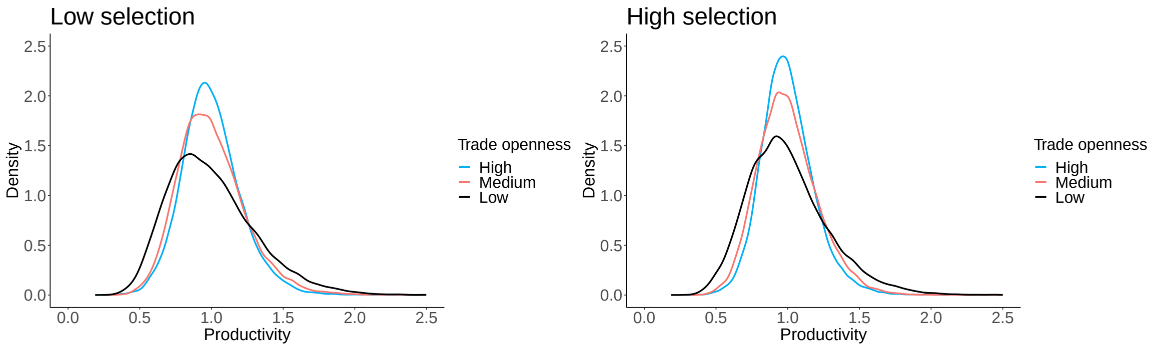

Figure 6: Kernels densities of firm productivity levels across competition regimes regimes. The densities are computed on the sample of firm productivity levels pooled over Monte Carlo simulations and normalized by the average productivity level.

Competition Regime		**Distribution Moments**		
Selection Intensity	*Trade Openness*	*Std. Dev.*	*Skewness*	*Kurtosis*
	Low	0.304	0.79	3.833
Low	Medium	0.235	0.743	4.287
	High	0.207	0.546	4.192
	Low	0.28	0.765	3.944
High	Medium	0.211	0.594	4.062
	High	0.185	0.572	4.186

Table 12: Standard deviation, skewness and kurtosis of the distribution of firm productivity levels (rescaled by the industry mean). Results are pooled over Monte Carlo simulations.

Furthermore, coherently with **SF12**, our model predicts that exporters are not a random subgroup of all firms in terms of productivity. Figure 7 shows that exporters are on average more productive than non-exporters in all competition regimes. However, the two distributions of exporters and non-exporters overlap, which indicates that high-productivity non-exporters co-exist with low-productivity exporters (cf. **SF11**). The foregoing properties are usually explained on the grounds of imperfect self-selection or unobserved heterogeneity at the firm level (e.g. because entry costs or financial constraints, see Impullitti et al., 2013; Mayer et al., 2014; Assenza et al., 2016; Guerini et al., 2021). Our model instead generates them from the interaction between imperfect market selection and firm learning. First, the existence of trade barriers and the fact that market selection rewards more productive firms implies that entry in foreign market depends on a productivity premium. Second, firm productivity changes over time because of learning. Third, firms are selected to compete on their basis of their size, not of their productivity. This three features imply that foreign firms whose productivity increases more relative to other firms in the market gain further market shares and thus participate more to the process of reallocation of customers. In contrast, foreign entrants whose productivity grows relatively less will keep small but stable market shares because they will not be selected for the reallocation of further customers.

The above considerations also help to understand how the two distributions of exporters and non-exporters change across competition regimes. Higher trade barriers increase the average productivity premium of exporters and contribute to widen the gap between the distribution of exporters and non-exporters (cf. the plots Figure 7 from right to left). This is explained by the fact that when trade barriers - and so iceberg costs - are high, less productive firms have more difficulties in entering foreign markets. Indeed, a low productivity firm already has a low likelihood of winning the pairwise match with another incumbent firm. When iceberg costs increase, such a probability decreases further.

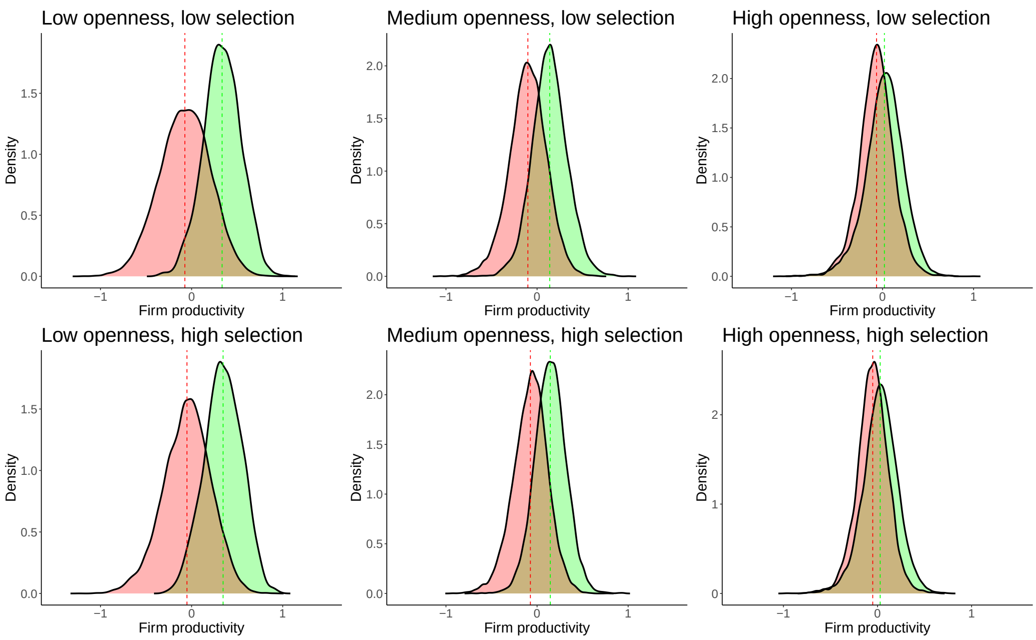

Figure 7: Log-centred productivity distribution conditional upon the export status (exporters in green; non-exporters in red). Dashed lines represent conditional averages. Results are pooled over Monte Carlo simulations.

Similar considerations apply to the relation between export status and firm size. The empirical evidence indicates that on average exporters are larger than non-exporters (see **SF14** in Table 1).

At the same time, there are some large non-exporting firms as well as small exporting firms (cf. **SF13**). Our model is also able to replicate this feature, which is presented in Figure 8. Furthermore, in line with di Giovanni et al. (2011), lower trade barriers increase the average size difference between exporters and non-exporters. This effect, can be explained by looking at the distributions of exporters' market shares generated by our model (see Figure 9). These distributions deviate from the Log-Normal benchmark in all competition regimes, which indicates that few firms are responsible for a large bulk of the export activity (a result in line with **SF15**). Notice that the deviation from the Log-Normal benchmark is stronger when trade openness is higher. As we already noticed above, removing trade barriers unleash competition among firms generating higher levels of market concentration. The result is that few and very productive firms are able to gain large market shares both in local as well as in foreign markets.

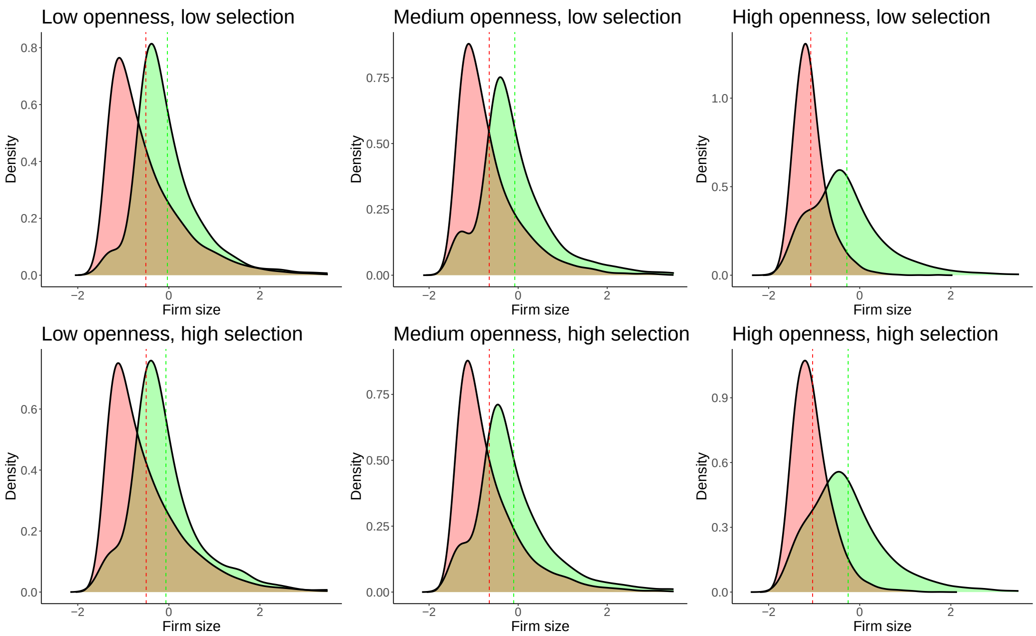

Figure 8: Log-normalized size densities (pooled from the last period of each simulation) of exporters (green) against non-exporters (red). The dashed lines are the averages.

A branch of the empirical trade literature has also investigated the relation between firm export activity and firm growth rate volatility, finding that these two variables are positively related (cf **SF17**). Table 13 reports the model-generated firm growth volatilities: those conditional upon export status (first two columns) and the unconditional one (last column). In line with the empirical evidence, exporters' growth is more volatile than the non-exporters one. However, the wedge between the two volatilities decreases with trade openness. Clearly, higher trade openness removes

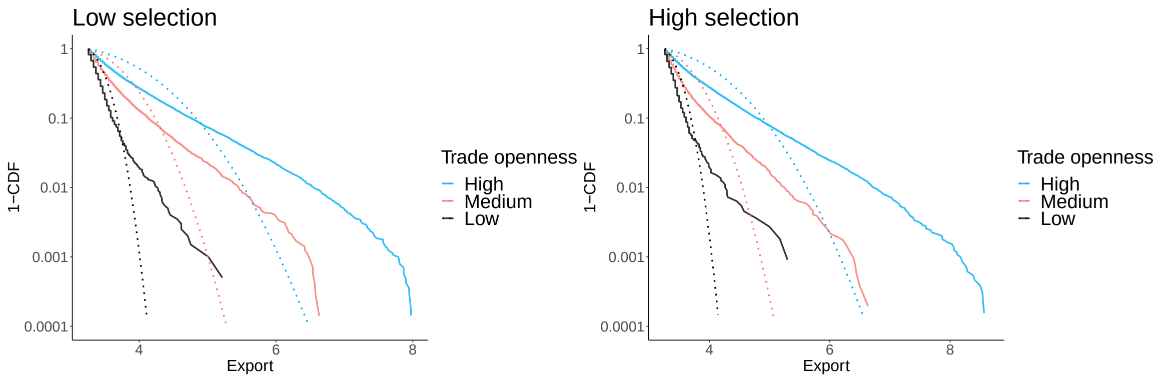

Figure 9: Pooled complementary cumulative distributions of exporters size in the local market (continuous lines) against the log-normal fit (dotted lines). Both the x and the y-axis are in log terms.

the shield from competition represented by iceberg costs. The result is that domestic firms have to compete with more foreign firms, which maps into a more more volatile growth process for every firm in the market.

Competition Regime		**Firm Growth Rate Volatility**		
Selection Intensity	*Trade Openness*	*Exporters Volatility*	*Non − exporters Volatility*	*All firms Volatility*
Low	Low	0.306 (0.008)	0.091 (0.007)	0.141 (0.02)
	Medium	0.306 (0.005)	0.142 (0.007)	0.232 (0.02)
	High	0.31 (0.008)	0.227 (0.02)	0.309 (0.019)
High	Low	0.339 (0.011)	0.099 (0.003)	0.144 (0.022)
	Medium	0.333 (0.011)	0.148 (0.01)	0.243 (0.019)
	High	0.33 (0.005)	0.233 (0.023)	0.324 (0.009)

Table 13: Monte Carlo averages of the firm growth volatility of the of exporters, non-exporters and all firms.

Finally, a large literature has analysed of the role that innovative activity plays in determining export performance. The main empirical conclusion is that innovation positively affects firms' participation in foreign markets as well as their performances (see **SF18**). In our model, the effects of innovation activity are captured by the learning process, which brings permanent shocks to the productivity level of a firm. Table 14 reports the correlation emerging in our model between innovation activity on the one hand and export participation and export volume on the other hand.[24] The Table indicates that innovation activities are positively correlated with both firm export participation and export volumes. Moreover, correlations increase in regimes with higher trade openness. As we already observed, the decrease of trade barriers removes the impediments to entry and growth in foreign markets. This implies that - for the same level of learning activity - a large share of firms is able to enter in foreign markets. These entrant firms are also able to reap a higher number of customers from incumbent firms. This suggests that the link between innovation activities and export participation and volumes become stronger in more open markets.

Competition Regime		**Correlation with Innovation Activity**	
Selection	*Openness*	*Export volume*	*Export participation*
Low	Low	0.328 (0.039)	0.659 (0.036)
	Medium	0.476 (0.06)	0.833 (0.021)
	High	0.542 (0.067)	0.913 (0.005)
High	Low	0.252 (0.041)	0.59 (0.042)
	Medium	0.431 (0.061)	0.793 (0.022)
	High	0.54 (0.051)	0.904 (0.005)

Table 14: Correlation between innovation activity (frequency of firm learning events), export volume (average export shares) and firm export participation (its average frequency of positive export events).

4.5 The dynamics of international leadership at the country level

The firm heterogeneity endogenously generated by our model maps, via aggregation, onto country heterogeneity. The result is that the two countries that are initially symmetric become different over time in terms of productivity level and export volume. In our two-country setting, this implies the presence of a leader country and of a laggard one. Accordingly, the model allows us to investigate the effects that competition exerts upon international leadership. It is indeed well known that the

[24] In our model innovation activity is not endogenously modelled but is implicit into the stochastic process of learning. Here we use as a proxy of innovation activity, the number of times an individual firm improves its productivity by obtaining a positive shock in equation (10).

Competition Regime		Length of Leadership Lock-In		Number of Leadership Catch-Ups	
Selection	*Openness*	*Productivity*	*Export*	*Productivity*	*Export*
Low	Low	21.373 (15.659)	15.937 (11.361)	16.38 (7.635)	21.95 (9.892)
	Medium	14.129 (7.12)	32.746 (34.336)	23.56 (7.359)	10.53 (5.489)
	High	5.156 (1.133)	38.061 (19.85)	57.58 (9.593)	7.98 (3.673)
High	Low	18.073 (9.782)	10.18 (4.947)	18.1 (6.15)	31.97 (9.224)
	Medium	12.1 (5.338)	27.665 (17.227)	25.7 (7.484)	12.56 (6.155)
	High	4.783 (0.855)	34.346 (13.908)	62.48 (8.406)	8.59 (2.836)

Table 15: Monte Carlo averages for the length of lock-ins and for the number of catch-up in both productivity and export.

international leadership of countries (see Abramovitz, 1986) and of their sectors (see Malerba and Nelson, 2011; Lee and Malerba, 2017) may suffer from episodes of catching-up by new players.

We study the results of the model in terms of productivity and export catch-ups in Table 15 by reporting the Monte Carlo averages of the length of lock-ins (average period of productivity/export superiority) and of the number of catch-up (number of times a country is able to overcome the other in terms of productivity level or export volumes) in both productivity level and export volumes.[25] Overall, the results suggest that our model is able to endogenously generate both persistent asymmetries in productivity and export volume across countries, as witnessed by the average length of lock-ins periods, and a dynamics of international catch-up, whose numbers are significantly larger than 0 in all the considered cases.

Table 15 also suggests that trade openness has contrasting effects on productivity and export catch-ups. On the one hand, higher trade openness allows for more catch-ups in terms of productivity. This is because higher trade openness is also associated to both higher business dynamism and extensive margins of export (see previous sections). On the other hand, higher trade openness decreases the number of catch-ups in export volume. Apparently counter-intuitive, this result is explained by the fact that it takes time for firms to transform productivity advantages into export share leadership. Technological competition is indeed mediated by size in the model. This implies that innovative entrants will compete with lower probability as they are smaller than incumbents.

[25]A country is the leader in terms of productivity (respectively, export volume) if the average firm productivity (respectively, the sum of firms' exports) of its firms is the greatest.

In presence of higher trade openness it will thus take more time for firms in laggard countries to reverse the export gap. Finally, the intensity of selection has a more straightforward relation with catch-ups. Indeed, higher intensity increases the number of catch-ups both in productivity and export volumes. This result is explained by the fact that Increases in selection intensity generate higher entry/exit dynamics and larger reallocation toward the international technological leader.

5 Concluding remarks

We have developed a simple dynamic model that aims at replicating the most important stylized facts of firm, industry and international trade dynamics on the basis of the interaction between simple processes of firm learning and market selection. In the model, firm learning takes place cumulatively and determines firm productivity and competitiveness. Market selection, is governed by a *finite pairwise Pólya urn process*, which incorporates dynamic increasing returns in firm size and rewards bigger and more productive firms.

We have proved that, absent firm learning, our model generates a market structure converging to a monopoly (either a national or an international one) held by the most productive firm. Next, we showed that an extended version of the model, coupling the finite pairwise Pólya urn selection with firm learning and entry and exit of firms, is able to *jointly* reproduce a wide ensemble of stylised facts of international trade, firm and industry dynamics. In addition, the model also generates which is able to *endogenously* generate asymmetries between countries. Furthermore, we documented how the intensity of market selection and the presence of barriers to trade affect the dynamics of firms, industries and international trade. Higher degrees of market concentration and volatility are observed in presence of a higher level of market selection and trade openness. These forces also affect the persistence of countries' international leadership as well as the ability of laggard countries to catch up international leaders.

The model could be extended to account for the effect of country asymmetries in both size and learning opportunities. A vast strand of empirical literature has shown that countries differ both in terms of industry size, demand levels and on the ability to generate new technological opportunities and to absorb foreign and advanced technological knowledge. These aspects might impact both on the selection and the learning mechanisms. A second extension could involve a generalization of the model to more than two countries and two sectors. This would allow one to take into account the heterogeneous spillovers between different pairs of country-sector combinations and the role of multi-product firms. Such a model, could also be calibrated by using the data from Input-Output Tables and would allow to better understand the importance of inter-sectoral knowledge flows.

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A Proofs of the propositions

In this appendix we report the proofs of the propositions stated in the paper

Proof of proposition 1. Consider the expected shock to sales $\xi_{i,k,t}$ of firm i at the kth assignment and time t when the market is not in monopoly:

$$\mathbb{E}(\xi_{i,k,t}) = \sum_{\forall i \neq j} z_{ij,k,t} p_{ij,k,t} \tag{A.1}$$

Given a continuous distribution of productivity levels a_{it} of the incumbent firms – defined as the firms with strictly positive size – there always exists a most productive incumbent $\bar{i}$ characterized by $a_{\bar{i}t} = \max\{a_{it}\}$ and a least productive incumbent $\underline{i}$ characterized by $a_{\underline{i}t} = \min\{a_{it}\}$. Thus, the following statements holds true:

1. $\mathbb{E}(\xi_{\bar{i},k,t}) > 0, \;\; \min(\xi_{\bar{i},k,t}) = 0$
2. $\mathbb{E}(\xi_{\underline{i},k,t}) < 0, \;\; \max(\xi_{\underline{i},k,t}) = 0$

The first statement tells us that the most productive firm has always positive expected shock and can only be hit by non-negative shocks. As a consequence, its size cannot reach 0 and the only fixed point of firm $\bar{i}$ is M (see Equation (6)). Also notice that the law of motion of size of all the firms different from $\bar{i}$ cannot reach the fixed point M by the same fact that firm $\bar{i}$ customers cannot be stolen by any of the other incumbents. According to the second statement, the least productive firm always experiences negative expected shock to sales and non-positive shocks. Since $\sum_i \mathbb{E}(\xi_{i,k,t}) = 0$, due to the symmetry between positive and negative shocks to firms, the system converges to monopoly by iterating the process for a sufficiently large value of K because firms are drawn to compete with positive probabilities. □

Proof of proposition 2. The productivity of an exporting firm may suffer from the presence of iceberg cost c as described by Equation (8). In the case *1* of the proposition, iceberg costs are null. Accordingly, the economies of the two countries are symmetric as if they were composed by the same $2 \cdot N$ domestic and foreign firms. In this case, Proposition 1 tells us that the market will converge to an international monopoly wherein shares are held by the same most productive firm.

For the proof of cases *2* and *3*, we introduce the following notation. Let us denote the maximum of firm productivity in country $h = 1, 2$ as M_h and the ith firm productivity as $a_{i,h,t}$.We define the probability of international monopoly as the sum of the probability of two events:

$$\text{P(International Monopoly)} = P(M_2 < (1-c)M_1) + P(M_1 < (1-c)M_2) \quad \text{(A.2)}$$

where $P(M_2 < (1-c)M_1)$ is the probability that the most productive firm belongs to country 1 and, similarly, $P(M_1 < (1-c)M_2)$ is the probability that it belongs to country 2, irrespectively of trade costs c. Given that Equation (A.2) describes the probability that the same firm is the most productive in both the markets, it is also the probability that the model converges to international monopoly (see proposition 1).

Let us now consider the case 2 ($c \in (0,1)$). We decompose the first component of Equation (A.2):

$$\begin{gathered} P\left(M_2 < (1-c)\cdot M_1\right) = \\ = P\left(a_{1,2,t} < (1-c)\cdot M_1,\ a_{2,2,t} < (1-c)\cdot M_1,\ \ldots,\ a_{N,2,t} < (1-c)\cdot M_1\right) = \\ = P\left(a_{1,2,t} < (1-c)\cdot M_1\right)\cdot P\left(a_{2,2,t} < (1-c)\cdot M_1\right)\cdot \ldots \cdot P\left(a_{N,2,t} < (1-c)\cdot M_1\right) \end{gathered} \quad \text{(A.3)}$$

Where each component can be rewritten as follows by taking into account that the draws $a_{i,h,t}$ are $i.i.d.$ and so are the differences (D_h) between them:

$$\begin{gathered} P\left(a_{1,2,t} < (1-c)M_1\right) = \\ P\left(a_{1,2,t} < (1-c)\cdot a_{1,1,t},\ a_{1,2,t} < (1-c)\cdot a_{2,1,t},\ \ldots,\ a_{1,2,t} < (1-c)\cdot a_{N,1,t}\right) = \\ P\left(a_{1,2,t} - (1-c)\cdot a_{1,1,t} < 0\right)\cdot P\left(a_{1,2,t} - (1-c)\cdot a_{2,1,t} < 0\right)\cdot, \ldots, \cdot P\left(a_{1,2,t} - (1-c)\cdot a_{N,1,t} < 0\right) = \\ P\left(D_1 < 0\right)\cdot P\left(D_1 < 0\right)\cdot, \ldots, \cdot P\left(D_1 < 0\right) = P\left(D_1 < 0\right)^N \end{gathered} \quad \text{(A.4)}$$

By plugging Equation (A.4) into Equation (A.3) we find:

$$P\left(M_2 < (1-c)M_1\right) = P\left(D_1 < 0\right)^{N^2} \quad \text{(A.5)}$$

Similarly, one can rewrite also the second term of Equation (A.2) as the product $P\left(D_2 < 0\right)^{N^2}$. Accordingly, we get the following final expression for the probability of convergence to an international monopoly:

$$\text{P(International Monopoly)} = P\left(D_1 < 0\right)^{N^2} + P\left(D_2 < 0\right)^{N^2} \quad \text{(A.6)}$$

By looking at Equation (A.6) we can see that the probability of international monopoly depends on two factors, namely the number of firms N and the difference between the productivity distributions considered.

First, as N grows bigger, the probability of international monopoly goes to 0 as it becomes easier to find highly productive firms in each of the two countries considered. Second, the probability that the differences D_1 and D_2 are smaller than 0 depends on the iceberg cost c. The higher the value of c, the lower the probability that the differences in productivity D_1 and D_2 are negative. One important remark is needed. We assumed symmetry in the firm productivity distributions of the two countries. In this respect, a productivity absolute advantage of a country vis-à-vis the other would yield the same effect of imposing an asymmetric iceberg cost c.

Finally, in case 3 ($c = 1$) Equation (A.2) becomes:

$$\text{P(International Monopoly)} = P(M_2 < 0) + P(M_1 < 0) = 0 \tag{A.7}$$

Because the firm productivity cannot take negative values, implying $P(M_2 < 0) = P(M_1 < 0) = 0$.

□