Entry dynamics as a solution to the puzzling behaviour of real marginal costs in the Ghironi-Melitz model

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Abstract

The work of Ghironi and Melitz (2005) is at the frontier of international real business cycle (IRBC) models with heterogeneous firms. In their model, the dynamic behaviour of real marginal costs is puzzling: a positive technology shock hitting the home country makes it permanently less cost-effective than the foreign economy. Wages grow more than profits during booms and the labour share in GDP is counterfactually procyclical. Entry by new firms is crucial in delivering this result. It is sufficient to posit that technology improvements are more efficacious in manufacturing than in the "production of new firms" for the labour share and real marginal costs to become countercyclical, consistently with empirical evidence. Once I introduce tradable capital goods and endogenous labour supply, the two models are on average equally good in replicating the empirical moments typically considered in the IRBC literature.

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1 Introduction

Since the mid-nineties, the empirical literature on international trade has exploited longitudinal plant or firm-level data to challenge the paradigm of firm homogeneity and establish the existence of large productivity differentials among exporters and non-exporters within narrowly defined industries (see Bernard, Jensen, Redding and Schott, 2011, for an extensive survey of the literature). Starting with the contributions of Bernard and Jensen (1995), Clerides, Lack and Tybout (1998), Bernard and Jensen (1999a) and Aw, Chung and Roberts (2000), a consensus has emerged that more productive firms self-select into export markets. Bernard and Jensen (1999b) further document that reallocation of resources towards more productive exporting plants accounts for 20 per cent of productivity growth in U.S. manufacturing.

Against this background, the model proposed by Ghironi and Melitz (2005), GM hereafter, is remarkable for several reasons. It features a dynamic extension of the seminal paper by Melitz (2003), which, together with Bernard, Eaton, Jensen and Kortum (2003), was the breakthrough in the modelling of international trade by heterogeneous firms.[1] GM develop an international real business cycle (IRBC) model: a dynamic stochastic, general equilibrium, two-country model that sheds new light on the interpretation of business cycle fluctuations in an open economy under the assumption of flexible prices. Interestingly, the model also generates persistent deviations from purchasing power parity (PPP) that would not exist without firm heterogeneity.

Two key features drive the mechanisms behind the response of international trade after a technology shock in GM. Firstly, the distinction between traded and nontraded goods fluctuates endogenously in their model, thanks to fixed export costs that only the most productive firms find it profitable to bear. Firms each produce a single differentiated good (variety) and move in and out of the exporting pool, triggering intra-industry reallocations of resources (towards relatively more productive plants) that do play a role in the propagation of business cycles. Secondly, new firms enter until all profitable opportunities are exploited, but entrants must wait one period before becoming productive. This turns the total number of firms into a state variable that evolves only sluggishly and introduces persistence in the system. Also, labour is required in order to "produce new firms", which entails a reallocation of resources from the production of the composite consumption good ("manufacturing").

In some detail, a transitory unexpected positive technology shock in the home economy initially spurs labour demand, wages (labour supply is fixed), consumption and demand for imports. Abroad, improved opportunities for exports trigger an increase in foreign labour demand, wages and consumption, which feeds back into higher demand for domestic firms; for some of them it now becomes profitable to serve the foreign market and enter the exporting sector. At home, increased consumption and exports boost current and expected future profits, which induces entry by new firms.[2] Labour demand for producing new firms adds to the existing increase in labour demand, so that wage growth outstrips technology (at all horizons): real marginal costs are procyclical. In relative terms, a positive technology shock hitting the home country makes it permanently less cost-effective than the foreign economy.

Here a puzzling aspect of the GM model arises. Rotemberg and Woodford (1999) acknowledge that "it is not easy to obtain measures of marginal cost of which one can be certain" (p. 1057). They find some evidence that real marginal costs are countercyclical. They also consider empirical definitions of marginal costs such that these are indeed procyclical in post-war U.S. data, but this is associated with countercyclical markups. It is therefore controversial for the GM model to deliver procyclical real marginal costs while postulating constant markups.[3] The argument is more transparent when expressed in terms of the labour share of income. In GM, procyclical marginal costs and the "excessive" responsiveness of wages, given that labour supply

[1] For surveys on the subject, see Helpman (2006), and Redding (2010).

[2] The entry cost also rises, being tied to wages, but not enough to prevent positive (net) entry.

[3] Rotemberg and Woodford (1999) define real marginal costs as nominal marginal costs over the own price (so that the markup is the inverse of real marginal costs), whereas GM use the aggregate price index as deflator.

is fixed, cause aggregate profits to rise less than wages during booms, so that the labour share of GDP is strongly procyclical. This contradicts the empirical evidence reported by Rotemberg and Woodford (1999), who find that in the U.S. the contemporaneous correlation between the labour share and GDP is negative for the private sector: it ranges from -0.47 to -0.19 depending on which definition of non-public sector is chosen.[4]

Recently, Ríos-Rull and Santaeulàlia-Llopis (2010) have also found that the labour share is countercyclical in U.S. data, the contemporaneous correlation with GDP being -0.24. They further document that the labour share lags output by roughly one year, and estimate the impulse response function to a Solow residual innovation: the labour share falls on impact 16 per cent below its steady state level, then starts growing. It overshoots its long-run average after five quarters, peaks around the fifth year and then slowly mean-reverts. In the GM model the dynamics are almost reversed: on impact the labour share jumps 11 per cent above its long-run average and then slowly mean-reverts.

GM is at the frontier of IRBC models. If theirs is to be the one we use to think about business cycles in open economy with flexible prices, it is worth attempting to reconcile its implications for the labour share with empirical evidence: our understanding of how exporters, non-exporters and international trade react to shocks should not ultimately turn on a controversial behaviour of real marginal costs. This is the aim of my paper, and I achieve it by assuming that technological advances are not as efficacious in the production of new firms as they are in manufacturing. This single modification to the baseline setting is sufficient to deliver the sought-for countercyclicality of the labour share and real marginal costs. The empirical predictions of my model improve further when I introduce endogenous labour supply and investment in physical capital.

In GM there is no capital accumulation in a standard sense, but their approach nonetheless entertains a notion of investment, associated with financing the creation of new firms (extensive margin). I shall maintain their interpretation while adding the usual component of investment in (tradable) physical capital goods in order to increase the capital stock in the production function of existing firms (intensive margin). This adds realism, makes the definition of investment more consistent with the real business cycle (RBC) literature, and also avoids the implicit assumption GM need to make that all capital goods be exogenously nontradable.

By adding a second input that is fixed in the short run (the capital stock), I introduce diminishing returns to labour (I use a standard Cobb-Douglas production function), and this does have the potential to prevent (at least in the short run) the "excessive" reaction of wages displayed by the GM model. Endogenizing labour supply may also be effective to this end. Yet, neither of these two features is sufficient (nor necessary, as already mentioned) to alter the cyclical properties of the labour share.

Turning to the crucial assumption for my results, I assume that an aggregate productivity Z_t enters the production function of consumption goods, while Z_t^{ψ}, $\psi \in (0,1)$, enters the "production function of new firms" (GM implicitly use $\psi = 1$). Denote manufacturing output as $Z_t y_{t,1}$ and production of new firms as $Z_t^{\psi} y_{t,2}$, where $y_{t,i}$, $i \in \{1,2\}$, is determined by a standard Cobb-Douglas production function $y_{t,i} = K_{t,i}^{\alpha_i} L_{t,i}^{1-\alpha_i}$, $\alpha_i \in [0,1]$. Consider now the log-linear version of the model: using sans-serif fonts to denote percentage deviations from steady state, manufacturing output is $\mathsf{Z}_t + \mathsf{y}_{t,1}$ while production of new firms is $\psi \mathsf{Z}_t + \mathsf{y}_{t,2}$. That is, in the log-linear equilibrium, a technology shock Z_t increases total factor productivity in manufacturing by Z_t (relative to its long-run average), compared with only $\psi \mathsf{Z}_t$ in the production of new firms: technological advances are not as efficacious in the production of new firms as they are in manufacturing.

It is important to clarify that I am not assuming that entrants are less productive than incumbents, due to the timing convention I borrow from GM. Specifically, entrants in period

[4] The "excessive" responsiveness of wages in GM is also at odds with the evidence on sticky wages (see Barattieri, Basu and Gottschalk, 2010, for a recent contribution on this subject). However, I shall not challenge the assumption of perfectly flexible prices.

t only start producing in period $t+1$, so that they spend period t in setting up their firm (in sustaining the sunk entry cost), given technology Z_t^{ψ}. In period $t+1$, all existing firms (that is, including previous-period entrants) produce facing a common technology shock Z_{t+1}. For these reasons, the standard interpretation of Z_t as a parable for a common shock across all domestic *producers* is still valid in my model. From a different angle, setting $\psi = 1$ (as in GM) is as arbitrary as setting $\psi \in (0,1)$, since I am not aware of any empirical study that tries to estimate how different the behaviour of the Solow residual is in the setting up of new firms relative to the actual production of manufactures. Indeed, it can be argued that $\psi \in (0,1)$ is the more plausible case: major historical advances towards cost-effectiveness in production (such as assembly lines, product standardization, mass production, just-in-time production strategies, and so on) do not appear to characterize the setting up of new firms.

The problem remains that ψ needs to be calibrated in order to simulate my model. I will take an agnostic approach and "solve" for the value of ψ (say $\bar{\psi}$) that is needed, given all other parameters, in order to match the correlation between the labour share and GDP as estimated by Ríos-Rull and Santaeulàlia-Llopis (2010). The approach is going to be: if you want the behaviour of the labour share to be realistic in a GM-type model, you need to be willing to believe that $\psi = \bar{\psi}$ in the data. Encouragingly, my calibration for ψ also does an excellent job at replicating the cyclical properties of net entry (defined as new incorporations minus failures), which is one of the variables most directly affected by the parameter ψ.

The main result is that the simple assumption $\psi \in (0,1)$ is successful in delivering a countercyclical labour share, consistently with empirical evidence, and likewise with real marginal costs. In relative terms, a technology shock hitting the home economy makes it the most cost-effective country in my model; the opposite holds in GM. Aside from these two variables, my model and GM's replicate equally well, on average, the empirical moments typically considered in the international real business cycle literature.

Farhat (2010) is also heavily influenced by the GM model, which he reinterprets to include endogenous labour supply and a standard notion of capital accumulation, but his aim and his approach are very different from mine. He is more interested in matching empirical moments than in the internal mechanisms of the GM model that drive the evolution of real marginal costs and the labour share. And in his set-up, he posits that heterogenous firms *à la* Ghironi-Melitz use labour as the only input to produce a differentiated *intermediate* good, which is traded internationally; intermediate goods are then used by a representative firm, together with non-traded physical capital, to produce a non-traded consumption good.[5] Farhat concludes that including both shocks to the production of intermediate goods and shocks to the production of final goods greatly improves the ability of the theoretical model in matching empirical moments, although this result crucially depends on the degree of labour supply elasticity. Unfortunately, it is not possible to understand from his presentation whether his model shares with GM's an "excessive" responsiveness of wages delivering the counterfactual predictions I highlighted earlier.

The rest of the paper is organized as follows. The next section lays down the model, defines equilibrium and describes the calibration strategy. Section 3 analyzes the dynamic properties of my model and compares them with those arising in GM. Section 4 concludes.

2 The model

The world consists of two structurally identical countries (home and foreign), so I describe the framework only for the home country. Foreign variables are denoted by an asterisk (*). All variables are expressed in real terms. Since my model is a straightforward extension of GM, the reader is referred to their paper for the few details I shall omit in my presentation.

[5] I abstract from intermediate goods, and allow for tradability of both consumption and capital goods (whose production requires the simultaneous use of labor and capital inputs).

2.1 Consumers, part I

There is a single (composite) consumption good, with a continuum of varieties. The representative household has the usual Dixit-Stiglitz preferences over varieties (with elasticity of substitution $\theta > 1$), so that his utility can be written in terms of the aggregate consumption bundle C_t (whose price P_t is the usual welfare-based aggregate of the prices of all individual varieties). The agent maximizes the expected present discounted value of utility

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{1}{1-\gamma} \left[C_s - \chi \frac{L_s^{1+\sigma^{-1}}}{1+\sigma^{-1}} \right]^{1-\gamma}, \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount factor, $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution, $\chi > 0$ parametrizes the disutility from working and $\sigma > 0$ is the Frisch elasticity of labour supply (L_t) to real wages. The utility function displays GHH preferences, after Greenwood, Hercowitz and Huffman (1988). I have chosen this representation because the work by Farhat (2009), who experiments with different utility functions so as to introduce endogenous labour supply into an otherwise standard GM model, documents that this is the only formulation delivering a reasonable amount of volatility in the series for hours. With GHH preferences, labour supply is an intertemporal decision that depends on wages only and displays no wealth effects.

Households own the capital stock K_t and rent it to firms. Investment in physical capital (I_t) requires the use of the same composite of all available varieties as the consumption basket.[6] It follows that aggregate demand by consumers is $D_t = C_t + I_t$. The usual law of motion applies: $K_{t+1} = (1-\delta_K)K_t + I_t$, where $\delta_K \in (0,1)$ is the depreciation rate.

Households have access to three financial assets: equity shares of domestic firms, domestic bonds and foreign bonds. Trading in foreign shares is ruled out. Agents in each country issue bonds that pay a risk-free real return in units of the country's consumption basket. International markets are therefore incomplete, since only risk-free bonds are traded across countries.

It is well known that open economy models with incomplete asset markets feature indeterminacy of steady-state net foreign assets, so that the equilibrium depends on initial conditions and equilibrium dynamics display nonstationarity. GM solve the problem by assuming that agents must pay a quadratic adjustment cost on bond holdings, symmetric between domestic and foreign bonds.[7] These financial fees (T_t^f) are then rebated back to consumers in an exogenous fashion.[8]

For expositional convenience, the presentation of the agent's budget constraint is put off to Section 2.5.

2.2 Firms

In each country, at the beginning of every period, there is a mass $N_{D,t}$ of heterogeneous active firms, indexed by their relative productivity z. After production and dividend distribution by active firms has occurred, a mass $N_{E,t}$ of new firms enters the market, as determined by the free entry condition detailed in Section 2.4. Entrants pay a sunk entry cost and then draw once and for all their relative productivity level z from a common, time-invariant probability distribution $G(z)$ with support $[z_{\min}, \infty)$. This relative productivity level remains fixed thereafter and is therefore an observable constant for all firms engaging in production. Entrants at time t will only start producing at time $t+1$. At the end of the current period, before entrants draw their z, all firms are hit with a "death" shock with probability $\delta \in (0,1)$. Therefore, a fraction δ of

[6] That is, the relative price between consumption and investment goods is exogenously set at unity.

[7] Uribe and Schmitt-Grohe (2003) survey the modifications proposed in the literature on small open economies to induce stationarity and conclude that they are largely equivalent.

[8] One may think of fees being paid out to a domestic financial intermediation sector that operates costlessly and is entirely owned by households. Fees are then refunded in the form of dividends.

entrants never gets to produce, and at the beginning of next period $N_{D,t+1} = (1-\delta)(N_{D,t}+N_{E,t})$ firms are active. Since the death shock is independent of z,[9] $G(z)$ also represents the productivity distribution of active firms in each period.

The production function in active firm z is $y_t(z) = \Phi Z_t z {k_t}^{\alpha} {l_t}^{1-\alpha}$, where $y_t(z)$ is individual output, $\Phi > 0$ is a scale parameter, Z_t is the aggregate productivity shock, k_t is the capital stock rented by the z^{th} firm, l_t is labour inputs and $\alpha \in (0,1)$.

Firms operate under monopolistic competition on the goods market, whereas the labour market is perfectly competitive. Firms can either produce for the domestic market or for the foreign one. Exporting is costly: it involves both an iceberg cost $\tau \geq 1$ and a fixed cost $f_X w_t/Z$ ($f_X > 0$ units of effective labour), where w_t is the real wage.[10] Firms maximize profits on a period-by-period basis. All of them produce to serve the domestic market, which entails no fixed costs. Exporting only occurs if it is profitable, that is, if revenues are sufficient to cover production (variable) costs, inclusive of the iceberg cost, and the fixed export cost.

As usual with Dixit-Stiglitz preferences, firm z faces a domestic demand for variety z given by $[\rho_{D,t}(z)]^{-\theta} D_t$, where $\rho_{D,t}(z)$ is the relative price charged by firm z (relative to the aggregate price level P_t, so that it is denominated in units of the domestic consumption bundle) and D_t is aggregate domestic demand. Export prices are denominated in the currency of the export market (local currency pricing): firm z faces a foreign demand given by $[\rho_{X,t}(z)]^{-\theta^*} D_t^*$, where D_t^* is foreign aggregate demand and $\rho_{X,t}(z)$ is the price in real terms, relative to the *foreign* aggregate price level P_t^*. Foreign demand denominated in units of the *home* composite good is therefore given by $Q_t[\rho_{X,t}(z)]^{-\theta^*} D_t^*$, where Q_t is the welfare-based real exchange rate (units of home consumption per unit of foreign consumption). Hereafter, I shall follow GM in assuming $\theta^* = \theta$.

Cost minimization dictates that, in order to produce an *arbitrary* quantity, the optimal capital-to-labour ratio is $k_t/l_t = \alpha w_t[(1-\alpha) r_{K,t}]^{-1}$, where $r_{K,t}$ is the rental rate of capital in real terms. This implies that all firms optimally choose the same capital-to-labour ratio, since they all face the same input prices. It also implies that production choices for serving the domestic market are independent of production choices for exporting: the two problems can be treated separately without loss of generality.[11]

The optimal prices entail the usual constant mark-up $\frac{\theta}{\theta-1} > 1$ over marginal cost:[12]

$$\rho_{D,t}(z) = \tfrac{\theta}{\theta-1} w_{cf,t} (\Phi Z_t z)^{-1}$$

$$\rho_{X,t}(z) = Q_t^{-1} \tau \tfrac{\theta}{\theta-1} w_{cf,t} (\Phi Z_t z)^{-1},$$

where

$$w_{cf,t} = \left(\tfrac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\tfrac{r_{K,t}}{\alpha}\right)^{\alpha}$$

can be thought of as the price of the "composite factor of production" resulting from renting one unit of labour endowed with k_t/l_t units of physical capital, where k_t/l_t is the optimal capital-to-labour ratio. The reason I use this notation is that it suffices to replace $w_{cf,t}$ with w_t in order to obtain the corresponding equations in the GM model without capital. The marginal cost is $w_{cf,t}(\Phi Z_t z)^{-1}$.

[9] This is a counterfactual assumption, but analytical tractability requires it.

[10] In order to sell one unit abroad, iceberg costs require that $\tau \geq 1$ units be produced. As for the fixed cost, note that it is uniform across all firms (i.e. it does not depend on z). It is paid on a period-by-period basis rather than sunk upon entry in the export market.

[11] Since the capital-to-labor ratio is independent of output levels, if it optimal for an exporter to produce $y_{D,t} = \Phi Z_t z {k_{D,t}}^{\alpha} {l_{D,t}}^{1-\alpha}$ for the home market and $y_{X,t} = \Phi Z_t z {k_{X,t}}^{\alpha} {l_{X,t}}^{1-\alpha}$ for the foreign market, then it is optimal for him to produce $y_{D,t} + y_{X,t} = \Phi Z_t z (k_{D,t} + k_{X,t})^{\alpha} (l_{D,t} + l_{X,t})^{1-\alpha}$ in order to serve both markets (and vice versa).

[12] Equivalently, the value of the marginal product of each factor of production is equated to a markup $\frac{\theta}{\theta-1}$ over its price.

Total profits in period t are given by $d_t(z) = d_{D,t}(z) + d_{X,t}(z)$, where $d_{D,t}(z)$ are optimal profits on the domestic market and $d_{X,t}(z)$ are optimal profits on the foreign market:

$$d_{D,t}(z) = \frac{1}{\theta}[\rho_{D,t}(z)]^{1-\theta} D_t$$

$$d_{X,t}(z) = \begin{cases} Q_t \frac{1}{\theta}[\rho_{X,t}(z)]^{1-\theta} D_t^* - w_t f_X Z_t^{-1} & \text{if firm } z \text{ exports} \\ 0 & \text{otherwise .} \end{cases}$$

Note the asymmetry: $d_{X,t}(z)$ is expressed in units of C_t, whereas $\rho_{X,t}(z)$ is denominated in units of C_t^*. As expected, more productive firms earn higher profits (relative to less productive firms), although they set lower prices.

As already mentioned, a firm chooses to export if and only if its productivity z is above the cutoff level $z_{X,t} \triangleq \inf\{z : d_{X,t}(z) > 0\}$. Fixed export costs will be parametrized so that $z_{X,t} > z_{\min}$. All firms $z < z_{X,t}$ constitute an endogenously determined, time-varying nontraded sector: the set of firms that could export but prefer not to. Also, as $z_{X,t}$ fluctuates, the range of varieties available to foreign consumers (via imports) changes. In conclusion, domestic and foreign consumers have access to different, time-varying sets of varieties.

From the capital-to-labour ratio it is straightforward to express labour demand as a function of profits. Consider firms only serving the domestic market: from $k_t/l_t = \alpha w_t[(1-\alpha) r_{K,t}]^{-1}$, we get $w_t l_t + r_{K,t} k_t = w_t l_t/(1-\alpha)$. It takes some textbook algebra to show that optimal real profits can be written as $d_{D,t}(z) = (w_t l_t + r_{K,t} k_t) / (\theta - 1)$. It follows that labour demand is $l_{D,t}(z) = (\theta - 1)(1-\alpha) d_{D,t}(z)/w_t$. In a similar fashion, for exporters one gets

$$l_{X,t}(z) = (\theta - 1)(1-\alpha)\left[\left(d_{X,t}(z)/w_t\right) + f_X Z_t^{-1}\right] + f_X Z_t^{-1},$$

where $f_X Z_t^{-1}$ is labour absorbed by fixed export costs.

2.3 Aggregation

In every period $N_{D,t}$ firms are active in the home country. All of them produce for the domestic market, while only $N_{X,t} = [1 - G(z_{X,t})] N_{D,t}$ of them also serve the foreign market. As mentioned above, the mass $N_{D,t}$ is distributed according to $G(z)$.

Since firms are heterogeneous, defining aggregate variables pertaining to the productive sector (aggregate domestic profits, for instance) requires integrating the relevant firm-level variable, $d_{D,t}(z)$ in the example, across the distribution $G(z)$. A major contribution of Melitz (2003) was to show how to circumvent such analytical complexity. By defining the appropriate average (across the firms' distribution), all aggregate quantities can be expressed as the product of that average and the corresponding mass of firms. Specifically, one needs to define two average productivity levels for each country: a time-invariant average $\tilde{z}_D$ for active firms (which all serve the domestic market), and an average $\tilde{z}_{X,t}$ for exporters:

$$\tilde{z}_D \triangleq \left[\int_{z_{\min}}^{\infty} z^{\theta-1} dG(z)\right]^{\frac{1}{\theta-1}},$$

$$\tilde{z}_{X,t} \triangleq \left[\int_{z_{X,t}}^{\infty} z^{\theta-1} dG(z \mid z \geq z_{X,t})\right]^{\frac{1}{\theta-1}} = \left[\frac{1}{1-G(z_{X,t})}\int_{z_{X,t}}^{\infty} z^{\theta-1} dG(z)\right]^{\frac{1}{\theta-1}},$$

where $\tilde{z}_{X,t}$ uses the conditional distribution $G(z \mid z \geq z_{X,t})$ and $z_{X,t}$ is the export cutoff level defined above.

Thanks to these special averages, the model becomes isomorphic, as far as aggregate variables are concerned, to one where $N_{D,t}$ identical firms with productivity level $\tilde{z}_D$ produce in the home country and $N_{X,t}$ identical firms with productivity level $\tilde{z}_{X,t}$ also export to the foreign market. Clearly, since $z_{X,t}$ fluctuates endogenously, so does $\tilde{z}_{X,t}$.

One is now able to define average profits $\tilde{d}_{D,t}$ of domestic firms on the domestic market as $\tilde{d}_{D,t} = d_{D,t}(\tilde{z}_D)$ and average profits $\tilde{d}_{X,t}$ of domestic exporters as $\tilde{d}_{X,t} = d_{X,t}(\tilde{z}_{X,t})$. Similarly, $\tilde{\rho}_{X,t} = \rho_{X,t}(\tilde{z}_{X,t})$ is the average price charged by domestic exporters,[13] and $\tilde{\rho}_{D,t} = \rho_{D,t}(\tilde{z}_D)$ is the average price charged by domestic firms on the domestic market. Finally, let $\tilde{d}_t$ denote average overall profits. The following relation for overall profits then holds: $N_{D,t}\tilde{d}_t = N_{D,t}\tilde{d}_{D,t} + N_{X,t}\tilde{d}_{X,t}$.

Average labour demand in firms operating only on the domestic market is

$$\tilde{l}_{D,t} = (\theta - 1)(1 - \alpha)\tilde{d}_{D,t}/w_t\,,$$

leading to aggregate labour demand

$$L_{D,t} = N_{D,t}\tilde{l}_{D,t}\,.$$

The corresponding aggregate output equals demand $N_{D,t}\left(\tilde{\rho}_{D,t}\right)^{1-\theta} D_t$ on the goods market. Similarly, aggregate labour demand in exporting firms is

$$L_{X,t} = N_{X,t}\tilde{l}_{X,t} = N_{X,t}(\theta - 1)(1 - \alpha)\left[\left(\tilde{d}_{X,t}/w_t\right) + f_X Z_t^{-1}\right] + f_X Z_t^{-1}.$$

2.4 The free entry condition

Prospective entrants correctly anticipate their future expected profits. With probability δ they are hit by the death shock and never produce. With probability $1-\delta$ they draw a relative productivity level z from the distribution $G(z)$, so that their expected profits at any future time s coincides with the average overall profits $\tilde{d}_s$. The present discounted value of their expected stream of profits $\left\{\tilde{d}_s\right\}_{s=t+1}^{\infty}$ is therefore

$$\tilde{v}_t \triangleq E_t \sum_{s=t+1}^{\infty} [(1-\delta)]^{s-t}[\beta^{s-t}U_1(C_s, L_s)/U_1(C_t, L_t)]\tilde{d}_s\,,$$

where $U_1(\cdot)$ denotes the marginal utility of consumption and entrants correctly anticipate households' stochastic discount factor $\beta^{s-t}U_1(C_s, L_s)/U_1(C_t, L_t)$, as it is customary to posit when intertemporal decisions are taken by firms.[14] Note that $\tilde{v}_t$ is also the ex-dividend average value of incumbents (evaluated before the death shock has hit).

Potential entrants are in unbounded mass, so that entry occurs until $\tilde{v}_t$ is equalized with the sunk entry cost.[15] New firms are "produced" by a competitive sector (the "E-sector"), using labour as the unique factor of production. Since there is only one representative firm in the E-sector, it is useless to introduce a relative productivity level z: setting up a new firm requires $f_E Z_t^{-\psi}$ units of effective labour, which cost $w_t f_E Z_t^{-\psi}$ units of the consumption-bundle, leading to the free entry condition $\tilde{v}_t = w_t f_E Z_t^{-\psi}$. This holds so long as the mass $N_{E,t}$ of entrants is positive. I follow GM in assuming that macroeconomic shocks are small enough for this condition to hold in every period.[16] Aggregate labour demand in the E-sector is $L_{E,t} = N_{E,t} f_E Z_t^{-\psi}$.

As mentioned in the introduction, GM use $\psi = 1$, whereas I will assume $\psi \in (0, 1)$. Also, they entertain the notion that the aggregate output of the E-sector, $N_{E,t}\tilde{v}_t$ units of the home consumption bundle, is representative of investment in physical capital, thus making the implicit assumption that capital goods can only be produced domestically: they are exogenously nontradable goods.

[13] Technically, $\left(\tilde{\rho}_{X,t}\right)^{1-\theta}$ is the average of $[\rho_X(z)]^{1-\theta}$ across the distribution of exporters.

[14] The above expression is the unique solution, absent speculative bubbles, of the consumer's Euler equation for shares (see Section 2.6 below), as the case should be in equilibrium.

[15] In a related paper, Bilbiie, Ghironi and Melitz (2007) show that it is not worth postponing entry, since the option value of waiting to enter is zero.

[16] This is the case in all my simulations. The condition is less stringent than it may appear, since, absent entry, the total numbers of firms in the home economy would fall at a rate δ due to the death shock.

I follow GM in assuming that fixed export costs and sunk entry costs only entail hiring labour. Whereas in GM this is consistent with labour being the only factor of production, it is not in my model, where the production of the consumption bundle requires both labour and capital. I have experimented with a different model where both inputs are required along the lines of Bilbiie, Ghironi and Melitz (2007), who consider a closed economy featuring product entry along the lines of GM, but disregard relative productivity levels. I share their findings that non-convergent solutions arise, unless unrealistic values for the depreciation rate of capital are picked (around 0.3, implying that half of the capital stock depreciates in two quarters).

2.5 Consumers, part II

GM assume that all $N_{D,t} + N_{E,t}$ home firms are owned by a domestic mutual fund, whose shares (of mass 1) are only tradable among home households.[17] The mutual fund collects all dividends, pays for the aggregate sunk entry cost and distributes what is left to shareholders. It is therefore consumers who ultimately finance the entry cost.

The ex-dividend value of the mutual fund is $\tilde{v}_t(N_{D,t} + N_{E,t})$, since the (average) value of new entrants coincides with the average ex-dividend value of the active firms, as noted in the previous section.

The budget constraint of the representative household is then given by

$$\begin{aligned} &R_t B_t + Q_t R_t^* B_{*,t} + T_t^f + w_t L_t + r_{K,t} K_t + \left[(N_{D,t}\tilde{d}_t - N_{E,t}\tilde{v}_t) + \tilde{v}_t(N_{D,t} + N_{E,t}) \right] x_t = \\ &= B_{t+1} + Q_t B_{*,t+1} + \tfrac{\eta}{2} \left(B_{t+1}\right)^2 + Q_t \tfrac{\eta}{2} \left(B_{*,t+1}\right)^2 + C_t + I_t + \tilde{v}_t(N_{D,t} + N_{E,t}) x_{t+1} \ . \end{aligned} \tag{2}$$

Real resources (denominated in units of home consumption) on the left-hand side consist of gross interest income on domestic bonds B_t, gross interest income on foreign-issued bonds $B_{*,t}$ (these pay a gross interest rate of R_t^* units of the foreign consumption bundle), financial fees rebated in the exogenous amount T_t^f, labour income, and rental proceeds on the capital stock. The last term is the resources associated with the mutual fund, proportional to the beginning-of-period equity share x_t. Dividends from home firms, $N_{D,t}\tilde{d}_t$, net of the aggregate entry costs $N_{E,t}\tilde{v}_t$, are rebated to the household. Finally, the consumer can sell its initial share position x_t of the mutual fund, whose value is $\tilde{v}_t(N_{D,t} + N_{E,t})$. Note that the term in square brackets simplifies to $\left[N_{D,t}\tilde{d}_t + \tilde{v}_t N_{D,t} \right]$.

Expenditure on the right-hand side consists of purchases of domestic bonds to be carried into next period B_{t+1}, purchases of foreign-issued bonds to be carried into next period $B_{*,t+1}$ (their price is one unit of the foreign consumption bundle, i.e. Q_t units of the home composite good), adjustment costs on bond holdings, consumption, investment and the purchase of equity holdings to be carried into next period x_{t+1}.

The agent maximizes the expected present discounted value of utility, equation [1], subject to [2]. He chooses consumption C_t, labour effort L_t, share holdings to be carried into next period x_{t+1}, domestic-bond holdings to be carried into next period B_{t+1}, foreign-bond holdings to be carried into next period $B_{*,t+1}$, investment in physical capital I_t. The usual first-order conditions are presented in the next section.

2.6 Equilibrium

I follow GM in parametrizing $G(z)$ as a Pareto distribution with lower bound $z_{\min}$ and shape parameter φ,[18] so that $G(z) = 1 - (z_{\min}/z)^{\varphi}$. Letting $\xi \triangleq \{\varphi / [\varphi - (\theta - 1)]\}^{1/(\theta-1)}$, the following relations hold:

[17] The presence of a mutual fund keeps consumers' policy function for shares from being indexed by z.

[18] GM use the notation k instead of φ.

- the average productivities $\tilde{z}_D$ and $\tilde{z}_{X,t}$ are given by $\tilde{z}_D = \xi z_{\min}$, $\tilde{z}_{X,t} = \xi z_{X,t}$;
- the share of home-exporting firms is $N_{X,t}/N_{D,t} = (\xi z_{\min}/\tilde{z}_{X,t})^{\varphi}$;
- from the definition of the export-cutoff profit $d_X(z_{X,t}) = 0$, average export profits satisfy

$$\tilde{d}_X = w_t f_{X,t} Z_t^{-1}[\xi^{\theta-1} - 1],$$

which requires $Q_t\left(\tilde{\rho}_{X,t}\right)^{1-\theta} D_t^* = \theta w_t f_X Z_t^{-1} \xi^{\theta-1}$ to hold from the definition of export profits.

My presentation uses 43 endogenous variables:

$B_t,\ B_t^*,\quad B_{*,t},\ B_{*,t}^*\quad C_t,\ C_t^*,\quad \tilde{d}_t,\ \tilde{d}_t^*,\quad \tilde{d}_{D,t},\ \tilde{d}_{D,t}^*,\quad \tilde{d}_{X,t},\ \tilde{d}_{X,t}^*,\quad N_{D,t},\ N_{D,t}^*,\quad N_{E,t},\ N_{E,t}^*,$

$N_{X,t},\ N_{X,t}^*,\quad \tilde{\rho}_{D,t},\ \tilde{\rho}_{D,t}^*,\quad \tilde{\rho}_{X,t},\ \tilde{\rho}_{X,t}^*,\quad Q_t,\quad R_t,\ R_t^*,\quad \tilde{v}_t,\ \tilde{v}_t^*,\quad w_t,\ w_t^*,\quad \tilde{z}_{X,t},\ \tilde{z}_{X,t}^*,$

$L_t,\ L_t^*;\quad I_t,\ I_t^*,\quad K_t,\ K_t^*\quad r_{K,t},\ r_{K,t}^*,\quad w_{cf,t},\ w_{cf,t}^*,\quad D_t,\ D_t^*.$

The first two lines (31 variables) are from the GM model; the remaining ones are used to introduce endogenous labour supply and physical capital. The following 43 equations characterize the equilibrium of the model.

Optimal pricing on the respective domestic markets at home and abroad:

$$\tilde{\rho}_{D,t} = \tfrac{\theta}{\theta-1} w_{cf,t} (\Phi Z_t \tilde{z}_D)^{-1}$$

$$\tilde{\rho}_{D,t}^* = \tfrac{\theta}{\theta-1} w_{cf,t}^* (\Phi^* Z_t^* \tilde{z}_D^*)^{-1}.$$

Optimal pricing on the respective export markets by home and foreign firms:

$$\tilde{\rho}_{X,t} = Q_t^{-1} \tau \tfrac{\theta}{\theta-1} w_{cf,t} (\Phi Z_t \tilde{z}_{X,t})^{-1}$$

$$\tilde{\rho}_{X,t}^* = Q_t \tau^* \tfrac{\theta}{\theta-1} w_{cf,t}^* (\Phi^* Z_t^* \tilde{z}_X^*)^{-1}.$$

Average overall profits in each economy:

$$\tilde{d}_t = \tilde{d}_{D,t} + (N_{X,t}\,/\,N_{D,t})\tilde{d}_{X,t}$$

$$\tilde{d}_t^* = \tilde{d}_{D,t}^* + (N_{X,t}^*\,/\,N_{D,t}^*)\tilde{d}_{X,t}^*.$$

Average export profits by home and foreign firms:

$$\tilde{d}_{X,t} = w_t f_X Z_t^{-1}[\xi^{\theta-1} - 1]$$

$$Q_t\left(\tilde{\rho}_{X,t}\right)^{1-\theta} D_t^* = \theta w_t f_X Z_t^{-1} \xi^{\theta-1}$$

$$\tilde{d}_{X,t}^* = w_t^* f_X^* Z_t^{*-1}[{\xi^*}^{\theta-1} - 1]$$

$$Q_t^{-1}\left(\tilde{\rho}_{X,t}^*\right)^{1-\theta} D_t = \theta w_t^* f_{X,t}^* Z_t^{*-1} {\xi^*}^{\theta-1}.$$

Profits from sales on the respective domestic market in each economy:

$$\tilde{d}_{D,t} = \tfrac{1}{\theta}\left(\tilde{\rho}_{D,t}\right)^{1-\theta} D_t$$

$$\tilde{d}_{D,t}^* = \tfrac{1}{\theta}\left(\tilde{\rho}_{D,t}^*\right)^{1-\theta} D_t^*.$$

The share of demand going to domestic goods and the share of demand going to foreign goods (i.e. imports) add up to one:

$$N_{D,t}\left(\tilde{\rho}_{D,t}\right)^{1-\theta} + N_{X,t}^*\left(\tilde{\rho}_{X,t}^*\right)^{1-\theta} = 1$$

$$N_{D,t}^*\left(\tilde{\rho}_{D,t}^*\right)^{1-\theta} + N_{X,t}\left(\tilde{\rho}_{X,t}\right)^{1-\theta} = 1.$$

Share of exporting firms in each economy:

$$N_{X,t} \ / \ N_{D,t} = (\xi \ z_{\min} \ / \ \tilde{z}_{X,t})^{\varphi}$$
$$N^*_{X,t} \ / \ N^*_{D,t} = (\xi^* \ z^*_{\min} \ / \ \tilde{z}^*_{X,t})^{\varphi^*}.$$

Free entry conditions:

$$\tilde{v}_t = w_t \ f_E Z_t{}^{-\psi}$$
$$\tilde{v}^*_t = w^*_t f^*_E Z^{*-\psi^*}_t.$$

Laws of motion for the total number of active firms:

$$N_{D,t+1} = (1-\delta)\left(N_{D,t} + N_{E,t}\right)$$
$$N^*_{D,t+1} = (1-\delta^*)\left(N^*_{D,t} + N^*_{E,t}\right).$$

Euler equations for shares,[19] where $MRIS(\cdot)$ indicates the marginal rate of intertemporal substitution, to be specified below:

$$\tilde{v}_t = \beta(1-\delta)E_t[MRIS(\cdot)(\tilde{v}_{t+1} + \tilde{d}_{t+1})]$$
$$\tilde{v}^*_t = \beta^*(1-\delta^*)E_t[MRIS^*(\cdot)(\tilde{v}^*_{t+1} + \tilde{d}^*_{t+1})].$$

Euler equations for bonds issued in the home country to be carried into next period, whose holdings by domestic (foreign) agents are denoted by B_{t+1} (B^*_{t+1}):

$$1 + \eta B_{t+1} = \beta R_{t+1} E_t[MRIS(\cdot)]$$
$$1 + \eta^* B^*_{t+1} = \beta^* R_{t+1} E_t[(Q_t/Q_{t+1})MRIS^*(\cdot)].$$

Euler equations for bonds issued in the foreign country to be carried into next period, whose holdings by domestic (foreign) agents are denoted by $B_{*,t+1}$ ($B^*_{*,t+1}$):

$$1 + \eta B_{*,t+1} = \beta R^*_{t+1} E_t[(Q_{t+1}/Q_t)MRIS(\cdot)]$$
$$1 + \eta^* B^*_{*,t+1} = \beta^* R^*_{t+1} E_t[MRIS^*(\cdot)].$$

Resource constraint for the home economy:[20]

$$B_{t+1} + Q_t B_{*,t+1} = R_t B_t + Q_t R^*_t B_{*,t} + w_t L_t + N_{D,t}\tilde{d}_t + r_{K,t}K_t - N_{E,t}\tilde{v}_t - C_t - I_t.$$

Equilibrium on the bond markets:

$$B_{t+1} + B^*_{t+1} = 0$$
$$B_{*,t+1} + B^*_{*,t+1} = 0.$$

Equilibrium on the labour markets:[21]

$$L_t = (\theta-1)\, w_t^{-1}(1-\alpha)\left(N_{D,t}\tilde{d}_{D,t} + N_{X,t}\tilde{d}_{X,t}\right) + \\ + \left[(\theta(1-\alpha)+\alpha)N_{X,t}\, f_X Z_t^{-1} + N_{E,t}\, f_E Z_t^{-\psi}\right]$$

$$L^*_t = (\theta-1)\, w_t^{*\,-1}(1-\alpha^*)\left(N^*_{D,t}\tilde{d}^*_{D,t} + N^*_{X,t}\tilde{d}^*_{X,t}\right) + \\ + \left[(\theta(1-\alpha^*)+\alpha^*)N^*_{X,t}\, f^*_X Z_t^{*-1} + N^*_{E,t}\, f^*_E Z_t^{*\,-\psi^*}\right].$$

The equations shown above define the equilibrium in the original GM model when α, $r_{K,t}$, I_t and K_t are set to zero, D_t is substituted for C_t, $w_{cf,t}$ is substituted for w_t and L_t is exogenously set to some constant L (and similarly for foreign variables). Importantly, $D_t = C_t + I_t$ needs to be

[19] I use $N_{D,t+1}/\,(N_{D,t} + N_{E,t}) = 1-\delta$ from the law of motion for $N_{D,t}$.

[20] In equilibrium demand for equity shares must equal the outstanding stock at all dates: $x_t = x_{t+1} = 1$.

[21] The left-hand side L_t is labour supply, the right-hand side is $L_{D,t} + L_{X,t} + L_{E,t}$.

replaced with $D_t = C_t$ to obtain the GM model, despite the fact that they entertain $N_{E,t}\tilde{v}_t$ as a notion of investment. Since D_t and D_t^* are the scale variables that activate imports and exports, this is another manifestation of the implicit assumption that in their model all capital goods are a special category of exogenously nontradable goods. I follow Bilbiie, Ghironi and Melitz (2007) in defining total aggregate investment TI_t as the sum of investment in tradable physical capital I_t and investment in new firms $N_{E,t}\tilde{v}_t$ (nontradable capital goods).[22] The remaining equations defining equilibrium follow.

Aggregate labour supply in each country:

$$L_t = (w_t/\chi)^{\sigma}$$
$$L_t^* = (w_t^*/\chi^*)^{\sigma^*}.$$

Euler equations for capital accumulation in each economy:

$$1 = \beta E_t[(1 + r_{K,t+1} - \delta_K) MRIS(\cdot)]$$
$$1 = \beta^* E_t[(1 + r_{K,t+1}^* - \delta_K^*) MRIS^*(\cdot)].$$

Laws of motion for the capital stock:

$$K_{t+1} = (1 - \delta_K)K_t + I_t$$
$$K_{t+1}^* = (1 - \delta_K^*)K_t^* + I_t^*.$$

Rental prices for the composite factor:

$$w_{cf,t} = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_{K,t}}{\alpha}\right)^{\alpha}$$
$$w_{cf,t}^* = \left(\frac{w_t^*}{1-\alpha^*}\right)^{1-\alpha^*} \left(\frac{r_{K,t}^*}{\alpha^*}\right)^{\alpha^*}.$$

Aggregate capital stock in each economy:[23]

$$K_t = \frac{\alpha}{1-\alpha}\frac{w_t}{r_{K,t}} \left(L_t - N_{X,t}\frac{f_{X,t}}{Z_t} - N_{E,t}\frac{f_{E,t}}{Z_t^{\psi}}\right)$$
$$K_t^* = \frac{\alpha^*}{1-\alpha^*}\frac{w_t^*}{r_{K,t}^*} \left(L_t^* - N_{X,t}^*\frac{f_{X,t}^*}{Z_t^*} - N_{E,t}^*\frac{f_{E,t}^*}{Z_t^{*\psi^*}}\right).$$

Aggregate demand in each country:

$$D_t = C_t + I_t$$
$$D_t^* = C_t^* + I_t^*.$$

To close the model, the marginal rates of intertemporal substitution are defined by

$$MRIS(\cdot) = \left[C_t - \chi\frac{L_t^{1+\sigma^{-1}}}{1+\sigma^{-1}}\right]^{\gamma} \left[C_{t+1} - \chi\frac{L_{t+1}^{1+\sigma^{-1}}}{1+\sigma^{-1}}\right]^{-\gamma}$$
$$MRIS^*(\cdot) = \left[C_t^* - \chi^*\frac{L_t^{*1+\sigma^{*-1}}}{1+\sigma^{*-1}}\right]^{\gamma^*} \left[C_{t+1}^* - \chi^*\frac{L_{t+1}^{*1+\sigma^{*-1}}}{1+\sigma^{*-1}}\right]^{-\gamma^*},$$

which simplify to $MRIS(\cdot) = (C_{t+1}/C_t)^{-\gamma}$, $MRIS^*(\cdot) = (C_{t+1}^*/C_t^*)^{-\gamma^*}$ in the GM model without endogenous labour supply.

Note that in the presentation proposed here the resource constraint for the foreign economy is redundant due to Walras' law. Similarly, the above equations imply that a country's borrowing

[22] Theirs is a closed-economy model (featuring product entry along the lines of GM but disregarding relative productivity levels), so they do not dwell on this distinction between tradable and nontradable capital goods.

[23] Since production of the composite good occurs with a common capital-to-labour ratio in all firms, the same ratio must hold between the aggregate capital stock and the aggregate labour inputs used in production of the composite good (that is, all workers L_t minus those absorbed by fixed export costs and those hired in the E-sector).

must equal the other country's lending, when expressed in the same units: $CA_t + Q_t CA_t^* = 0$, where CA_t is the current account.

It is well known that when preferences over varieties display constant elasticity of substitution, the welfare-based aggregate price index P_t can be decomposed into a component reflecting averages prices $\tilde{P}_t$ and product variety N_t: $P_t = N_t^{1/(1-\theta)} \tilde{P}_t$, where $N_t = N_{D,t} + N_{X,t}^*$ is the mass of varieties available at home (via domestic production and imports). As noted by GM, the price index $\tilde{P}_t$ is a geometric weighted average of domestic and imported goods prices and thus corresponds more closely to empirical measures such as the CPI than the welfare-based index P_t. This being the case, GM define a new set of "empirical real variables", using $\tilde{P}_t$ as deflator in the place of P_t. I do the same and define, from a generic real variable X_t obtained using P_t as deflator, the corresponding "empirical real variable" $X_{R,t} \triangleq X_t P_t / \tilde{P}_t = X_t N_t^{1/(1-\theta)}$. This also applies to the real exchange rate; following GM, I define the empirical real exchange rate as the ratio between the CPI (i.e. average prices expressed in a common currency) in the two countries: $Q_{R,t} = \varepsilon_t \tilde{P}_t^* / \tilde{P}_t$, where ε_t is the nominal exchange rate.[24]

Finally, I just mention that my model features a unique symmetric steady state: its computation is only slightly more complicated than the derivation in the appendix of the GM paper.

2.7 Calibration

I use log-utility ($\gamma = 1$) and borrow all parameter values from GM. Thus, I set $\beta = 0.99$, $\delta = 0.025$, $\theta = 3.8$, $\varphi = 3.4$, $\tau = 1.3$, $f_E = 1$, $z_{\min} = 1$, $\bar{Z} = 1$, where an overbar denotes steady-state levels; the fixed export cost f_X is set equal to 23.5 per cent of the per-period, amortized flow value of the entry cost: $f_X = 0.235 \left[1 - \beta (1 - \delta)\right] / \left[\beta (1 - \delta) f_E\right]$. The remaining parameters are calibrated as follows: Φ and χ are selected so as to deliver the same $\bar{C}$ and $\bar{L}$ as in GM. This requires setting $\Phi = 0.55524$ and $\chi = 2.93087$ (GM implicitly use $\Phi = 1$ and $\chi = 0$, which are the values I use in replicating their model). The depreciation rate for capital, δ_K, is set to 0.025 as customary in quarterly RBC models.

As for the exponents in the Cobb-Douglas production function, I set $\alpha = 0.221$ so as to match a consumption-to-GDP ratio (one of the "great ratios" in the RBC literature) of 0.75. This also implies that in steady state the labour share of GDP is 0.649, which is in line with the estimate of 0.679 by Ríos-Rull and Santaeulàlia-Llopis (2010). This compares with a consumption-to-GDP ratio of 0.848 and a labour share of GDP of 0.787 in GM.

I experiment with three values for the elasticity of labour supply to wages: $\sigma \in \{0.5,\ 1.0,\ 1.5\}$. These values fall in the range [0.3, 2.2] that Greenwood, Hercowitz and Huffman (1988) found in the empirical evidence they report. The extensive survey by Blundell and MaCurdy (1999) indicates a wider range, with $\sigma = 0.5$ looking like a reasonable average: the contributions that use micro data tend to find low labour supply elasticities (below 0.5), whereas researchers focusing on a macro perspective tend to find higher elasticities (between 1 and 2). Keane and Rogerson (2011) survey the literature that seeks to reconcile these conflicting micro and macro views; they lean towards the larger macro estimates. I will refer to the case $\sigma = 1$ as my baseline parametrization.

The parameter for the adjustment cost on bond-holdings, η, is set so as to match the standard deviation of total real investment $TI_{R,t}$ relative to the standard deviation of real output $Y_{R,t}$. In the model versions I will present, η varies from 0.00760 to 0.01235 (GM use $\eta = 0.00250$). Similarly, the parameter ψ I focus on is set so as to match the contemporaneous correlation of the labour share with GDP: it varies from 0.133 to 0.339 (GM implicitly use $\psi = 1$). All foreign parameters are set equal to their domestic counterparts, so that the model is symmetric.

For the parametrization of the stochastic process for the vector $[Z_t,\ Z_t^*]$ I follow GM, who in turn borrow it from Backus, Kehoe and Kydland (1992). Using sans-serif fonts to denote

[24] GM use the notation $\tilde{Q}_t$ instead of $Q_{R,t}$.

percentage deviations from steady state, I posit

$$\begin{bmatrix} \mathsf{Z}_{t+1} \\ \mathsf{Z}^*_{t+1} \end{bmatrix} = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix} \begin{bmatrix} \mathsf{Z}_t \\ \mathsf{Z}^*_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{Z,t+1} \\ \varepsilon_{Z^*,t+1} \end{bmatrix},$$

where $\varepsilon_{Z,t+1}$ and $\varepsilon_{Z^*,t+1}$ are bivariate normally distributed, zero-mean innovations (the standard deviation is set to 0.852 per cent, the correlation coefficient is set to 0.258). Note the presence of (small) symmetric spillover effects between Z_t and Z^*_{t+1}.

3 International real business cycles

I log-linearize and solve the model.[25] In this section I present the impulse response functions (IRFs) to a technology shock at home and discuss in detail how it propagates through the home and the foreign economies. In Section 3.3 I will present the model-generated moments in comparison with their empirical counterparts.

It is important to note that intuition regarding the mechanisms at work and the behaviour of the model economy are deduced from the welfare-based real variables (which use P_t as deflator), whereas the statistical performance of the model in replicating empirical moments will be based on the empirical real variables, which use $\tilde{P}_t$ as deflator and are identified by a subscript R.

3.1 Impulse response functions

I start by discussing the dynamic properties of my model in its baseline parametrization ($\sigma = 1$), as detailed in Section 2.7. Figure 1 presents the IRFs for the home economy after an unexpected, temporary positive technology innovation, while Figure 2 presents the dynamics for the foreign economy. The IRFs of all the empirical real variables are qualitatively similar to those of the corresponding welfare-based real variables, with the exception of $Q_{R,t}$, which is the only empirical real variable appearing in the figures.

As detailed by GM, since the adjustment cost on bond holdings is symmetric across bonds, in the log-linear equilibrium it is optimal to spread the cost evenly and choose $B_{t+1} = B_{*,t+1}$. Therefore, I only report the IRF for overall end-of-period assets, defined as $A_t = B_{t+1} + Q_t B_{*,t+1}$.

All variables that are nil in steady state (asset holdings A_t, the trade balance TB_t, the current account CA_t) are log-linearized around the level of steady state consumption. This is especially relevant for the interpretation of the IRF for the trade balance: it is not obtained as the difference between the IRF of exports X_t and the IRF of imports M_t, which are log-linearized around the respective (equal) steady-state levels (roughly 30 per cent of $\bar{C}$).

It is crucial to start by understanding how the positive technology spillover across countries works. The IRF for Z_t looks like a standard $AR(1)$ process: it jumps upward on impact and then it mean-reverts in a concave fashion. Instead, Z^*_t remains unchanged on impact, then moves in a hump-shaped trajectory.

It follows that the behaviour of home households at $t = 0$ is the standard one: they want to consume, import and invest more, and they want to finance capital accumulation by borrowing, in accordance with the standard consumption-smoothing argument.[26] Foreign households, instead, learn that productivity will increase for some time in the future relative to today. They anticipate that their consumption will eventually rise above its long-run average, so they want to consume more today because of their desire for consumption-smoothing. Next, consider that in the symmetric steady state $\bar{R} = \bar{R}^* = 1 + \bar{r}^*_K - \delta^*_K$, i.e. the net return on bonds for foreigners is equal to the net rental rate of capital. Increased bond supply by domestic agents in order to finance consumption drives R_0 up, so that abroad it becomes more profitable to invest in bonds

[25] All variable are in logs except for interest rates, the rental rate of capital and shares of GDP.

[26] Output Y_t increases on impact thanks to the technology shock, but savings S_t rise less than investment I_t.

rather than in physical capital.[27] Foreigners end up increasing their consumption at the expense of investment in physical capital[28] while lending to home agents at the same time. Lending by foreigners entails that abroad the drop in investment is bigger than the increase in consumption, so that overall internal demand D_0^* falls. This is a temporary phenomenon, to be reversed as early as the next period, thanks to output Y_t^* growing and to investment in physical capital quickly rising after the initial drop.

In conclusion, C_0 and I_0 jump upward, while I_0^* falls and C_0^* rises a little. The increase in foreign consumption is partly financed by higher labour income, which results from increased labour demand in the foreign exporting sector (in order to serve the higher import demand from the home economy) and from the fall in investment below its steady-state level, which decreases the capital stock and drives up the marginal product of labour.[29] The opposite reaction of investment in the two economies is not peculiar to my model; the same happens in Backus, Kehoe and Kydland (1992). In their words, "Roughly speaking, resources are shifted to the more productive location, the home country" (p. 761).

I have experimented with increasing η, the parameter driving adjustment costs on bond holdings, all other parameters equal. The impact responses of I_t and I_t^* decrease (in absolute value), precisely because international lending and borrowing is now costlier. In this sense, η can be thought of as indirectly capturing the effects of adjustment costs on physical capital. More interestingly, a natural interpretation arises where η is regarded as a way to model a risk premium (over the risk-free interest rate) on the rental rate of capital. In steady state all bond holdings are zero, there are no adjustment costs and the return on bonds is equal to the rental rate of capital, consistently with the fact that the risk premium should be zero in steady state, where no uncertainty exists. When shocks hit the two economies and agents try to lend and borrow across borders, η introduces a risk premium in that it drives the net interest earned on risk-free bonds below $R_t - 1$, and similarly in the foreign country. Pursuing this interpretation further, such risk premium has the desirable property that it increases with uncertainty, as parametrized by the standard deviation of the structural innovations $\varepsilon_{Z,t}$ and $\varepsilon_{Z^*,t}$. The reason is that the bigger the jump in Z_0, the larger are international borrowing and lending, and the higher is the adjustment cost on bond-holdings. Clearly, the risk premium fluctuates with the stocks of bond holdings, which may be an undesirable property. The size of the adjustment cost on bond holdings introduced by my calibration for the value of η is very small. Relative to the highest level of bond holdings that would arise with η reasonably close to zero ($\eta = 10^{-6}$), the value chosen in my baseline calibration ($\eta = 0.0094$) entails an adjustment cost worth 0.02 basis points, compared with a steady-state level of the (net) quarterly interest rate on risk-free bonds of 1.0101 per cent.

Turning back to the IRFs for the home economy, the fall in export demand on impact (via the decrease in D_0^*) is not sufficient to match the increase in home demand and in labour productivity: wages w_t and labour supply L_t start to rise. In equilibrium, the jump in w_0 and in the composite-factor price $w_{cf,0}$ is smaller than Z_0, which ultimately results in marginal costs slightly falling. The opposite happens abroad, so that relative marginal costs in domestic production tilt in favour of the home country. These are defined as the ratio of real marginal costs in domestic production in the two countries, expressed in a common consumption bundle, the home one:

$$RMC_{D,t} \triangleq Q_t \left[w_{cf,t}^* / \left(\Phi^* Z_t^* \tilde{z}_D^* \right) \right] / \left[w_{cf,t} / \left(\Phi Z_t \tilde{z}_D \right) \right] = Q_t \left[w_{cf,t}^* / Z_t^* \right] / \left[w_{cf,t} / Z_t \right],$$

where the second equal sign follows from the symmetric parametrization of the two countries. $RMC_{D,t}$ (last panel in Figure 1) will be a pivotal variable when I compare my model with GM

[27] The rental rate $r_{K,0}^*$ falls abroad.

[28] Reducing investment is not so detrimental, anyway, since productivity will only start growing, in a gradual fashion, starting in $t = 1$. In fact, labour substitutes for physical capital and foreign output Y^* sligthly rises on impact.

[29] Recall that the foreign country experiences no technological change on impact.

in the next section.

In the home country, the internal price of domestic production $\tilde{\rho}_{D,t}$ also falls on impact (it is proportional to marginal costs), which increases the share $s_{D,t} = N_{D,t} \left(\rho_{D,t}\right)^{1-\theta}$of domestic demand being allocated to domestic production (the state variable $N_{D,t}$ does not move on impact, and $1-\theta < 0$). This effect reinforces the increase in demand faced by home firms. In conclusion, the domestic sector enjoys the biggest benefits on impact from the technological shock: hiring, production and average domestic profits $\tilde{d}_{D,t}$ rise strongly, and in later periods remain above their long-run average while mean-reverting, since persistent quantity effects (demand remains above its long-run average) prevail over temporary reallocation and cost-effectiveness effects.[30]

By contrast, the exporting sector is initially hurt by the fall in foreign demand, despite enjoying slightly lower export prices $\tilde{\rho}_{X,t}$. The marginal firms exit the export market, since it is no longer profitable to sell abroad: the export cutoff $z_{X,0}$ rises slightly and so does the average productivity $\tilde{z}_{X,0}$ of the exporting sector,[31] while the mass of exporters $N_{X,0}$ marginally declines. This is a temporary phenomenon, to be reversed as early as the next period. Even though only relatively more productive and profitable firms survive in the export sector, average export profits $\tilde{d}_{X,t}$ fall a little on impact. In fact, they remain below their long-run average for a few quarters, since in subsequent periods the export cutoff falls below its steady-state level, so that relatively less productive firms (re-)enter the export market, which hurts the average profits of the sector (and average prices $\tilde{\rho}_{X,t}$).

The temporary increase in the cutoff $z_{X,0}$ is entirely driven by the fall in foreign investment in physical capital I^* (which is missing in GM) and is immaterial; it could have been avoided by slightly increasing η, but I have preferred the less arbitrary choice of calibrating it so as to match the relative standard deviation of total investment. The message remains that in my model the cutoff $z_{X,t}$ is permanently below its long-run average, while the opposite holds for the number of exporters $N_{X,t}$ and aggregate real exports X_t.

The minor fall in average export profits does not prevent average overall profits $\tilde{d}_t$ from rising almost 2 per cent on impact, thanks to higher average profits $\tilde{d}_{D,t}$ in production for the domestic market. In subsequent periods, they remain above their steady-state level. This represents a more favourable environment for potential entrants, but the sunk entry cost also rises,[32] thanks to the assumption that technology improvements are not as efficacious in the E-sector as in manufacturing ($\psi = 0.219 < 1$ in my baseline calibration). That is, in the log-linear equilibrium w_t rises on impact less than Z_t, but more than Z_t^{ψ}. Differently from GM, entry costs jump upward on impact: entry is hurt to a minor extent ($N_{E,0}$ is 0.1 per cent smaller than its long-run average). Once again, this is a temporary effect: by the next period the IRF for $N_{E,t}$ turns positive and gradually evolves in a hump-shaped fashion.[33] This is a major difference with GM, as I will detail later: in their model entry costs grow gradually, so that entry surges on impact.

Finally, as aggregate output Y_t rises on impact in the home economy, so do wages and employment, but eventually the wage bill increases less than one to one with GDP and the labour share ls_t falls.

In periods $t > 0$, investment in both countries quickly moves closer to its steady-state level and home consumption slowly increases, with foreign consumption catching up at a faster rate. Home GDP remains on average flat for several periods, despite the decay in technology, thanks to the persistence introduced by capital accumulation and entry by new firms. For the same two

[30] Respectively, the share of demand spent on domestic varieties drops below its long-run average after 13 quarters, while marginal costs increase above their long-run average after 6 quarters.

[31] Recall that $\tilde{z}_{X,t}$ is proportional to $z_{X,t}$ ($\tilde{z}_{X,t} = \xi z_{X,t}$).

[32] In equilibrium, entry costs equal $\tilde{v}_t$.

[33] I have also experimeted with a smaller intertemporal elasticity of substitution ($\gamma = 2$). All IRFs are qualitatevly identical to those presented here. The only significant difference is that, due to the bigger desire for consumption smoothing, consumption in each country rises more on impact, to remain flatter thereafter. As a consequence, in $t = 0$ foreign demand falls less than with log-utility, ultimately leading to positive entry in the home country also on impact.

reasons, foreign GDP increases, lagging the pattern of delayed technology improvement in the foreign country. Wages and labour supply tend to track the trajectory of GDP in both countries. As foreign demand rises above its long-run average, so do home exports X_t, while home imports M_t initially follow the sharp decrease in home investment and then slowly grow for several periods. The pattern of the home trade balance TB_t follows: after running a deficit on impact (exports temporarily fall and imports rise), it comes back into equilibrium in the subsequent period, then turns into surplus for roughly three years and finally mean-reverts while remaining slightly negative.

As the technology shock decays at home while picking up momentum abroad, relative marginal costs in domestic production tend to converge. The home economy is more cost-effective for the first four years, after which a negligible competitiveness advantage arises for foreigners. As the cost structures converge, so do average overall profits. At home, however, the mass of available varieties and domestic demand are permanently higher than abroad, so the same applies to aggregate overall profits.[34]

In the long run all variables mean-revert to the stationary long-run equilibrium.

As a final note, I have also experimented with two alternative values of the Frisch elasticity of labour supply to real wages, as mentioned in Section 2.7. All IRFs are qualitatively similar to my baseline case ($\sigma = 1$), so I do not report them.

3.2 Comparison with GM

Figure 3 displays the IRFs for the home economy in the original GM model. What makes my model inherently different from theirs? The main differences lie in the behaviour of the relative marginal costs in domestic production, in real terms, and the associated effects on entry.

Many of the intuitions developed by GM revolve around the trajectory of what they call "terms of labour", defined as $Q_t(w_t^*/Z_t^*)/(w_t/Z_t)$. This is just the ratio of real marginal costs in domestic production in the two countries, expressed in a common consumption bundle, the home one. In my model, this variable is represented by $RMC_{D,t}$, as defined in the previous section. In both models, the variable under examination is equal to relative average domestic prices $Q_t\tilde{\rho}^*_{D,t}/\tilde{\rho}_{D,t}$.

$RMC_{D,t}$ measures the cost-effectiveness in domestic production of home firms relative to foreign enterprises: if $RMC_{D,t}$ falls below one, its long-run average, a firm with given productivity z could produce any amount of output at lower cost in the foreign country than at home. The same does not hold exactly for the exporting sector, since there averages also fluctuate endogenously with the cutoff productivity levels $z_{X,t}$ and $z^*_{X,t}$, but it does hold for the *variable* costs of any two firms z and z^* whose exporting status does not change over time.

In GM, $RMC_{D,t}$ falls permanently with the technology shock (Figure 3, second-to-last panel): the home economy becomes the least cost-effective. Marginal costs rise in both economies, but slightly more so at home. This is driven by wages rising more than proportionately with the technology shock, which ultimately leads to procyclical real marginal costs at home (see next section). This is a controversial prediction given that markups are constant, as explained in the introduction. Instead, in my model the rise in $w_{cf,t}$ is initially smaller than the rise in Z_t, which is mainly driven by wages increasing less than in the GM model:[35] $RMC_{D,t}$ rises above its long-run average for about four years after impact (Figure 1, last panel), so that home is the country more cost-effective. This leads to the home economy being more profitable than in the GM model, which, coupled with the smaller increase in wages, eventually delivers the sought-for countercyclicality of the labour share.

Figure 4 depicts the IRF of the labour share in my model and in GM against the IRF

[34]Internal demand tends to overwhelm external demand, since in steady state exports are less than one third of aggregate domestic demand.

[35]Abroad, the gradual growth of w_t^* in the GM model is very similar to the response of $w^*_{cf,t}$ in my model.

estimated by Ríos-Rull and Santaeulàlia-Llopis (2010) from U.S. data.[36] My model correctly predicts an initial fall in the labour share, which later rises above its long-run average (after three quarters; half a year later in the empirical IRF), then peaks and finally mean-reverts. It can be surmised from the figure that the labour share in my model is not going to be quite as volatile as in the data. In GM the dynamics are almost reversed: the labour share jumps upward on impact and stays permanently above its long-run average.

Ultimately, the question is: why do home real wages grow so much in the GM model relative to mine?

The answer does not rest upon labour supply being endogenous in my model. I have experimented with introducing labour supply in an otherwise standard GM model,[37] and their main findings stand.[38] The result is robust to different specifications of the labour supply, as documented by Farhat (2009): he introduces endogenous labour supply in an otherwise standard GM model, and finds that in all preference parametrization he proposes the "effective wage" w_t/Z_t is permanently above its long-run average after a positive technology shock.

Nor does the answer rest upon my model featuring endogenous capital accumulation. In GM, production functions are linear in labour, so that there are no diminishing returns that restrain firms in increasing labour demand when technology improves. Instead, a predetermined capital stock has the potential to deliver a trajectory for wages that is only gradually increasing, since it introduces diminishing returns to labour in the short run. In turn, this may cause the effective wage w_t/Z_t to fall in the initial periods. It does not, however: I have experimented with setting $\psi = 1$ in my model (the only other significant difference with respect to GM, having clarified that endogenous labour supply is not one) and once again the behaviour of the labour share and effective wages in the home country are very similar to the baseline GM model.

What drives the difference between my results and GM's is the assumption that technology improvements are not as efficacious in the production of new firms as they are in manufacturing,[39] which affects the behaviour of entry, the associated labour demand and, ultimately, wages.

Specifically, in GM domestic demand rises on impact less than in my model: besides the reactive component of investment in physical capital, my consumers anticipate relatively higher GDP growth (thanks to endogenous labour supply and capital accumulation), and this wealth effect lifts the entire path of consumption. Also, in GM foreign demand for home goods (i.e. foreign consumption) rises on impact; D_t^* falls in my model following the contraction in investment I_t^*, but the consequences for the home economy are limited by exports being only roughly one fourth of domestic aggregate demand. Note that the concept of investment in GM, $N_{E,t}\tilde{v}_t$, does behave like investment in physical capital I_t in my model: in the initial periods, it jumps upward at home and drops abroad.[40] However, these are exogenously nontradable capital goods, so "disinvestment" abroad does not directly feed back on the home economy via reduced import demand.

Overall demand for goods produced in the home economy ultimately grows more in my model relative to GM. Together with the rise in their home marginal costs, this results in a smaller increase in overall average profits in their model. Therefore, the environment is not as profitable for entrants, but entry costs are also lower due to the fact that they fully benefit from the technological improvement: they gradually rise mildly together with effective wages w_t/Z_t, whereas in my model they jump upward together with w/Z_t^{ψ}. This entails entry increasing almost 4 per cent on impact in GM, then mean-reverting in a concave trajectory, whereas it gradually increases in a hump-shaped fashion in my model. The result is that in GM there is

[36] I am grateful to the two authors for sharing their estimates with me.

[37] Specifically, I have used GHH preferences, with $\sigma = 1$; Φ and χ are calibrated so that in steady state labour supply and consumption equal the corresponding values in the GM model.

[38] Similarly, making labour supply exogenous in my model ($\sigma = 0$) does not alter my findings.

[39] I have experimented also with setting $\psi \in (0, 1)$ in an otherwise standard GM model.

[40] Not surprisingly, the international correlation of investment is negative in both models, a counterfactual prediction I will come back to in the next section.

a large reallocation of labour away from the domestic manufacturing sector into the production of new firms (the exporting sector also sheds jobs, but in a small amount). This is indicative of a large increase in labour demand within the E-sector, which is ultimately what drives the growth of effective wages.[41]

In conclusion, in GM the "excessive" responsiveness of wages is responsible for the counterfactual behaviour of marginal costs and the labour share in the home country, and it is driven by an "excessive" absorption of labour in the sector producing new firms. In my model, diminishing returns to labour and endogenous labour supply do play a role in reducing the initial response of wages and marginal costs, but the crucial difference is in the gradual entry of new firms induced by the parameter ψ being smaller than unity. In a sense, ψ can be seen as a smoothing parameter that strikes a compromise between the unbounded mass of potential entrants featured by GM and the ad hoc assumption made by Chaney (2008), where the mass of potential entrants is set exogenously.

My results are robust to varying the Frisch elasticity of labour supply to real wages by 50 per cent. As I detail in the next section, my version of the process of entry of new firms also delivers a more plausible correlation between real GDP and net entry than the GM model.

In both my model and in GM, in the initial periods the export cutoff $z_{X,t}$ falls as marginal firms profitably move into the export market. In my model this is driven by relative marginal costs falling, whereas in GM it mainly follows from the increase in foreign demand. In their model, however, the relative cost-effectiveness of home firms ($RMC_{D,t}$) keeps deteriorating for some time, ultimately reversing the dynamics of the cutoff $z_{X,t}$: after falling on impact it starts growing, and it rises above its steady-state level after roughly three years. As mentioned in the previous section, in my model the cutoff $z_{X,t}$ is permanently below its long-run average after the initial period.

The behaviour of relative marginal costs also shapes the trajectory of the empirical real exchange rate $Q_{R,t}$ (Figures 2 and 3, last panel). In my model, marginal costs at home fall in the initial period while rising abroad. Not surprisingly, then, the home average price (the CPI) falls relative to the foreign one and $Q_{R,t} = \varepsilon_t \tilde{P}_t^* / \tilde{P}_t$ rises on impact (real depreciation) and then slowly mean-reverts. In GM the foreign economy is permanently more cost-effective than the home country, and this is mirrored in a persistent appreciation of the empirical real exchange rate. In both models, deviations of $Q_{R,t}$ from its long-run average ($\bar{Q} = 1$) represent endogenous deviations from purchasing power parity, which only arise thanks to firm heterogeneity.

3.3 Simulated moments

The statistical performance of the model in replicating empirical moments is based on the empirical real variables, which use the CPI ($\tilde{P}_t$) as deflator and are identified by a subscript R. Table 1 reports the main results. The first column displays the empirical moments based on U.S. and European data, mainly taken from Backus, Kehoe and Kydland (1992), the same benchmark used by GM. The remaining columns are devoted to the model-generated moments of the Hodrick-Prescott-filtered variables:[42] the second column refers to the GM model, the third to my baseline model ($\sigma = 1$), the fourth to my model with $\sigma = 0.5$, and the last to my model with $\sigma = 1.5$ (see Section 2.7 on calibration for details). Recall that in all versions of my model, parameter values for ψ and η are calibrated so as to match the contemporaneous correlation of real output $Y_{R,t}$ with the labour share and the relative standard deviation of total investment $TI_{R,t}$ (relative to the standard deviation of GDP).

[41] GM themselves point out that "The effect of endogenous entry is crucial, as the labor demand generated by a greater number of home firms translates into an appreciation of home labor units" (p. 889). In his lecture notes on the GM model, Chaney (2012) is even more explicit: "Absent entry of domestic firms which drive the domestic wages up, a productivity gain at home would actually depreciate effective domestic labor" (p. 13).

[42] These are obtained as the average of the corresponding moments from 2500 simulations of 200 observations each.

Special attention will be devoted to three "stylized facts" of the international trade literature. The first is the "consumption-output anomaly", from Backus, Kehoe and Kydland (1992) delivering the counterfactual prediction that consumption is more strongly correlated across countries than GDP. The second is the "consumption-real exchange rate anomaly": Backus and Smith (1993) show that IRBC models with complete asset markets entail a perfect positive correlation between the real exchange rate and relative real consumption levels, but they document that no clear pattern emerges from the data. Instead, Chari, Kehoe and McGrattan (2002) report evidence of a negative correlation emerging for the U.S. relative to Europe. The last stylized fact is the classic Feldstein-Horioka puzzle that the saving rate and the investment rate tend to be highly correlated within countries even though they need not be in open economy (Feldstein and Horioka, 1980).

Finally, the models under examination will also be evaluated for their ability to predict the cyclical properties of net entry (the difference between new incorporations and failures) and profitability, which Bilbiie, Ghironi and Melitz (2007) show to be strongly procyclical in U.S. data.

My model does an excellent job at replicating the volatility of real GDP (1.71 in U.S. data), which is too low in GM (0.91). This is thanks to capital accumulation, a highly volatile component of aggregate output, and to endogenous labour supply. Varying the Frisch elasticity σ of labour supply to real wages has a non-negligible effect on the volatility of output: smaller values of σ clearly reduce it, as we move closer to the exogenous-labour-supply case.

Turning to relative standard deviations, that of real consumption $C_{R,t}$ (0.49 in the data) is better predicted by the GM model (0.57), while mine tends to overestimate it under all parametrizations (around 0.70). Introducing habit formation in consumption, an avenue I have not explored, would likely improve my baseline model here.

In GM, the real output of the E-sector ($N_{E,t}\tilde{v}_{R,t}$), i.e. sunk entry costs, is interpreted as a proxy for investment in physical capital and displays excess volatility, consistently with what I have argued is the "excessive" reactiveness of this sector in their model. The relative standard deviation is 4.26 (3.15 in U.S. data). In my model, $N_{E,t}\tilde{v}_{R,t}$ is only one third more volatile than output (across all values of σ), thanks to the assumption $\psi < 1$. The relative volatility of the real capital stock $K_{R,t}$ in my model (0.44) and the relative volatility of the proxy for capital in GM ($N_{D,t}\tilde{v}_{R,t}$), 0.32, are evenly spread around the empirical value of 0.37.

Despite choosing GHH preferences for their potential to avoid excessive smoothness in the series for hours, aggregate labour is only half as volatile as output in my model, the relative standard deviation being 0.86 in the data. The failure to replicate the volatility of hours is a standard result in the RBC literature, known at least since the indivisible-labour contribution of Hansen (1985). Conversely, in my model hours are perfectly positively correlated with GDP, whereas the empirical correlation is only 0.86.

As expected, the volatility of hours is increasing in σ. As anticipated in the previous section, in my model the labour share displays far too little volatility. Since changes in the elasticity σ of hours to wages also feed back on the volatility of aggregate output, not surprisingly σ has no effect on the relative standard deviation of the labour share. This is around 0.08 in all my models, only slightly larger in GM (0.13), and definitely greater in U.S. data (0.42, as estimated by Ríos-Rull and Santaeulàlia-Llopis, 2010).

Both my model and GM greatly overestimate the relative standard deviation of the trade-balance-to-GDP ratio ($TB_{R,t}/Y_{R,t}$), which is 0.26 in the data and above 0.88 in all models presented in Table 1. Symmetrically, all the models predict that the ratio should be almost perfectly negatively correlated with contemporaneous output (-0.99 in GM, around -0.88 in the various versions of my model), whereas the correlation is only -0.28 in U.S. data.

My model slightly underestimates the contemporaneous correlation between real total investment $TI_{R,t}$ and real output (around 0.70 in the simulations), whereas in the GM model the proxy for investment ($N_{E,t}\tilde{v}_{R,t}$) perfectly matches the empirical correlation (0.90). As for the capital stock, all the models correctly predict that it is uncorrelated with GDP, with GM doing a slightly better job with their proxy $N_{D,t}\tilde{v}_{R,t}$ at replicating the empirical correlation of 0.01. Conversely, all the models overestimate the procyclicality of real consumption $C_{R,t}$: the correlation with real GDP is 0.76 in the data, above 0.90 in all simulations. Introducing habit formation in consumption would probably improve the models' performance in this area too.

The labour share of GDP (ls_t) is counterfactually procyclical in GM, and strongly so: the correlation with contemporaneous output is 0.85 in their model. It is -0.24 in U.S. data as estimated by Ríos-Rull and Santaeulàlia-Llopis (2010), who also report the labour share lagging real output by roughly one year. This is entirely consistent with my findings:[43] in the bottom portion of Table 1, the highest correlation between current output and leads-and-lags of the labour share occurs after three to four quarters. In my baseline parametrization ψ is set to 0.219; it can be increased up to around 0.33 before turning the labour share into a procyclical variable.[44]

Turning to the stylized facts of the international trade literature, all the models under consideration share the consumption-output anomaly pointed out by Backus, Kehoe and Kydland (1992). While my model does a better job than GM in overestimating the cross-country correlation of real consumption (0.46 in U.S.-Europe data), their model delivers a slightly higher, yet insufficient, cross-country correlation of real GDP (0.70 in U.S.-Europe data). The broad intuitive answer as to why the international correlation of output is smaller than the international correlation of consumption in both models is that investment is negatively correlated across countries, as mentioned in the previous section.[45]

As for the consumption-real exchange rate anomaly, the GM model is closer to the finding by Chari, Kehoe and McGrattan (2002) of a negative correlation coefficient (-0.35) emerging between U.S. and Europe, although the simulated correlation is much larger in absolute value (-0.78). Instead, my model entails a correlation around 0.83, closer to the Backus and Smith (1993) finding of a perfect positive correlation between the real exchange rate $Q_{R,t}$ and relative real consumption levels $C_{R,t}/C^*_{R,t}$, although my model features incomplete asset markets. Chari, Kehoe and McGrattan (2002) further document that the real exchange rate is highly volatile and highly persistent: the standard deviation is 7.52 and the first-order autocorrelation is 0.83. While all models in Table 1 replicate the large persistence equally well, they all fail with volatility: the standard deviation is 0.04 in GM, around 0.30 in my model.

GM feature an extreme version of the Feldstein-Horioka puzzle, with the correlation between the saving rate $S_{R,t}/Y_{R,t}$ and the investment rate (as proxied by $N_{E,t}\tilde{v}_{R,t}/Y_{R,t}$) equal to 0.98. My model considers the total investment rate $TI_{R,t}/Y_{R,t}$ and delivers a correlation with the saving rate around 0.77, in line with the one found in U.S.-Europe data (0.68). I mentioned earlier that introducing endogenous labour supply and investment in physical capital not only makes my model more realistic, but also improves its empirical predictions. Most notably, absent these two assumptions, my model would deliver a negative correlation between savings and investment.

The "excessive" role of entry in the GM model is reflected in the contemporaneous correlation between real GDP and net entry (new incorporations minus failures, $N_{E,t} - \delta\left(N_{D,t} + N_{E,t}\right)$) being 0.89 in their model, while my version of the E-sector delivers a correlation around 0.40, perfectly in line with the estimates by Bilbiie, Ghironi and Melitz (2007) on U.S. data. Conversely, the countercyclicality of marginal costs in my model is mirrored in real profits being almost perfectly correlated with output (the correlation coefficient is 0.97). The GM model does a better job at delivering a correlation closer to the value found in U.S. data (0.75; 0.82 in GM).

[43] The parameter ψ is only calibrated to match the *contemporaneous* correlation between GDP and the labour share.

[44] All other parameters equal except for η, which I keep calibrating so as to match the relative standard deviation of $TI_{R,t}$.

[45] Backus, Kehoe and Kydland (1992) do not estimate the international correlation of investment. Farhat (2010) updates their estimates and reports a correlation coefficient of 0.37 based on U.S.-Europe data. Some variants of his model are able to deliver a positive correlation of investment across countries.

4 Conclusions

The GM model (Ghironi and Melitz, 2005) features exogenous labour supply and it requires the unrealistic assumption that all capital goods are nontradable. Its main predictions are robust to the introduction of endogenous labour supply and investment in tradable capital goods.

Unfortunately, so is the puzzling behaviour of real marginal costs: a positive technology shock hitting the home country makes it less cost-effective than the foreign economy. Real marginal costs are procyclical despite constant mark-ups, whereas the mixed evidence on procyclical marginal costs is associated with countercyclical mark-ups (Rotemberg and Woodford, 1999). Definitely counterfactual are the ultimate consequences for the splitting of income: wages grow more than profits during booms, so that the labour share of GDP is procyclical. This is at odds with the evidence presented by Rotemberg and Woodford (1999) and Ríos-Rull and Santaeulàlia-Llopis (2010).

The GM paper is at the frontier of IRBC models. If it is to be the one we use to think about business cycle fluctuations in open economy with flexible prices, it is worth attempting to reconcile its implications for the labour share with empirical evidence: our understanding of how exporters, non-exporters and international trade react to shocks should not ultimately turn on a controversial behaviour of real marginal costs. This is the aim of my paper, and I achieve it by assuming that technological advances are not as efficacious in the production of new firms as they are in manufacturing; after all, major historical advances towards industrial cost-effectiveness (such as assembly lines, product standardization, mass production, just-in-time production strategies, and so on) do not appear to characterize the setting up of new firms. This single modification to the baseline setting is sufficient to deliver the sought-for countercyclicality of the labour share and real marginal costs.

Specifically, I assume that an aggregate productivity Z_t enters the production function of consumption goods, while Z_t^{ψ}, $\psi \in (0, 1)$, enters the "production function of new firms" (GM implicitly use $\psi = 1$). In my baseline parametrization $\psi = 0.219$, meaning that in the log-linear version of the model technology improvements are only about one fourth as efficacious in the production of new firms. Encouragingly, my calibration for ψ also does an excellent job at replicating the cyclical properties of net entry (defined as new incorporations minus failures), which is one of the variables most directly affected by the parameter ψ.

What makes the GM setting at odds with the countercyclicality of the labour share observed in U.S. data is their model for entry by new firms. In their equilibrium, wages display an "excessive" reactiveness to technology shocks, which can be traced back to an "excessive" increase in labour demand by the sector devoted to the "production of new firms". GM entertain a notion of investment, associated with the output of this sector: although it does not impinge on import demand (these are inherently nontradable capital goods), it is as responsive as investment in physical capital usually is in RBC models. This is also mirrored in an excessive role of entry by new firms, as highlighted by the contemporaneous correlation between real GDP and net entry being twice as big as in U.S. data.

Introducing endogenous labour supply and decreasing marginal returns to labour in the short run (thanks to the predetermined capital stock) has the potential to reduce the "excessive" reactiveness of real wages in the GM model, but it is not sufficient (nor necessary): an adequately small value for ψ is needed. The assumption $\psi < 1$ tightens the link between wages and the sunk entry cost. As the former start to grow, the latter follows closely and entry by new firms is slower than in GM, which feeds back into a smaller increase in labour demand and a more gradual wage growth. The ultimate result is that the labour share becomes countercyclical, consistently with empirical evidence. So do real marginal costs: in relative terms, a technology shock hitting the home economy makes it the most cost-effective country.

Aside from the labour share and real marginal costs, overall my model and GM's do an equally good job at replicating the empirical moments typically considered in the IRBC literature.

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Figure 1: IRFs for the home economy, baseline model

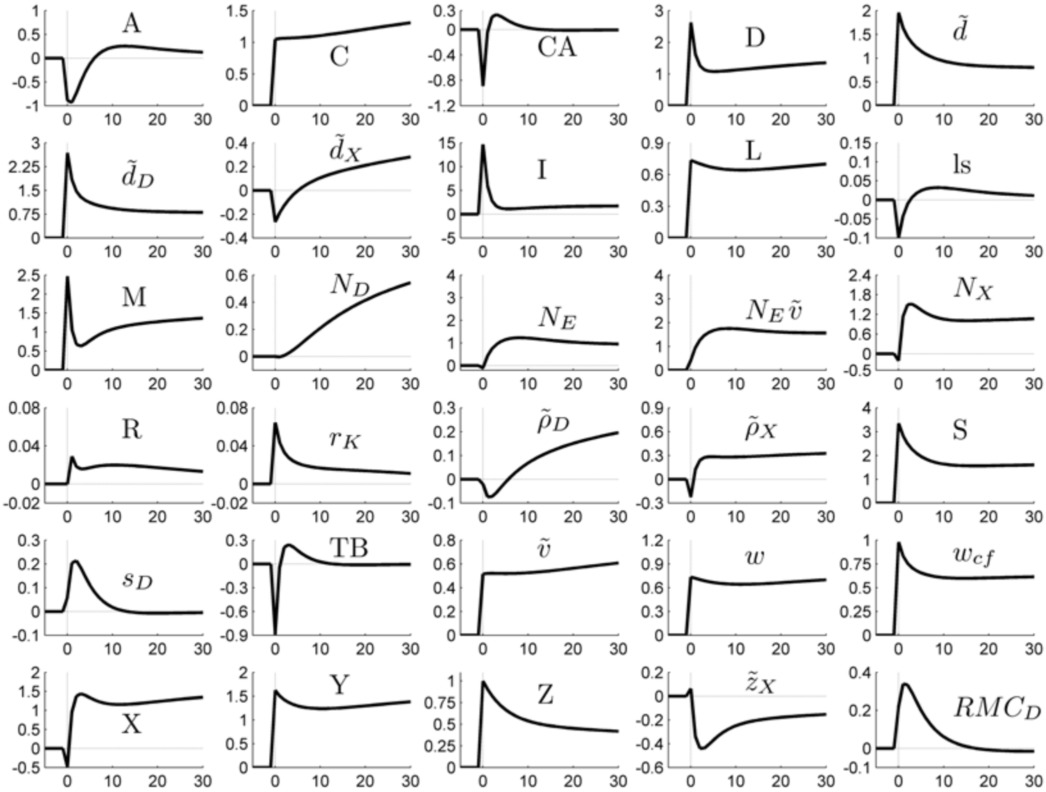

Figure 2: IRFs for the foreign economy, baseline model

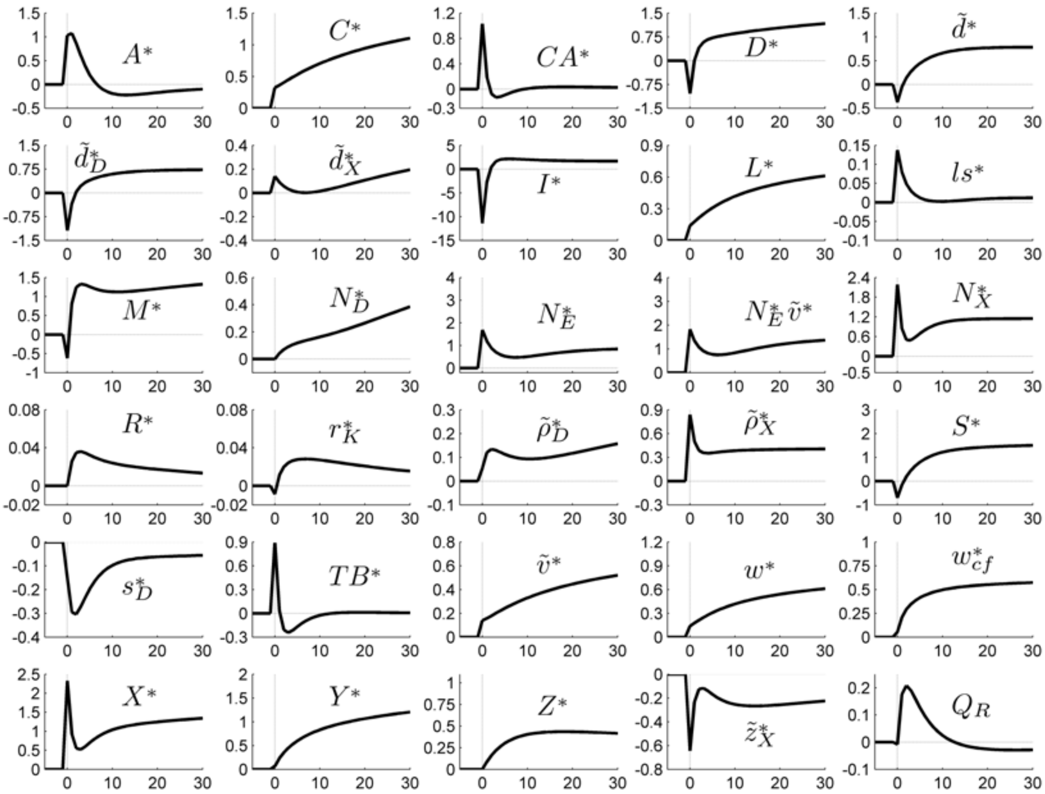

Figure 3: IRFs for the home economy, GM model

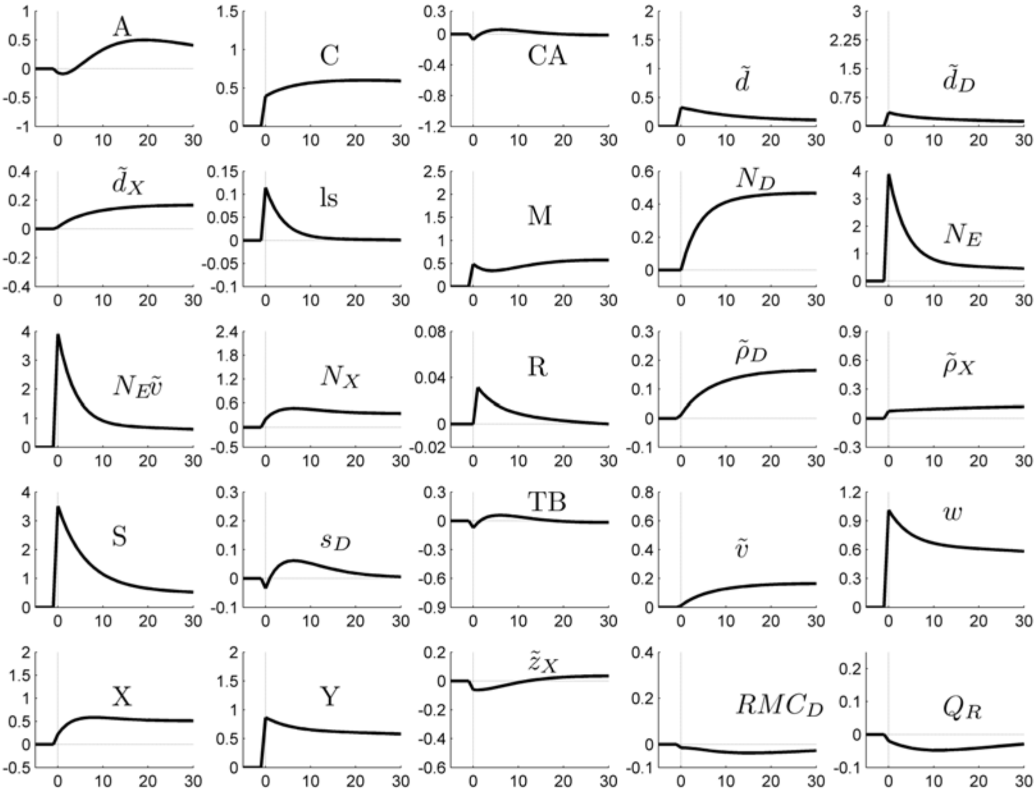

Figure 4: Empirical and model-generated IRFs for the labour share

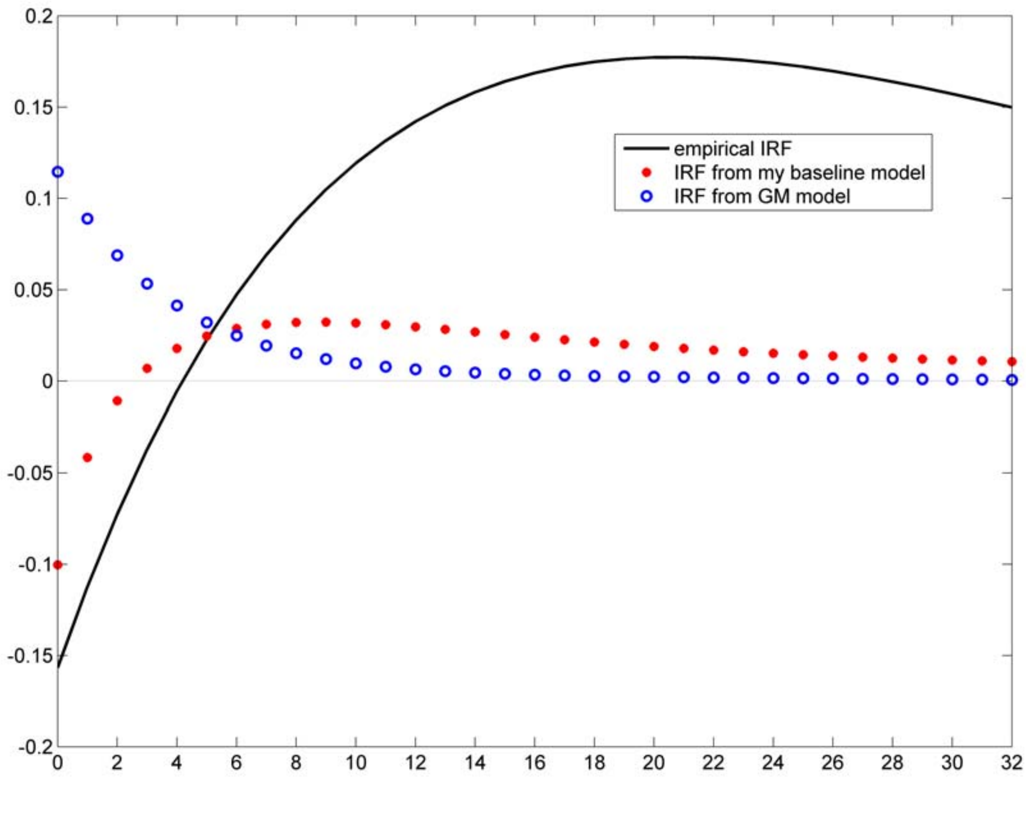

Table 1: EMPIRICAL AND MODEL-GENERATED MOMENTS

Variable	Empirical moments(1)	GM	My model $\sigma=1$	$\sigma=0.5$	$\sigma=1.5$
			Standard deviation		
Y_R	1.71	0.91	1.63	1.36	1.87
C_R	0.84	0.51	1.18	0.91	1.43
TI_R	5.38	3.86(2)	5.15	4.27	5.90
K_R	0.63	0.29(3)	0.72	0.61	0.83
L	1.47	0.00	0.84	0.46	1.16
ls	0.72(4)	0.12	0.14	0.11	0.16
TB_R/Y_R	0.45	0.80	1.55	1.25	1.82
Q_R	7.52(5)	0.04	0.31	0.25	0.35
			Standard deviation relative to Y_R		
Y_R	1.00	1.00	1.00	1.00	1.00
C_R	0.49	0.57	0.72	0.67	0.76
TI_R	3.15	4.26(2)	3.15(6)	3.15(6)	3.15(6)
K_R	0.37	0.32(3)	0.44	0.45	0.44
L	0.86	0.00	0.51	0.34	0.62
ls	0.42(4)	0.13	0.08	0.08	0.08
TB_R/Y_R	0.26	0.88	0.95	0.92	0.97
			Contemporaneous cross correlations		
Y_R, Y_R^*	0.70	0.40	0.34	0.31	0.37
C_R, C_R^*	0.46	0.92	0.63	0.64	0.63
C_R/C_R^*, Q_R	−0.35(5)	−0.78	0.84	0.88	0.80
S_R/Y_R, TI_R/Y_R	0.68	0.98(7)	0.77	0.80	0.76
			Other contemporaneous correlations with real GDP		
Labour share	−0.24(4)	0.85	−0.24(8)	−0.24	−0.24
Net entry	0.40(9)	0.89	0.40	0.36	0.44
Real profits	0.75(9)	0.82	0.97	0.97	0.97
			First-order autocorrelation for the real exchange rate Q_R		
$corr(Q_{R,t}, Q_{R,t-1})$	0.83(5)	0.89	0.78	0.80	0.76

(1) Source: Backus, Kehoe and Kydland (1992) unless otherwise noted.
(2) Standard deviation of $N_{E,t}\tilde{v}_{R,t}$.
(3) Standard deviation of $N_{D,t}\tilde{v}_{R,t}$.
(4) Source: Ríos-Rull and Santaeulàlia-Llopis (2010).
(5) Source: Chari, Kehoe and McGrattan (2002).
(6) The parameter η is calibrated so as to match this value.
(7) The investment rate is defined as $N_{E,t}\tilde{v}_{R,t}/Y_R$.
(8) The parameter ψ is calibrated so as to match this value.
(9) Source: Bilbiie, Ghironi and Melitz (2007). Approximate value surmised from their graphs.

Correlations between $Y_{R,t}$ and variables dated $t+j$, $j=-5,...,5$, baseline model

Variable	−5	−4	−3	−2	−1	0	1	2	3	4	5
Y_R	−.05	.07	.23	.44	.70	1.00	.70	.44	.23	.07	−.05
C_R	−.08	.04	.20	.40	.66	.97	.74	.51	.31	.14	.01
TI_R	.04	.12	.23	.37	.53	.70	.22	.03	−.06	−.11	−.13
K_R	−.28	−.26	−.22	−.14	−.02	.14	.48	.53	.47	.38	.30
L	−.06	.06	.22	.43	.69	.99	.70	.46	.26	.10	−.02
ls	−.05	−.08	−.12	−.16	−.21	−.24	.05	.18	.22	.22	.19
TB_R/Y_R	.01	−.10	−.24	−.42	−.64	−.88	−.42	−.20	−.06	.02	.09
Q_R	−.08	−.04	.02	.11	.23	.38	.56	.45	.27	.12	−.01