Consumer Behavior, Monopolistic Competition, and International Trade*

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Abstract

We argue that monopolistic competition trade models featuring pro-competitive effects, such as the model in Krugman (1979) and, more generally, models assuming linear demand systems, build on dubious assumptions about consumer behavior, and are therefore potentially misleading. Specifically we show that, in this class of models, the pro-competitive effect is entirely driven by the assumption that the substitutability between varieties is *decreasing* in the level of individual consumption. Under the alternative and more plausible assumption that the elasticity of substitution is instead *increasing* in the consumption level, these models yield an anti-competitive effect. We study the robustness of this surprising result in alternative settings featuring, e.g., consumers' preference for an ideal variety, strategic interactions and heterogenous firms. We find that a pro-competitive effect may plausibly arise only in the presence of a *small* number of firms, an assumption ad odds with monopolistic competition. Otherwise, markups are either increasing or unrelated to trade opening. We also find that trade-induced selection effects *à la* Melitz (2003) are instead generally robust to the assumptions about consumer behavior. Overall, our results cast doubts on existing attempts to depart from the standard monopolisitc competition trade framework featuring CES preferences and constant markups.

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1 Introduction

In a celebrated paper, Krugman (1979) developed a simple model providing an appealing explanation for trade in horizontally differentiated varieties between similar countries, a widespread phenomenon which could not be easily explained by the traditional paradigm. Krugman showed that, under the assumption of additive and symmetric preferences on the demand side, and monopolistic competition on the supply side, a trade-induced expansion in market size leads to higher welfare (as consumers can enjoy a greater variety of goods at a lower price) and lower markups (the pro-competitive effect of trade liberalization).

Crucially, Krugman (1979) assumes that $\varepsilon'(c_i) < 0$, where c_i is individual consumption of variety i and ε is the price elasticity of demand for an individual firm. Under the Dixit and Stiglitz (1977, henceforth DS) assumption that firms ignore price interactions with other firms, ε can be shown to equal the elasticity of substitution between any two varieties when they are consumed in the same amount. The inequality $\varepsilon'(c_i) < 0$ can therefore be stated as an assumption that preferences feature an elasticity of substitution that is decreasing in the level of individual consumption (henceforth, DES preferences). Krugman claims (p. 476) that: "*This assumption [..] seems plausible. In any case, it seems necessary if this model is to yield reasonable results, and I make the assumption without apology.*" In this paper we argue, instead, that the assumption $\varepsilon'(c_i) < 0$ is implausible, and that its implications are no more reasonable, in general, than the implications of the opposite and at least as plausible assumption that $\varepsilon'(c_i) > 0$.

To motivate our analysis, we propose the following exercise of introspection. Consider a situation in which you are endowed with two red pencils and two blue pencils, and compare it with a situation in which you are endowed instead with ten red and ten blue pencils. The key question is: do you perceive a red and a blue pencil as more substitutable in the former or in the latter situation? If you think, as we do, that varieties become less substitutable when consumption of each shrinks, then the assumption $\varepsilon'(c_i) < 0$ is violated. In this case, $\varepsilon'(c_i) > 0$ and preferences exhibit an increasing elasticity of substitution in the level of individual consumption (henceforth, IES preferences).

In Section 2, we explore the implications of IES preferences in a framework *à la* Krugman (1979) and find that a trade-induced expansion in market size leads to higher markups, an implication strongly at odds with the conventional wisdom. IES preferences also imply, however, that an expansion in market size due to productivity growth is pro-competitive, whereas DES preferences imply the opposite. To gain more intuition, we propose a specific functional form for IES preferences. In particular, we show that symmetric and additive preferences of the type $U = \sum_i u(c_i)$, where $u(c_i) = \gamma\left(c_i^\rho/\rho\right) + c_i$ is a weighted average of a CES and a linear sub-utility function, feature an increasing elasticity of substitution. This formulation is amenable to a simple economic interpretation, according to which consumers perceive differentiated varieties of some product as being midway between heterogenous and homogeneous goods,[1] possibly capturing a specific feature of intra-industry product differentiation (and trade). These preferences also imply that, as in Krugman's model, trade is always welfare increasing, as the anti-competitive effect associated with a trade-induced expansion in market size cannot offset the standard gains from trade.

In Section 3, we explore the robustness of the trade-induced anti-competitive effect generated by IES preferences. A possible counter-argument in defense of Krugman's model is that, although it may build on implausible assumptions, it nonetheless captures, in reduced form, a pro-competitive effect arising from other mechanisms. Specifically, a trade-induced expansion in market size may be pro-competitive because it increases the number of firms n, and n has a positive *direct* impact on the price elasticity of demand. Building on the trade literature, we therefore consider three different setups in which n may directly affect ε. First, we relax the assumption that preferences are additive, as additivity implies that the elasticity of substitution between any two varieties is independent of n (the Appendix provides a formal discussion of this point). Even with non-additive preferences, however, there is no compelling reason for the elasticity of substitution to be directly increasing in n. To make the point, we consider the quasi-linear quadratic preferences used in Melitz and Ottaviano (2008), a prominent example of non-additive preferences yielding pro-competitive effects and thus perceived as an appealing alternative to CES preferences. We show that, perhaps surprisingly, quasi-linear quadratic preferences imply that, for given level of individual consumption, the elasticity of substitution is *decreasing* in n. This means that in Melitz and Ottaviano (2008), just as in Krugman (1979), the trade-induced pro-competitive effect is entirely driven by the *indirect* negative impact of n on individual consumption, namely, by the assumption that $\varepsilon'(c_i) < 0$.

Another reason why the demand elasticity may positively depend on the number of firms is that a trade-induced increase of n tends to "crowd" the variety space, thereby making varieties closer substitutes. This effect cannot be captured by Krugman model's, as it implicitly assume that the number of characteristics/varieties is the same as the number of firms. We therefore revert to Lancaster's (1979) ideal variety approach to monopolistic competition, where the space of characteristics is fixed and finite and a trade-induced

[1] A possible rationalization is that, by their very nature, differentiated varieties of some product can be used to perform either generic or more specific tasks. For instance, a blue pencil can be used either to write down a laundry list (for which a red pencil would be equally appropriate) or, jointly with a red pencil, to mark different types of comments on an exam paper. Hence, a fall in the symmetric endowment of varieties, by reducing the opportunity to use varieties to perform specific tasks, may also reduce their substitutability.

increase in the number of firms makes available varieties closer to one another in the variety space (represented by a unit circle). We show that, even in this framework, the demand elasticity is *not*, in general, increasing in n: indeed, in order for Lancaster's framework to deliver a pro-competitive (or an anti-competitive) effect, additional and rather *ad hoc* assumptions unrelated to the basic framework are required.

Finally, a trade-induced pro-competitive effect may naturally arise by simply relaxing the Dixit and Stiglitz (1977) assumption that firms are small enough to ignore the impact of their pricing decisions on the price index. The reason is that in this case, for given elasticity of substitution, n has a positive direct impact on the demand elasticity ε. To make the point, we consider a Cournot variant of our basic setup with IES preferences and find that the results are in general ambiguous. This is because a trade-induced expansion in market size has now a pro-competitive effect due to the positive direct impact of n on ε, in addition to the anti-competitive effect due to IES preferences. The pro-competitive effect can indeed prevail when the number of competitors is small. For n sufficiently large, however, the anti-competitive effect necessarily prevails.

The above results are derived from a setup with homogenous firms. In Section 4, we show the robustness of the anti-competitive effect of IES preferences in a framework *à la* Melitz (2003) allowing for firm heterogeneity in productivity and fixed costs of exporting. We also find that, under costless trade integration, the flip side of the anti-competitive effect is an anti-selection effect whereby less productive firms can survive in a larger market, due to higher markups. A richer set of results arises instead in the presence of costly trade, as in this case the anti-selection effect due to IES preferences interacts with the standard selection effect due to fixed costs of exporting. Importantly, we show that the selection effect always prevails when fixed costs of exporting induce a partitioning of firms into exporters and non-exporters, i.e., arguably, in the empirically relevant case.

Section 5 briefly concludes. We draw two main messages from our analysis. The first is that a trade-induced expansion in market size is likely to bring about an anti-competitive effect. Our results suggest that this effect, so far ignored in the theoretical and empirical trade literature, is plausible enough to be taken seriously. This does not mean, however, that in our view trade is necessarily anti-competitive. As mentioned above, the anti-competitive mechanism illustrated in this paper may be neutralized by other pro-competitive forces, e.g., when firms interact strategically, and the absence of any pro- or anti-competitive effect is the most natural implication of a framework à la Lancaster (1979). This leads to the second main message of the paper: given our current ignorance of how to model the relationship between trade and competition, departing from DS monopolistic competition with CES preferences, often criticized for the implied invariance of markups to market size, may be premature. Potentially, this conclusion stands in sharp contrast to that of a recent and influential paper by Arkolakis, Costinot and Rodriguez-Clare (2011): it argues that with CES preferences (and provided that some other conditions are satisfied) the number of sources of gains from trade does not affect (sic!) overall estimates of these gains, conditional on observed trade flows. Our paper suggests, however, that abandoning CES preferences to embrace demand systems yielding trade-induced pro-competitive effects may turn out to be a move in the wrong direction. We view this warning as the most important message of our paper.

Our paper is related to the vast theoretical literature on monopolistic competition and international trade, initiated by Dixit and Stiglitz (1977), Krugman (1979, 1980), Lancaster (1979), Helpman (1981), and whose early contributions are systematized in Helpman and Krugman (1985). More recent contributions include Bertoletti (2006) and Behrens and Murata (2007), which discuss specific functional forms consistent with Krugman's (1979) type of preferences.[2] Our paper is also closely related to Zhelobodko et al. (2010). In their independent work, posterior to the first version of our manuscript, they argue against the plausibility of CES preferences, as these represent a knife-edge between cases yielding opposite results. They do not discuss, however, the plausibility of these alternative cases, a key contribution of our paper, and which leads us to reach opposite conclusions. Finally, our paper is related to the recent heterogeneous-firm extensions of the monopolistic competition trade model, and in particular to Melitz (2003), which assumes CES preferences, and Melitz and Ottaviano (2008), which builds instead on quasi-linear quadratic preferences and assumes away the fixed costs of exporting.

2 Monopolistic Competition with IES Preferences

Consider an economy populated by L workers, whose wage is $w = 1$. Consumers share the same additive and symmetric preferences, represented by the following utility function:

$$U = \sum_{i=1}^{N} u(c_i), \tag{1}$$

where c_i is consumption of variety i, defined over a large number N of potential varieties. Only varieties indexed by $i = 1, ..., n$, with $n < N$, are actually produced. The sub-utility function $u(\cdot)$ is strictly increasing and concave, and is at least thrice continuously

[2] See also Neary (2004) for a critical survey of monopolistic competition in international trade theory. He also shares Krugman's view that the assumption $\varepsilon'(c_i) < 0$ is plausible and argues, instead, that the main reason why most scholars opted for a CES utility function is that preferences embedding the assumption $\varepsilon'(c_i) < 0$ proved hardly tractable.

differentiable. In particular, we assume that $u'(\cdot) > 0$, $u''(\cdot) < 0$ and $u(0) = 0$.[3]

Firm i produces a differentiated variety with the total cost function $TC_i = \alpha + \beta q_i$, where $q_i = c_i L$ is its output, and α and β are the fixed and marginal cost, both in terms of labor. Firms are symmetric on the cost and demand side, and therefore solve the same problem. In the following, we therefore drop the variety index i.

Utility maximization subject to a budget constraint implies $u'(c) = \lambda(\mathbf{p})p$, where p is the price charged by an individual firm, $\lambda > 0$ is the marginal utility of income, and $\mathbf{p} = [p_1, ..., p_n]$ is the price vector. Preference additivity implies the elasticity of λ with respect to each price p to be of the same order of magnitude as $1/n$, provided that prices are not disproportionate.[4] Thus, if n is large enough, one can assume, as in Dixit and Stiglitz (1977), that each firm treats λ as a constant, thereby ignoring the price interaction with other firms. Under this assumption, the (inverse) individual demand for a variety is:

$$p = \frac{u'(c)}{\lambda}, \tag{2}$$

and the price elasticity of demand for an individual firm is

$$\varepsilon(c) = -\frac{p(c)}{p'(c)c} = -\frac{u'(c)}{u''(c)c}. \tag{3}$$

Note that $\varepsilon(c)$ equals the reciprocal of the elasticity of marginal utility with respect to individual consumption and that, for given c, is independent of λ and L. Importantly, as shown in the Appendix, $\varepsilon(c)$ equals the elasticity of substitution between any two varieties when they are consumed in the same amount c. Krugman (1979) assumes that $\varepsilon'(c) < 0$, namely, that the elasticity of substitution between any two varieties is decreasing in the level of consumption (DES preferences).[5] As argued in the Introduction, this assumption seems at odds with introspection. In this paper, we therefore explore the implications of the opposite (and at least equally plausible) assumption that $\varepsilon'(c) > 0$, namely, that preferences feature an increasing elasticity of substitution in consumption (IES preferences).

The revenue of an individual firm is:

$$R(c) = p(c)cL = \frac{u'(c)}{\lambda}cL. \tag{4}$$

[3] Note that $U = nu(c)$ for a symmetric consumption pattern, with $nu(c) > u(nc)$: thus, U embeds a Chamberlinian "taste for variety". Moreover, U is well defined over the positive orthant of the relevant Euclidean space: according to standard results, this implies regular and well-behaved demand functions for strictly positive prices and income.

[4] See, e.g., Deaton and Muellbauer (1980), Section 5.3.

[5] Note, also, that $\varepsilon(\cdot)$ equals the reciprocal of the "coefficient of relative risk aversion" of $u(\cdot)$. Krugman's assumption is thus formally equivalent to assuming that preferences feature *increasing relative* risk aversion.

We denote marginal revenue and the derivative of marginal revenue, respectively, by $R'(c) = r'(c)/\lambda$ and $R''(c) = r''(c)/(\lambda L)$, where

$$\begin{aligned} r'(c) &= u'(c) + u''(c)c, \quad (5) \\ r''(c) &= 2u''(c) + u'''(c)c. \quad (6) \end{aligned}$$

The first-order condition for profit maximization implies:

$$r'(c) = \lambda\beta. \quad (7)$$

To obtain a unique and well-behaved solution to the problem of profit maximization, we assume that the marginal revenue is everywhere positive and decreasing. That is, we assume that $r'(c) > 0$ and $r''(c) < 0$ for all c, with $\lim_{c\to\infty} r'(c) = 0$ and $\lim_{c\to 0} r'(c) = \infty$.

Equations (2), (3), (5) and (7) imply that the profit maximizing price can be written as

$$p = \frac{u'(c)}{r'(c)}\beta = \frac{\varepsilon(c)}{\varepsilon(c) - 1}\beta = m(c)\beta, \quad (8)$$

where $m(c) > 1$ is the price-marginal cost markup, and

$$m'(c) = -\frac{\varepsilon'(c)}{[\varepsilon(c) - 1]^2}. \quad (9)$$

Evidently, unlike Krugman's model, where $\varepsilon'(c) < 0$ and the markup is increasing in c, IES preferences imply markups to be decreasing in individual consumption.[6] Differentiating (3) and using (5) and (6) yields the following expression for $\varepsilon'(c)$:

$$\varepsilon'(c) = \frac{u'(c)r''(c) - u''(c)r'(c)}{[u''(c)c]^2}, \quad (10)$$

which implies that

$$\varepsilon'(c) \gtrless 0 \Leftrightarrow \frac{r'(c)u''(c)}{r''(c)u'(c)} = \frac{\eta(c)}{\varepsilon(c)} \gtrless 1, \quad (11)$$

where $\eta(c) = -\frac{r'(c)}{r''(c)c} > 0$ is the reciprocal of a measure of the revenue function curvature.

Free entry implies zero equilibrium profits:

$$\pi = \pi_v - \alpha = (p - \beta)cL - \alpha = 0,$$

where π and π_v denote total and variable profits. Using (5) and (8), the free-entry condi-

[6] As is well known, markups are instead constant with CES preferences.

tion can be written as

$$\pi_v(c) = [m(c) - 1]\,\beta Lc = \left[\frac{u'(c)}{r'(c)} - 1\right]\beta Lc = \left[-\frac{u''(c)}{r'(c)}c\right]\beta Lc = \alpha. \tag{12}$$

Differentiating π_v with respect to c yields:

$$\frac{\partial \pi_v}{\partial c} = \beta L\left[\frac{u''(c)r'(c) - u'(c)r''(c)}{r'(c)^2}c - \frac{u''(c)}{r'(c)}c\right] = \beta L\frac{m(c)}{\varepsilon(c)} > 0. \tag{13}$$

Variable profit is therefore monotonically increasing in individual consumption, which ensures that the equilibrium is unique. Finally, full employment implies that labor demand, $n(\alpha+\beta cL)$, equals labor supply, L (equivalently, equilibrium in the product market implies that $n = 1/pc$):

$$n = \frac{L}{\alpha + \beta Lc} = \frac{L}{\alpha\varepsilon(c)} = \frac{1}{m(c)c\beta}, \tag{14}$$

where the latter equalities follow from (12). Note that (12) implicitly defines the level of individual consumption consistent with profit maximization and free entry as a function $c(L, \beta, \alpha)$ of market size L, marginal cost β, and the fixed cost α. The equilibrium number of firms, $n(L, \beta, \alpha)$, is instead determined recursively by (14).

We can now show how individual consumption depends on the model's parameters. Differentiating (12) with respect to L, β and α yields:

$$\frac{\partial \pi_v}{\partial c}\frac{\partial c}{\partial L} + \frac{\partial \pi_v}{\partial L} = \frac{\partial \pi_v}{\partial c}\frac{\partial c}{\partial \beta} + \frac{\partial \pi_v}{\partial \beta} = 0 \text{ and } \frac{\partial \pi_v}{\partial c}\frac{\partial c}{\partial \alpha} = 1.$$

Noting that $\frac{\partial \pi_v}{\partial L}L = \frac{\partial \pi_v}{\partial \beta}\beta = \pi_v = \alpha$, we obtain the following

Lemma 1 *Individual consumption is decreasing in market size L and marginal cost β, and increasing in the fixed cost α, with:*

$$\frac{\partial c}{\partial L}\frac{L}{c} = \frac{\partial c}{\partial \beta}\frac{\beta}{c} = -\frac{\partial c}{\partial \alpha}\frac{\alpha}{c} = -\frac{r'(c)u''(c)}{r''(c)u'(c)} = -\frac{\eta(c)}{\varepsilon(c)} \lesseqgtr -1 \Leftrightarrow \varepsilon'(c) \gtreqless 0. \tag{15}$$

Lemma 1 and (9) immediately imply the following

Proposition 1 *With IES preferences, markups are increasing in market size L and marginal costs β, and decreasing in the fixed cost α. In contrast, with DES preferences, markups are decreasing in L/α and β.*

Finally, from (11) and (15) IES preferences imply that the elasticity of individual consumption with respect to market size L is greater than one in absolute value. It follows that firm size, $q(c) = cL$, is decreasing in market size. The intuition is that the reduction in individual consumption induced by a rise in market size, by increasing markups, requires a reduction in firm size in order for the zero-profit condition to be restored. Firm size is instead increasing in market size with DES preferences.

2.1 Discussion

Propositions 1 implies that, with IES preferences, frictionless trade integration between identical countries, which in this model is isomorphic to a rise in market size, leads to higher markups. This result is the opposite of the pro-competitive effect delivered by Krugman's model and by virtually all monopolistic competition trade models departing from CES preferences. The internal logic behind this result is simple: at given prices, trade integration allows individuals to spread their consumption across a larger number of varieties. If varieties become less substitutable when consumption of each falls, then firms face less elastic demands, charge higher markups and make profits. In turn, positive profits induce entry and a reduction in firm size in order for the free-entry condition to be restored.

It is hard to dismiss IES preferences as implausible because of these implications. Aside from the fact that they build on at least as plausible assumptions about consumer behavior as DES preferences, Propositions 1 suggests that the overall implications of IES preferences are not obviously more implausible than those generated by DES preferences. Consider, in particular, the effects of a fall in the marginal cost β. In the trade literature, it is standard to interpret $1/\beta$ as a productivity measure. In this respect, with IES preferences, a market size expansion due to productivity growth is pro-competitive. That economic development is associated with more competitive markets seems plausible, yet DES preferences imply the opposite.

Next, consider the effects of an equiproportional fall in both α and β. In this case, the number of firms increases proportionally (see 14). Moreover, (12) implies that individual consumption, markups and firm size are unchanged, and hence that the sign of $\varepsilon'(c)$ plays no role. However, there is no obvious reason to expect technical change to affect fixed and marginal costs in exactly the same proportion. For instance, in monopolistic competition trade models endogenizing technology it is standard to assume that a lower marginal cost requires a higher fixed cost, e.g., in terms of R&D expenditures [see, among others, Yeaple (2005), Bustos (2011a, 2011b), Costantini and Melitz (2007)]. In this latter case, IES preferences would imply an even stronger pro-competitive effect of technical change, as both a rise of fixed costs and a fall of marginal costs would lead to higher individual consumption. This type of technical change would instead imply stronger anti-competitive effects in Krugman's model.

2.2 Welfare

We now show how the sign of $\varepsilon'(c)$ affects the welfare effects of a change in the model's parameters. In a symmetric equilibrium, welfare equals

$$U(L, \beta, \alpha) = n(L, \beta, \alpha)u(c(L, \beta, \alpha)). \tag{16}$$

Differentiating (16) with respect to L, β and α yields:

$$\frac{\partial U}{\partial L}\frac{L}{U} = \left[\phi(c) - 1 + \frac{1}{\varepsilon(c)} - \frac{1}{\eta(c)}\right]\frac{\partial c}{\partial L}\frac{L}{c} = -\frac{\partial U}{\partial \alpha}\frac{\alpha}{U}, \tag{17}$$

$$\frac{\partial U}{\partial \beta}\frac{\beta}{U} = \left[\phi(c) - \frac{\varepsilon'(c)c}{\varepsilon(c)}\right]\frac{\partial c}{\partial \beta}\frac{\beta}{c}, \tag{18}$$

where $\phi(c) = u'(c)c/u(c) < 1$ due to $u(0) = 0$ and the strict concavity of $u(c)$ and $\frac{\partial c}{\partial L}\frac{L}{c} = \frac{\partial c}{\partial \beta}\frac{\beta}{c} < 0$ from (15). Note that, for $\varepsilon' \leq 0$, the expression in brackets is negative in (17) and positive in (18), implying that welfare is increasing in market size and decreasing in fixed and marginal costs. Instead, for $\varepsilon' > 0$ the sign of the expressions in brackets is ambiguous. Therefore, with IES preferences the welfare effects of a change in the model's parameters are in general ambiguous. This is because a rise in L (or a fall in α) has a positive welfare effect due to the induced rise in n (the standard love for variety effect), and a negative welfare effect due to the rise in markups. Conversely, a rise in β has a negative welfare effect due to the fall in the real wages (also due to the rise in markups), and a positive welfare effect due to the induced rise in n.

Interestingly, a sufficient condition for a rise in market size to be welfare increasing is that $\phi' > 0$.[7] Recall that ϕ is the ratio of $u'c$ to u, where the former is proportional to firm revenue and the latter captures the contribution of each variety to consumer welfare. Thus, as suggested by Dixit and Stiglitz (1977: pp. 303-4), ϕ is a sort of "appropriability ratio": if $\phi' > 0$, at the margin each firm finds it profitable to produce more than the socially optimal quantity, and the free-entry equilibrium therefore entails too few varieties. In this case, a social planner would introduce more varieties and sell them at a higher price (to cover the higher unit costs). To see this, consider the constrained (no lump-sum transfers) social optimum that maximizes U under the resource constraint $L = n(\alpha + \beta cL)$. The first-order conditions for this problem imply that:

$$\left[\frac{1}{\phi(c)} - 1\right]\beta Lc = \alpha. \tag{19}$$

[7] To see this, note first that $\frac{\phi' c}{\phi} = \frac{1}{m} - \phi$, which implies that the condition $\phi' > 0$ can equivalently be written as $m\phi < 1$. Second, note that the expression in square brackets in (17) is equivalent to $\phi - \frac{1}{m} - \frac{1}{\eta}$.

Note that $\phi' > 0 \Leftrightarrow m\phi < 1$ implies that the LHS of (19) is increasing in c and larger than the LHS of (12), thereby implying that the optimal c (n) is smaller (larger) than in the free-entry equilibrium. Thus, the intuition for why, with IES preferences and $\phi' > 0$, a rise in market size is welfare increasing is that the induced rise in markups has an impact on entry that partially compensates for the fact that there are too few firms in the market equilibrium. Conversely, a sufficient condition for a rise in marginal cost to be welfare decreasing is that $\phi' < 0$.[8] This is becasue the induced rise in the number of varieties cannot compensate for the reduction in consumption since in this case there are too many firms in the market equilibrium.

In the next section, we formulate a specific functional form for IES preferences with an appealing economic interpretation and standard welfare implications.

2.3 IES Preferences: An Example

Consider the following additive and symmetric utility function:[9]

$$U = \sum\nolimits_{i=1}^{N} \left(\gamma \frac{c_i^{\rho}}{\rho} + c_i \right), \quad \rho \in (0,1), \quad \gamma > 0. \tag{20}$$

The sub-utility function $u(c_i) = \gamma \frac{c_i^{\rho}}{\rho} + c_i$ is a weighted average of a CES and a linear utility function, where the weight of the CES component is regulated by the parameter γ. This formulation implies that varieties are treated partly as different goods and partly as homogeneous goods, possibly capturing the idea that differentiated varieties of a given good are in a sense midway between homogeneous and heterogenous products.[10]

Straightforward calculations using (20) imply that:

$$\varepsilon(c_i) = \frac{1}{1-\rho} \left(1 + \frac{c_i^{1-\rho}}{\gamma} \right). \tag{21}$$

Hence, $\varepsilon(c_i)$ is increasing in individual consumption and ranges from $1/(1-\rho) > 1$ to infinity. The reason is that, for higher c_i, the linear component of $u(c_i)$ tends to prevail

[8] To see this, it is sufficient to note that the expression in square brackets in (18) is equivalent to $\phi - 1/m + u''/r''$.

[9] The utility function in (20) is a special case of the more general formulation

$$U = \sum\nolimits_{i=1}^{N} \left(\gamma \frac{c_i^{\rho}}{\rho} + \frac{c_i^{\delta}}{\delta} \right),$$

with $\gamma > 0$, $\rho, \delta \in (0,1]$ and $\rho \neq \delta$. The results illustrated in this section hold also in this more general IES case.

[10] Insofar as these preferences capture an important characteristic of differentiated products, they may fit nicely into the new trade theory paradigm, originally developed to explain intraindustry trade in similar products by similar countries.

over the concave component, which captures product differentiation, thereby implying that varieties become more substitutable for higher levels of individual consumption.

Note that, unlike CES preferences, where the elasticity of substitution is governed only by the parameter ρ, here also γ plays a role. In particular, a higher γ implies a higher weight on the concave component of $u(c_i)$, and therefore a lower elasticity of substitution for given c and ρ. Note also that the inverse demand function associated to (20) is $p = u'(c_i)/\lambda = \left(\rho\gamma c_i^{\rho-1} + 1\right)/\lambda$, thereby suggesting that a higher γ is naturally associated to a higher level of demand. Thus, the above generalization of CES preferences implies that a higher γ should produce a fall in the elasticity of substitution and a rise in the level of demand for individual firms. These preferences may therefore prove useful to model a framework in which quality differentiation (as captured by a higher γ) naturally leads to higher demand and higher markups.

Welfare The above preferences also imply that, as is the case with $\varepsilon' \leq 0$, a rise in L/α is welfare increasing. To see this, note that:

$$\phi = \frac{\gamma c^{\rho} + c}{\gamma \frac{c^{\rho}}{\rho} + c}, \; m = \frac{1 + \gamma c^{\rho-1}}{1 + \rho\gamma c^{\rho-1}}, \; \frac{\varepsilon' c}{\varepsilon} = \frac{(1-\rho)\, c^{1-\rho}}{(\gamma + c^{1-\rho})},$$

thereby implying:

$$\phi m = \frac{(\gamma c^{\rho} + c)\left(1 + \gamma c^{\rho-1}\right)}{\left(\gamma \frac{c^{\rho}}{\rho} + c\right)\left(1 + \rho\gamma c^{\rho-1}\right)} < 1.$$

Finally, a rise of in the marginal cost is welfare reducing if and only if:

$$\phi - \frac{\varepsilon' c}{\varepsilon} = \frac{(\gamma c^{\rho} + c)\left(\gamma + c^{1-\rho}\right) - (1-\rho)\, c^{1-\rho}\left(\gamma \frac{c^{\rho}}{\rho} + c\right)}{\left(\gamma \frac{c^{\rho}}{\rho} + c\right)\left(\gamma + c^{1-\rho}\right)} > 0,$$

for which a *sufficient* condition is $\rho > 1/3$.

3 Alternative Environments

Our previous results are derived from a simple setup featuring, as in Krugman (1979), additive and symmetric preferences on the demand side, and Dixit-Stiglitz monopolistic competition on the supply side. A crucial implication of these assumptions is that in a symmetric equilibrium the demand elasticity equals the elasticity of substitution, and the latter does not directly depend on the number of firms. Thus, trade opening can only affect the demand elasticity and markups by affecting individual consumption, which in turn directly affects the elasticity of substitution. It follows that, if the substitutability across varieties is increasing in the consumption level, as it should, then trade is necessarily anti-competitive.

In this Section, we consider alternative environments. In particular, building on examples drawn from the received trade literature, we discuss the implications of relaxing the assumptions that preferences are additive, that the space of characteristics/varieties is not finite and fixed,[11] and that firms ignore price interactions with other firms. We consider, in particular, the following setups: a) quasi-linear quadratic preferences, as in Melitz and Ottaviano (2008); b) the ideal variety approach to monopolistic competition, as in Lancaster (1979); c) strategic interactions *à la* Cournot. A common denominator of these different frameworks is that they potentially allow for a trade-induced increase in the number of firms to directly affect the demand elasticity.

3.1 Quasi-Linear Quadratic Preferences

When preferences are additive and symmetric, as in (1), the elasticity of substitution σ_{ij} between varieties i and j is independent of the number of varieties n available for consumption (see the Appendix for a proof). This result is easily understood when recalling that, if $U(\cdot)$ is additive, the marginal rate of substitution between any two varieties is unaffected by consumption of other varieties.

Matters are different, however, if preferences are non-additive. For instance, at a symmetric consumption pattern ($c_i = c$, $i = 1, ..., n$), consumption of other varieties may affect the elasticity of substitution between varieties i and j through a direct impact of n on σ_{ij} (see the Appendix). Hence, non-additive preferences potentially allow for a sort of (pro- or anti-competitive) *externality* of n on the elasticity of substitution. The sign and the interpretation of this externality are not obvious, however.[12] To illustrate, consider the quasi-linear quadratic preferences $U(c_0, u(\mathbf{c}_{-0})) = c_0 + u(\mathbf{c}_{-0})$, where c_0 is consumption of a numeraire good, $\mathbf{c}_{-0} = [c_1, c_2, ..., c_n]$ is consumption of n varieties of some product and

$$u(\mathbf{c}_{-0}) = \alpha \sum_{j=1}^{n} c_j - \frac{\gamma}{2} \sum_{j=1}^{n} c_j^2 - \frac{\eta}{2} \left(\sum_{j=1}^{n} c_j \right)^2 . \tag{22}$$

The above non-additive preferences have been recently used in an influential paper by Melitz and Ottaviano (2008). Their monopolistic competition framework with heterogeneous firms is widely perceived as an appealing alternative to the original Melitz (2003)

[11] See also Dixit and Stiglitz (1979) on this point.

[12] In this respect, it is suggestive that in their discussion of "diversity as a public good", Dixit and Stiglitz (1975: section 4.4) consider a non-additive case in which n enters the utility function without however affecting the marginal rate of substitution between any two varieties.

model, as it delivers a trade-induced pro-competitive effect.

Maximization of (22) with respect to c_i subject to a budget constraint yields:[13]

$$p_i = \frac{\partial u}{\partial c_i} = \alpha - \gamma c_i - \eta \sum_{j=1}^{n} c_j, \qquad i = 1, ..., n. \tag{23}$$

Thus, summing (23) across varieties:

$$\sum_{j=1}^{n} p_j = n\alpha - (\gamma + n\eta) \sum_{j=1}^{n} c_j \Rightarrow \sum_{j=1}^{n} c_j = \frac{n\alpha - n\overline{p}}{\gamma + n\eta}, \tag{24}$$

where $\overline{p} = \frac{1}{n}\sum_{j=1}^{n} p_j$ is the average price of a variety. Using (24) in (23) yields an expression for the inverse demand function:

$$p_i = \frac{\alpha\gamma}{\gamma + n\eta} - \gamma c_i + \frac{n\eta\overline{p}}{\gamma + n\eta}.$$

The direct uncompensated demand function for variety i, $c_i(\mathbf{p})$, which coincides with the compensated demand $\widetilde{c}_i(\mathbf{p})$, is therefore given by:[14]

$$\widetilde{c}_i(\mathbf{p}) = c_i(\mathbf{p}) = \frac{\alpha}{\gamma + n\eta} - \frac{p_i}{\gamma} + \frac{n\eta}{\gamma + n\eta}\frac{1}{\gamma}\overline{p}. \tag{25}$$

Note that the demand perceived by firm i, under the assumption that it takes both n and $\overline{p}$ as given, is *linear* in p_i, with constant slope $1/\gamma$; accordingly, the demand elasticity equals $\frac{1}{\gamma}\frac{p_i}{c_i}$.

As detailed in the Appendix, the expression for the elasticity of substitution is $\sigma_{ij} = \widetilde{\varepsilon}_{ji} - \widetilde{\varepsilon}_{ii}$, where $\widetilde{\varepsilon}_{ij} = \widetilde{c}_{ij}\,(p_j/\widetilde{c}_i)$ and $\widetilde{\varepsilon}_{ii} = \widetilde{c}_{ii}\,(p_i/\widetilde{c}_i)$ are the compensated demand-price elasticities, and $\widetilde{c}_{ii} = \partial\widetilde{c}_i/\partial p_i$, $\widetilde{c}_{ij} = \partial\widetilde{c}_i/\partial p_j$ are the corresponding derivatives, whose expression is given by:

$$\widetilde{c}_{ii} = -\frac{1}{\gamma} + \frac{\eta}{\gamma + n\eta}\frac{1}{\gamma} = \widetilde{c}_{ij} - \frac{1}{\gamma}, \qquad i, j = 1, 2.., n \text{ and } i \neq j.$$

Hence, for $p_i = p_j$ (which implies $c_i = c_j$) we have:

$$\sigma_{ij} = \widetilde{\varepsilon}_{ji} - \widetilde{\varepsilon}_{ii} = \frac{p_i}{c_i}\,(\widetilde{c}_{ji} - \widetilde{c}_{ii}) = \frac{1}{\gamma}\frac{p_i}{c_i} = \frac{1}{\gamma c_i}(\alpha - \gamma c_i - \eta \sum_{h=1}^{n} c_h). \tag{26}$$

[13] The following discussion assumes that an internal solution arises in equilibrium (i.e., $c_0 > 0$).

[14] Recall that, with quasi-linear preferences, there are no income effects on the demand for non-numeraire goods.

In a quasi-symmetric equilibrium, i.e., for $c_h = c$, $h = 1, ..., n$, $i \neq h \neq j$, we obtain:

$$\sigma_{ij} = \frac{1}{\gamma}\left[\frac{\alpha}{c_i} - \gamma - \eta(2 + (n-2)\frac{c}{c_i})\right]. \tag{27}$$

Note that, for given consumption levels c_i and c, the elasticity of substitution is actually *decreasing* in the number of available varieties n. Note also that σ_{ij} is decreasing in c_i and c and that, for $c_i = c$ (i.e., in a fully symmetric equilibrium) its expression boils down to

$$\sigma = \frac{1}{\gamma}\left[\frac{\alpha}{c} - \gamma - \eta n\right]. \tag{28}$$

The above results show that, in the most popular example of non-additive preferences used in the monopolistic competition trade literature, the number of firms has a *negative* direct impact on the elasticity of substitution. This implies that the pro-competitive effect delivered by the model in Melitz and Ottaviano is entirely driven by the linearity of the demand function perceived by the firms, hence by the fact that, just as in Krugman (1979), the elasticity of substitution is decreasing in the level of individual consumption.

3.2 Ideal Variety Approach to Monopolistic Competition

In our baseline setting, borrowed from Krugman (1979), the introduction of new varieties does not crowd the variety space, as the number of characteristics/varieties is the same as the number of firms. One may argue, however, that a pro-competitive effect may naturally arise in a framework in which a trade-induced increase in the number of available varieties reduces their distance in the fixed characteristics space, thereby increasing their substitutability.

In this Section we show that, surprisingly, this needs not be the case. To make the point, we consider Lancaster's (1979) "ideal variety" approach to monopolistic competition. In this setting, consumer preferences are heterogeneous and the aggregate demand for each variety arises from diversity of tastes. In particular, each consumer has a most preferred ("ideal") variety. As described in Helpman and Krugman (1985, pp. 120-21), on which we build in this section, ideal variety means that "when the individual is offered a well-defined quantity of the good but is free to choose any potentially possible variety, he will choose the ideal variety independently of the quantity offered and independently of the consumption level of other goods. Moreover, when comparing a given quantity of two different varieties, the individual prefers the variety that is closest to his ideal product".

These assumptions are formalized by assuming that each variety is represented by a point ω on the unit length circumference Ω of a circle, and that preferences for the ideal product are uniformly distributed over Ω. L is the size (and density) of the population. The utility function of a consumer with ideal variety $\widetilde{\omega}$ is assumed to be:

$$U = \sum_{\omega \in \Omega} \frac{c(\omega)}{h(\delta(\omega, \widetilde{\omega}))}, \tag{29}$$

where $\delta(\omega, \widetilde{\omega})$ is the shortest arc distance between ω and $\widetilde{\omega}$, and $h(\delta)$ is the so-called Lancaster's compensation function, assumed to be positive, non decreasing and generally normalized so that $h(0) = 1$ (see Lancaster, 1975). Moreover, it is generally assumed (see, Helpman and Krugman, 1985) that $h(\delta)$ is strictly increasing and convex, and that $h'(0) = 0$.

We now show that the above assumptions are insufficient to deliver a pro-competitive impact of entry in this setting.[15] To this purpose, note first that preferences as in (29) are of the "perfect substitute" type, with constant marginal rate of substitution (MRS) between any two varieties $\dot{\omega}$ and $\ddot{\omega}$ given by:

$$MRS(\dot{\omega}, \ddot{\omega}) = \frac{h(\delta(\ddot{\omega}, \widetilde{\omega}))}{h(\delta(\dot{\omega}, \widetilde{\omega}))}. \tag{30}$$

The above assumptions on $h(\cdot)$ imply that $MRS(\widetilde{\omega}, \omega) = h(\delta(\omega, \widetilde{\omega}))$ is an increasing convex function of the arc distance between $\widetilde{\omega}$ and ω. Utility maximization then implies:

$$c(\omega) = \begin{cases} \frac{1}{p(\omega)} & \omega = \omega' \\ 0 & \omega \neq \omega' \end{cases},$$

where $\omega' = \arg\min_{\omega \in \Omega} p(\omega) h(\delta(\omega, \widetilde{\omega}))$. In the Appendix we derive the aggregate demand function, $q(\omega)$, for a firm selling variety ω at the price $p(\omega)$, with contiguous competitors ω_l and ω_r charging prices $p(\omega_l)$ and $p(\omega_r)$. In a symmetric equilibrium in which $p(\omega) = p(\omega_l) = p(\omega_r)$ and $d = \delta(\omega_l, \omega) = \delta(\omega_r, \omega) = \frac{1}{n}$, the price elasticity of the aggregate demand is given by:

$$\varepsilon(q(\omega)) = -\frac{\partial q(\omega)}{\partial p(\omega)} \frac{p(\omega)}{q(\omega)} = 1 + \frac{h(d/2)}{2(d/2)h'(d/2)} = 1 + \frac{1}{2\epsilon_h(d/2)},$$

where ϵ_h is the elasticity of the compensation function, and crucially affects the relationship between ε and n. Specifically, if $\epsilon_h' > 0$, a rise in n increases ε. In this case, the model implies that a trade-induced increase in the number of firms is pro-competitive. Instead, if $\epsilon_h' < 0$, a rise in n decreases ε and is therefore anti-competitive. Finally, if $\epsilon_h' = 0$ (i.e., if $h(\cdot)$ is isoelastic), ε is independent of n, just as in the "love for variety" approach when

[15] See also Helpman (1981) on this point.

preferences are CES. We therefore conclude that a pro-competitive effect is unwarranted even in a framework in which a trade-induced increase in the number of firms crowds the variety space. This is because the ideal variety approach does not impose sufficient restrictions on $h(\cdot)$ to pin down the properties of ϵ_h, and therefore the relationship between ε and n.[16]

Is the assumption $\epsilon_h'(\cdot) > 0$ plausible? Note that $MRS(\widetilde{\omega}, \omega) = h(\delta(\omega, \widetilde{\omega}))$ implies that, in order for Lancaster's model to deliver a pro-competitive effect, consumer preferences must feature an ever increasing distance elasticity of the marginal rate of substitution between $\widetilde{\omega}$ and ω. It is hard to provide a rationale for this assumption, which seems no more plausible that the opposite assumption of a decreasing distance elasticity. In this latter case, however, Lancaster's model would deliver an anti-competitive effect. If follows that in this framework the most natural assumption is that is $h(\cdot)$ isoelastic, which implies the absence of any pro- or anti-competitive effects.

3.3 Cournot Competition

In our setting, the assumption that each firm treats λ (the marginal utility of income) as a constant removes a direct channel whereby a trade-induced increase in the number of firms may raise the perceived demand elasticity ε. Under the alternative assumption that firms properly treat λ as a function of the price vector (i.e., $\lambda = \lambda(\mathbf{p})$), the demand elasticity ε no longer coincides with the elasticity of substitution σ. In the case of symmetric consumption c, its expression is actually given by (see (64) in the Appendix):

$$\varepsilon(c, n) = \sigma(c) - \frac{\sigma(c) - 1}{n}. \tag{31}$$

It follows that a trade-induced increase in the number of firms has a positive direct impact on ε and, with IES preferences, an indirect negative impact through σ. The net effect is therefore in general ambiguous, possibly making a case for the standard assumption of a constant markup. Note, however, that:

$$\begin{aligned} \frac{\partial \ln \varepsilon}{\partial \ln \sigma} &= \frac{(n-1)\,\sigma}{(n-1)\,\sigma + 1} \Rightarrow \lim_{n\to\infty} \frac{\partial \ln \varepsilon}{\partial \ln \sigma} = 1, \\ \frac{\partial \ln \varepsilon}{\partial \ln n} &= -\frac{\sigma - 1}{(n-1)\,\sigma + 1} \Rightarrow \lim_{n\to\infty} \frac{\partial \ln \varepsilon}{\partial \ln n} = 0, \end{aligned}$$

[16] Only in the limit case in which d goes to zero, and due to the (rather ad hoc) assumptions $h(0) = 1$ and $h'(0) = 0$, the aggregate demand elasticity is increasing in n (and goes to infinite). This requires a situation, unfeasible under a positive fixed cost, in which the circumference of the circle is full and the aggregate demand for each firm is infinitesimal.

which suggests that the direct pro-competitive impact of n on ε weakens as the number of firms grows larger, and vanishes in the limit. Thus, for n large enough, the anti-competitive effect induced by IES preferences should always prevail.

The robustness of this conclusion can be confirmed by considering a Cournotian extension of our baseline setting, in which firms correctly perceive the demand functions they face, but strategically interact with their competitors by setting their production levels. Multiplying both sides of the first-order conditions for utility maximization ($u'(c_i) = \lambda p_i$) by c_i and adding up yields:

$$\lambda = \sum\nolimits_j u'(c_j)c_j. \tag{32}$$

Using (32) in the first-order conditions yields the following inverse demand system:

$$p_i(\mathbf{c}) = \frac{u'(c_i)}{\sum_j u'(c_j)c_j}, \quad i = 1, ...n. \tag{33}$$

Let $R_i(\mathbf{c}) = p_i(\mathbf{c})c_i L$ be firm i's revenue. Marginal revenue is therefore given by:

$$\frac{\partial R_i(\mathbf{c})}{\partial c_i}\frac{1}{L} = \frac{r'(c_i)\left(\sum_{j\neq i} u'(c_j)c_j\right)}{\left(\sum_j u'(c_j)c_j\right)^2}, \tag{34}$$

and is decreasing in c_i under our assumptions that $r'' < 0$ and $r' = u''c + u' > 0$. In a Nash equilibrium, each firm chooses its quantity to satisfy the first-order condition $\frac{\partial R_i}{\partial c_i}\frac{1}{L} = \beta$ under a correct conjecture about the quantities produced by its competitors. Then, (34) and (33) imply that, in any symmetric Nash equilibrium:

$$c = \frac{(n-1)\,r'(c)}{n^2 u'(c)\beta} = \frac{n-1}{n^2 m(c)\beta}, \tag{35}$$

$$p = \frac{1}{nc} = \frac{n}{n-1}m(c)\beta. \tag{36}$$

Note, from (36), that the markup $\frac{n}{n-1}m(c)$ depends on both n and c (through m).[17] Moreover, (35) uniquely pins down the equilibrium relationship $c(n)$ between c and n, with $c'(n) < 0$.[18] Accordingly, a rise in the number of firms is anti-competitive if and only

[17] Note that (36) implies that Cournotian firms behave "as if" they perceived a demand elasticity equal to $\frac{\sigma n}{\sigma+n-1}$, which is lower than the expression in (31). This is an implication of the fact that Cournotian firms are quantity setters. In a model with Bertrand competition (and product differentiation), the relevant demand elasticty would be just the one in (31). In both cases, however, the demand elasticity approaches σ for $n \to \infty$.

[18] This relationship would be unaffected by explicitly considering a free-entry condition which, by endogenizing n (as in our baseline setting), would redefine the equilibrium value of c as a function of L/α and β.

if the elasticity of $c(n)$ is greater than one in absolute value (so that $p = (nc)^{-1}$ increases):

$$\left|\frac{d\ln c}{d\ln n}\right| = \frac{(n-2)}{(n-1)}\frac{m(c(n))}{m'(c(n))c(n) + m(c(n))},$$

where $\frac{m}{m'c+m} > 1$ (as $m' < 0$) and $\lim_{n\to\infty}\frac{n-2}{n-1} = 1$. It follows that, for n large enough, a trade-induced increase in the number of firms can be anti-competitive even when firms interact strategically.

We summarize our main results in this section in the following

Proposition 2 *a) When preferences are quasi-linear and quadratic, as in Melitz and Ottaviano (2008), the number of firms has a negative direct impact on the elasticity of substitution, and the pro-competitive effect of a trade-induced expansion in market size is entirely driven by firms' perceived linearity of demand; b) When preferences are heterogeneous across consumers and are of the ideal variety type, as in Lancaster (1979), a trade-induced increase in the number of firms is pro-competitive only when the compensation function features an increasing distance elasticity; c) When firms are Cournotian competitors, and preferences are IES, a trade-induced increase in the number of firms can be pro-competitive only when the initial number of competitors is small.*

4 Heterogeneous Firms

So far, we have only considered setups with homogeneous firms, thereby ignoring the recent literature on heterogeneous firms in international trade. In this section, we therefore complement our previous analysis by studying how IES preferences interact with firm heterogeneity. Our basic setup is the same as in Section 2.1, except that we now allow for a continuum rather than a discrete number of varieties. More importantly, following Melitz (2003), we depart from the model in Section 2 by assuming that, upon paying a fixed entry cost α_e, firms draw their marginal cost $\beta \in [\underline{\beta}, \infty)$, with $\underline{\beta} > 0$, from a continuous cumulative distribution $G(\beta)$ with density $g(\beta)$. In the following, we index firms by their marginal cost β and denote by $c(\beta)$ the individual demand for their products.

Firm productivity, size and markups Denote by $p(\beta) = m(c(\beta))\beta$ and $\pi_v(\beta) = [m(c(\beta)) - 1]\,\beta L c(\beta)$ the price and variable profit of a β-firm. The first-order and second-order conditions for profit maximization, $r'(c) = \lambda\beta$ and $r''(c) < 0$, imply that:

$$\begin{aligned}
\frac{c'(\beta)\beta}{c(\beta)} &= \frac{\lambda\beta}{r''(c(\beta))c(\beta)} = -\eta(c(\beta) < 0, && (37)\\
\pi_v'(\beta) &= -c(\beta)L < 0, && (38)\\
\frac{p'(\beta)\beta}{p(\beta)} &= 1 - \frac{1}{c(\beta)}\left(1 - \frac{\eta(c(\beta))}{\varepsilon(c(\beta))}\right) \gtrless 1 \Leftrightarrow \varepsilon'(c(\beta)) \gtrless 0, && (39)\\
-\frac{\pi_v''(\beta)\beta}{\pi_v'(\beta)} &= \eta(c(\beta)). && (40)
\end{aligned}$$

Thus, as in Melitz (2003), high-productivity (low-β) firms are larger and more profitable. Unlike the Melitz model, however, where preferences are CES and markups are constant, with IES preferences larger firms charge lower markups. With DES preferences, larger firms would instead charge lower markups. Hence, in this simple yet quite general setup, the sign of $\varepsilon'(c)$ has clear-cut (and potentially testable) implications for the cross-sectional relationship between firm size and markups. Finally note that, as shown by (40), η can also be interpreted as a measure of the variable profit function curvature.

Zero cutoff profit condition Denote by β^* the marginal cost cutoff, namely, the value of β satisfying the zero cutoff profit condition $\pi(\beta^*) = 0$. This condition implicitly defines the individual demand for the cutoff firm, $c^* = c^*(\beta^*, L, \alpha)$. In particular, using (12) we can write:

$$\pi_v(\beta^*) = [m(c^*) - 1]\,\beta^* L c^* = \left[-\frac{u''(c^*)}{r'(c^*)}c^*\right]\beta^* L c^* = \alpha. \tag{41}$$

Differentiating with respect to c^*, L, β^* and α yields:

$$\frac{\partial c^*}{\partial L}\frac{L}{c^*} = \frac{\partial c^*}{\partial \beta^*}\frac{\beta^*}{c^*} = -\frac{\partial c^*}{\partial \alpha}\frac{\alpha}{c^*} = -\frac{\eta(c^*)}{\varepsilon(c^*)} < 0. \tag{42}$$

Evidently, c^* is decreasing in $(L\beta^*/\alpha)$, with an elasticity whose value depends on the sign of ε'.

Individual demand for a β-firm Profit maximization implies $r'(c^*) = \lambda\beta^*$. Solving for λ and substituting into $r'(c) = \lambda\beta$ yields:

$$r'(c) = r'(c^*)\frac{\beta}{\beta^*}. \tag{43}$$

Thus, the marginal revenue of a β-firm is proportional to the marginal revenue of the cutoff firm. Equation (43) is key to the characterization of the equilibrium, as it implicitly defines the individual demand for a β-firm, $c\,(\beta) = c(\beta; \beta^*, c^*)$. Moreover, having shown how c^*

depends on β^*, L and α, we can now show how $c(\beta) = c(\beta; \beta^*, c^*(\beta^*, L, \alpha)) = c(\beta; \beta^*, L, \alpha)$ varies with β^*, L and α. Differentiating (43) with respect to β^* yields:

$$r''(c)\frac{\partial c}{\partial \beta^*} = \frac{\beta}{\beta^{*2}}\left[r''(c^*)\frac{\partial c^*}{\partial \beta^*}\beta^* - r'(c^*)\right].$$

Using (42) and rearranging terms yields:

$$\frac{\partial c}{\partial \beta^*}\frac{\beta^*}{c} = \frac{\eta(c)}{m(c^*)} > 0.$$

Hence, as in the Melitz model, individual consumption is increasing in β^*. Similarly, differentiating (43) with respect to L and using (42) yields:

$$\frac{\partial c}{\partial L}\frac{L}{c} = -\frac{\partial c}{\partial \alpha}\frac{\alpha}{c} = -\frac{\eta(c)}{\varepsilon(c^*)} < 0.$$

Thus, as in the homogeneous-firm setup, individual consumption is decreasing in market size L and increasing in the fixed cost α (for given β^*). Finally, note that with IES preferences $\varepsilon(c^*) < \varepsilon(c)$ for $c^* < c$; hence, using (11):

$$\frac{\partial c}{\partial L}\frac{L}{c} = -\frac{\eta(c)}{\varepsilon(c^*)} < -\frac{\eta(c)}{\varepsilon(c)} < -1.$$

Thus firm size, $q = cL$, is decreasing in L for given β^*. In contrast, with DES preferences $\varepsilon(c^*) > \varepsilon(c)$ for $c^* < c$, and hence firm size is increasing in L:

$$\frac{\partial c}{\partial L}\frac{L}{c} = -\frac{\eta(c)}{\varepsilon(c^*)} > -\frac{\eta(c)}{\varepsilon(c)} > -1.$$

The following lemma summarizes

Lemma 2 *$c(\beta; \beta^*, L, \alpha)$ is increasing in β^* and α and decreasing in L, with*

$$\frac{\partial c}{\partial \beta^*}\frac{\beta^*}{c} = \frac{\eta(c)}{m(c^*)} > 0, \quad \frac{\partial c}{\partial L}\frac{L}{c} = -\frac{\partial c}{\partial \alpha}\frac{\alpha}{c} = -\frac{\eta(c)}{\varepsilon(c^*)} \lesseqgtr -1 \Leftrightarrow \varepsilon'(c) \gtreqless 0. \tag{44}$$

Free-entry condition Free entry implies that expected profits,

$$\pi^E = \int_{\underline{\beta}}^{\beta^*} \pi(\beta) dG(\beta),$$

equal the fixed cost of entry α_e. Integrating π^E by parts yields:

$$\pi^E = \pi(\beta^*)G(\beta^*) - \pi(\underline{\beta})G(\underline{\beta}) - \int_{\underline{\beta}}^{\beta^*} \pi'(\beta)G(\beta)d\beta = -\int_{\underline{\beta}}^{\beta^*} \pi'(\beta)G(\beta)d\beta,$$

since $\pi(\beta^*) = G(\underline{\beta}) = 0$. Using (38), the free-entry condition can therefore be written as:

$$\pi^E = \int_{\underline{\beta}}^{\beta^*} c(\beta)LG(\beta)d\beta = \alpha_e. \tag{45}$$

Differentiating π^E with respect to β^* yields:

$$\frac{\partial \pi^E}{\partial \beta^*} = c(\beta^*)LG(\beta^*) + \int_{\underline{\beta}}^{\beta^*} \frac{\partial c(\beta)}{\partial \beta^*} LG(\beta)d\beta > 0, \tag{46}$$

where the inequality follows from Lemma 2. Hence, as in Melitz (2003), expected profits are increasing in β^* and the free-entry condition (45) uniquely pins down the equilibrium value of β^*, thereby defining the equilibrium value of $c(\beta) = c(\beta; L, \alpha, \alpha_e)$.

Finally, the measure of active firms n is pinned down by the budget constraint (or, equivalently, by the full employment condition), requiring average expenditure to equal $1/n$, and thus:

$$n = \left[\int_{\underline{\beta}}^{\beta^*} m(c(\beta))\beta c(\beta)\frac{dG(\beta)}{G(\beta^*)}\right]^{-1}. \tag{47}$$

4.1 Comparative Statics

Consider first the effect of a rise in market size L. Totally differentiating (45) with respect to L yields:

$$\frac{d\pi^E}{dL} = \frac{\partial \pi^E}{\partial L} + \frac{\partial \pi^E}{\partial \beta^*}\frac{d\beta^*}{dL} = 0, \tag{48}$$

where $\partial \pi^E/\partial \beta^* > 0$ from (46) and, using Lemma 2,

$$\frac{\partial \pi^E}{\partial L} = \int_{\underline{\beta}}^{\beta^*} \left[\frac{\partial c(\beta)}{\partial L}L + c(\beta)\right] G(\beta)d\beta = \int_{\underline{\beta}}^{\beta^*} \left(-\frac{\eta(c(\beta))}{\varepsilon(c^*)} + 1\right) c(\beta)G(\beta)d\beta. \tag{49}$$

It follows that with IES preferences $\partial \pi^E/\partial L < 0$, as firm size is decreasing in market size for given β^*. Thus, $d\beta^*/dL > 0$: a rise in market size leads to a rise in β^* and a consequent *anti-selection* effect, i.e., less productive firms can survive in a larger market. Lemma 2 and (49) also imply that, with DES preferences, a rise in market size leads instead to a standard selection effect.

Next, using (46) and (49) in (48) and rearranging yields:

$$c(\beta^*)LG(\beta^*)\frac{d\beta^*}{dL} + \int_{\underline{\beta}}^{\beta^*} c(\beta)G(\beta)d\beta + L\int_{\underline{\beta}}^{\beta^*} \frac{dc}{dL}G(\beta)d\beta = 0. \tag{50}$$

Note that the first two terms in (50) are positive, thereby implying that the last term is negative. Moreover, using Lemma 2:

$$\frac{dc}{dL}\frac{L}{c} = \left[\frac{\partial c}{\partial L} + \frac{\partial c}{\partial \beta^*}\frac{d\beta^*}{dL}\right]\frac{L}{c} = -\frac{\eta(c)}{\varepsilon(c^*)}\left[1 - (\varepsilon(c^*) - 1)\frac{d\beta^*}{dL}\frac{L}{\beta^*}\right]. \tag{51}$$

Note that $\eta(c) < 0$ for all β, and that the sign of the term in square brackets is independent of β. It follows that the sign of dc/dL is the same for all firms, and must therefore be negative according to (50). Thus, as in the baseline model with homogeneous firms, a rise in market size leads to a fall in individual consumption and a consequent rise of markups with IES preferences.[19]

Next, consider the effects of a rise in the fixed production cost α. Totally differentiating (45) with respect to α yields:

$$\frac{d\pi^E}{d\alpha} = \frac{\partial \pi^E}{\partial \alpha} + \frac{\partial \pi^E}{\partial \beta^*}\frac{d\beta^*}{d\alpha} = 0,$$

where, using Lemma 2,

$$\frac{\partial \pi^E}{\partial \alpha} = \int_{\underline{\beta}}^{\beta^*} \frac{\partial c(\beta)}{\partial \alpha} L G(\beta) d\beta = \frac{L}{\alpha \varepsilon(c^*)}\int_{\underline{\beta}}^{\beta^*} \eta(c(\beta))c(\beta)G(\beta)d\beta > 0.$$

Thus, independent of the sign of $\varepsilon'(c)$, a rise of α leads to a fall of β^*. Moreover, proceeding as above, it is possible to show that $dc/d\alpha > 0$ for all firms, implying that a rise in α is pro-competitive with IES preferences and anti-competitive with DES preferences, just as in the baseline model with homogeneous firms.

Finally, (45) and (46) immediately imply that, independent of the sign of $\varepsilon'(c)$, a rise in the fixed entry cost α_e leads to a rise of β^*. Moreover, since

$$\frac{dc}{d\alpha_e} = \frac{\partial c}{\partial \beta^*}\frac{d\beta^*}{d\alpha_e} > 0,$$

it follows that markups decrease with IES preferences and rise with DES preferences.

The main results are recorded in the following

Proposition 3 *With IES preferences, a rise in market size induces an anti-selection effect, whereby less productive firms survive in a larger market, and an anti-competitive effect, whereby firms charge higher markups. The converse is true with DES preferences, i.e., market size expansion leads to standard selection and pro-competitive effects. A rise in the fixed production cost α, or in the fixed entry cost α_e, are pro-competitive with IES preferences, and anti-competitive with DES preferences. In both cases, a rise of α leads to*

[19] The fall in individual consumption leads instead to a fall of markups when preferences are DES.

a fall in the marginal cost cutoff, whereas a rise of α_e induces a rise in the marginal cost cutoff.

4.2 Fixed Costs of Exporting

In our setup, frictionless trade integration between two identical countries is equivalent to a doubling of market size. In this respect, Proposition 3 suggests that, with IES preferences, trade opening leads to anti-competitive and anti-selection effects. We now show how the above results change in the presence of costly trade. Specifically, we assume that exporting to an identical foreign market involves an additional fixed cost α_x. Denoting by a subscript x variables related to the export market, and by no subscript those related to the domestic market, we have that profits of a β-firm active in both markets are given by:[20]

$$\pi(\beta) = \pi_v(\beta) - \alpha, \quad \pi_x(\beta) = \pi_v(\beta) - \alpha_x, \tag{52}$$

where $\pi_v(\beta) = [m(c(\beta)) - 1]\, \beta c(\beta) L$, and L denotes the size of each country. The marginal cost cutoff for exporters, β_x^*, is implicitly given by:

$$\pi_x(\beta_x^*) = [m(c(\beta_x^*)) - 1]\, \beta_x^* c(\beta_x^*) L - \alpha_x = 0, \tag{53}$$

where $c_x^* = c(\beta_x^*)$ is individual foreign demand for the cutoff exporter.

Total expected profits are now given by:

$$\pi^E = \int_{\underline{\beta}}^{\beta^*} \pi(\beta) g(\beta) d\beta + \int_{\underline{\beta}}^{\beta_x^*} \pi_x(\beta) g(\beta) d\beta. \tag{54}$$

Integrating (54) by parts and using the envelope theorem, the free-entry condition can be written more conveniently as:

$$\pi^E = \int_{\underline{\beta}}^{\beta^*} c(\beta) L G(\beta) d\beta + \int_{\underline{\beta}}^{\beta_x^*} c(\beta) L G(\beta) d\beta = \alpha_e. \tag{55}$$

Finally, the measure of active firms is given by:

$$n = \left[\int_{\underline{\beta}}^{\beta^*} m(c(\beta)) \beta c(\beta) \frac{dG(\beta)}{G(\beta^*)} + \int_{\underline{\beta}}^{\beta_x^*} m(c(\beta)) \beta c(\beta) \frac{dG(\beta)}{G(\beta_x^*)} \right]^{-1}. \tag{56}$$

[20] Note that a domestic firm producing *only* for the foreign market would incur an overall fixed cost equal to $\alpha + \alpha_x$, which implies that this case cannot arise in equilibrium. For analytical convenience, we can therefore apportion the fixed cost α to domestic profits, in this following the heterogeneous-firms literature.

We can now study how trade opening between two identical countries affects the marginal cost cutoff β^* in the presence of fixed costs of exporting. Consider first the case $\alpha_x > \alpha$, which implies that $\pi_x(\beta) < \pi(\beta)$ for all β. In this case, not all active firms can profitably export and hence $\beta^* > \beta_x^*$. For $\alpha_x \to \infty$, we are back in autarky; in this case, $\beta_x^* = \underline{\beta}$ and the second integral in (55) disappears. Next, note that (53) implicitly defines $\beta_x^* = \beta_x^*(\beta^*, L, \alpha_x)$. Thus, differentiating (53) and using (38), (13) and Lemma 2 yields:

$$\frac{\partial \beta_x^*}{\partial \alpha_x} = \frac{1}{\pi_x'(\beta_x^*)} < 0, \quad \frac{\partial \beta_x^*}{\partial \beta^*} = -\frac{[m'(c(\beta_x^*))c(\beta_x^*) + m(c(\beta_x^*)) - 1]\,\beta_x^* L}{\pi_x'(\beta_x^*)} \frac{\partial c(\beta_x^*)}{\partial \beta^*} > 0. \quad (57)$$

Thus, as in the Melitz model, a fall in the fixed costs of exporting leads to a rise in the marginal cost cutoff for exporters β_x^*. It follows that, for α_x sufficiently low, $\beta_x^* > \underline{\beta}$ and some high-productivity firms can export. In this case, the second integral in (55) is strictly positive and increasing in β^* and β_x^* according to (57) (recall that c is also increasing in β^*). It follows that a move from autarky to costly trade shifts the LHS of (55) upwards, thereby leading to a lower equilibrium value of β^*. Hence, as in Melitz (2003), and independent of the sign of ε', trade opening leads to a selection effect when $\alpha_x > \alpha$, namely, when fixed costs of exporting induce a partitioning of firms into exporters and non-exporters. Moreover, still as in the Melitz model, a fall of α_x in this range, by increasing β_x^* requires a decrease in β^* according to (55), and therefore yields a selection effect.

Consider, finally, the effects of trade opening when fixed costs of exporting are in the range $0 \leq \alpha_x \leq \alpha$. In this case, $\pi_x(\beta) \geq \pi(\beta)$ and all active firms export, implying that $\beta^* = \beta_x^*$. Thus, the zero cutoff profit condition and the free-entry condition boil down to:

$$\begin{aligned} \pi_v(\beta^*) &= [m(c^*) - 1]\,\beta^* L' c^* = \alpha + \alpha_x, & (58) \\ \pi^E &= \int_{\underline{\beta}}^{\beta^*} c(\beta) L' G(\beta) d\beta = \alpha_e, & (59) \end{aligned}$$

where $L' = 2L$. Note first that, for $\alpha_x = 0$, we are back in the case of costless trade integration, implying a rise of β^* relative to autarchy according to Proposition 3. Instead, for $\alpha_x = \alpha$, (58) is the same as in autarchy, and therefore c^* and $c(\beta)$ are unaffected for given β^*. It follows that β^* must fall relative to autarchy, due to the direct impact of the rise of L in (59). Moreover, note that a rise of α_x in the range $[0, \alpha]$ is equivalent to a rise in the fixed production cost in a closed economy of size L', and therefore implies a fall of β^* according to Proposition 3. Finally, given that β^* is higher than in autarchy for $\alpha_x = 0$, less than in autarchy for $\alpha_x = \alpha$, and monotonically decreasing in between, it follows that there must be a value $\alpha_x^* \in (0, \alpha)$ such that β^* is the same as in the autarchic equilibrium.

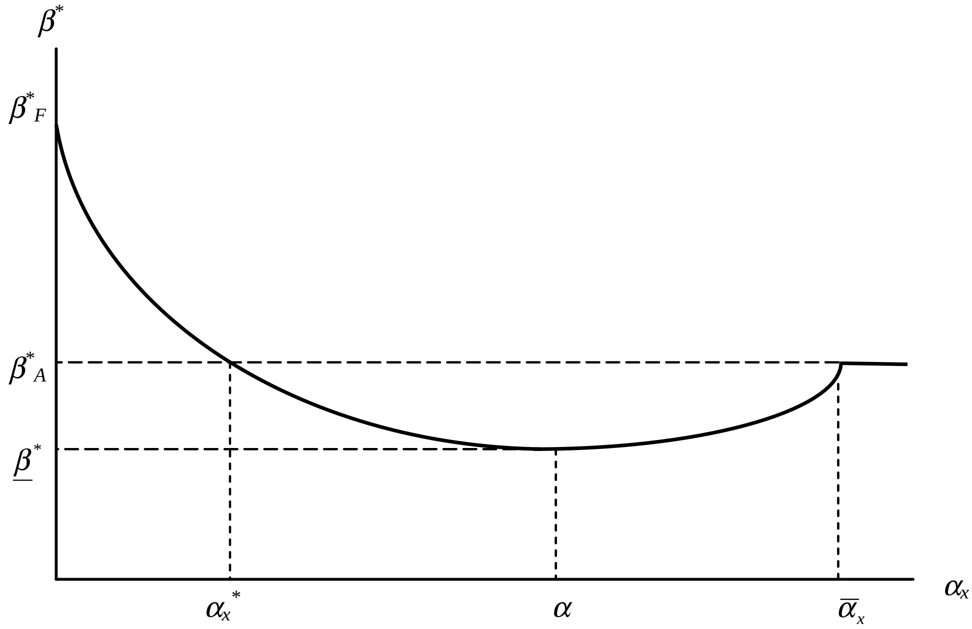

Figure 1 - Firm Selection with IES Preferences and Fixed Costs of Exporting

Figure 1 summarizes. It reports the marginal cost cutoff β^* on the vertical axis as a function of the fixed cost of exporting α_x on the horizontal axis. For $\alpha_x \geq \overline{\alpha}_x$, fixed costs of exporting are prohibitively high and β^* is therefore constant at the autarchic level β^*_A. For $\alpha_x \in (\alpha^*_x, \overline{\alpha}_x)$, $\beta^* < \beta^*_A$ and thus trade opening between two identical countries leads to a selection effect. Trade has instead an anti-selection effect for $\alpha_x \in [0, \alpha^*_x)$, as $\beta^* > \beta^*_A$ in this range and reaches a maximum at β^*_F for $\alpha_x = 0$, i.e., in free trade. Finally, note that a fall in the fixed costs of exporting has a non-monotonic impact on β^*, as it leads to a selection effect when the initial value of α_x is high (i.e., for $\alpha_x > \alpha$), and to an anti-selection effect when the initial level of α_x is low (i.e., for $\alpha_x \leq \alpha$). The selection effect is therefore strongest for $\alpha_x = \alpha$, namely, when fixed costs of exporting leave the overall ratio of fixed costs $(\alpha + \alpha_x)$ to market size L' unchanged relative to autarchy.

Consider, finally, how markups are affected by trade opening in the presence of fixed costs of exporting. We know that markups equal the autarchic level for $\alpha_x = \overline{\alpha}_x$, the free trade level for $\alpha_x = 0$, and that they are higher in free trade than in autarchy according to Proposition 3. Moreover, for $\alpha_x \in [0, \alpha]$, a rise in the fixed costs of exporting is equivalent to a rise in the fixed production cost in a closed economy of size $L' = 2L$. It is therefore pro-competitive according to Proposition 3. Finally, when $\alpha_x \in [\alpha, \overline{\alpha}_x]$, a rise in the fixed costs of exporting affects individual consumption only by increasing β^*. Therefore, according to Lemma 2, individual consumption rises and this leads to lower markups. The model thus implies that markups are monotonically decreasing in the fixed costs of exporting.

We record our main results in the following

Proposition 4 *a) In the presence of fixed costs of exporting α_x, trade opening between two identical countries induces an anti-selection effect for low values of α_x, and a selection effect otherwise; b) A sufficient condition for the selection effect to hold is that fixed costs of exporting induce a partitioning of firms by export status; c) The marginal cost cutoff β^* is non-monotonically related to fixed costs of exporting, as it is first decreasing and then increasing in α_x; d) Markups are monotonically decreasing in α_x.*

To conclude, in this section we have shown that, with heterogenous firms, IES preferences imply a trade-induced anti-selection effect, in addition to the anti-competitive effect discussed earlier. Although this anti-selection mechanism always weakens the selection effect *à la* Melitz, in our setup it prevails only for low (or zero) fixed costs of exporting, i.e., when the latter are insufficient to induce a partitioning of firms according to their export status. Yet, such a partitioning seems empirically omnipervasive, and hence our results are not inconsistent with the evidence in support of Melitz-type trade-induced selection effects.[21]

5 Conclusion

In this paper, we have shown that the pro-competitive effect of market size expansion delivered by existing monopolistic competition trade models seems to build on implausible and/or ad hoc assumptions about consumer behavior. We have therefore explored the implications of alternative assumptions and found that, under fairly general and plausible scenarios, trade opening and markups may turn out to be roughly unrelated. We also found that Melitz-type selection effects, i.e., those induced by the interaction between firm heterogeneity and fixed costs of exporting, are instead generally robust to the assumptions about consumer behavior. We view these results as important points in favor of the standard monopolistic competition trade model with CES preferences. The latter is however under attack, not only for the assumed invariance of markups and firm size to the trade regime (which may turn out to be a strength rather than a weakness, according

[21] See, e.g., the works cited in Section 2 of Redding (2011)'s recent survey on the heterogeneous-firms literature.

to our results), but also for the implied "conditional" invariance of gains from trade to the number of sources of these gains (Arkolakis, Costinot and Rodriguez-Clare, 2011). In this respect, our results suggest that, even if a revisitation of the basic tools used by the international trade theory is probably in order, that will not be an easy task, as opting for non-CES preferences delivering pro-competitive effects may turn to be a move in the wrong direction.

6 Appendix

6.1 Demand Elasticity and Elasticity of Substitution

Consider a setting in which consumers share the same preferences, represented by the utility function $U(c_0, u(\mathbf{c}_{-0}))$, where c_0 is consumption of a numeraire good, $\mathbf{c}_{-0} = [c_1, c_2, ..., c_n]$ is consumption of n varieties of some good, and $u(\cdot)$ is symmetric and concave in its arguments. Denote the associated system of compensated (Hicksian) demand by $\widetilde{\mathbf{c}} = [\widetilde{c}_0, \widetilde{\mathbf{c}}_{-0}] = \widetilde{\mathbf{c}}(\mathbf{p}, U)$, where $\mathbf{p} = [p_0, \mathbf{p}_{-0}]$ is the price vector. Moreover, denote by $\widetilde{c}_{ij} = \partial \widetilde{c}_i / \partial p_j$ $(i, j = 0, 1, ...n)$ the compensated demand-price derivatives, and by $\widetilde{\varepsilon}_{ij} = \partial \ln \widetilde{c}_i / \partial \ln p_j$ the corresponding elasticities. The *Morishima*'s[22] elasticity of substitution between goods i and j $(i \neq j)$ is given by

$$\sigma_{ij} = \widetilde{\varepsilon}_{ji} - \widetilde{\varepsilon}_{ii}, \tag{60}$$

and measures the substitutability between any *two* goods *at a given consumption vector* $\mathbf{c} = [c_0, \mathbf{c}_{-0}]$. Specifically, σ_{ij} measures how the marginal rate of substitution between i and j, $MRS_{ij} = (\partial U / \partial c_i) \, / \, (\partial U / \partial c_j)$, varies with the consumption ratio c_i / c_j.

The elasticity of substitution is a key ingredient of the demand elasticity. This can be seen by manipulating the Slutsky equation, which decomposes the price effect on demand into a substitution and an income effect:

$$c_{ij} = \widetilde{c}_{ij} - c_{iY} c_j, \tag{61}$$

where $c_i(\mathbf{p}, Y)$ is the Marshallian (uncompensated) demand for commodity i, Y is income, $c_{ij} = \partial c_i / \partial p_j$ and $c_{iY} = \partial c_i / \partial Y$. When expressed in elasticity terms, (61) implies:

$$\begin{aligned} \varepsilon_{ii} &= \widetilde{\varepsilon}_{ii} - \varepsilon_{iY} \theta_i, \\ \varepsilon_{ji} &= \widetilde{\varepsilon}_{ji} - \varepsilon_{jY} \theta_i, \end{aligned} \tag{62}$$

[22] See Blackorby and Russell (1989) for a discussion of other definitions of elasticity of substitution.

where $\varepsilon_{ii} = \partial \ln c_i / \partial \ln p_i$ is the (own-price) demand elasticity of commodity i,[23] $\varepsilon_{ij} = \partial \ln c_i / \partial \ln p_j$ is the cross-price demand elasticity of commodity i with respect to commodity j, $\theta_i = p_i c_i / Y$ is good i's expenditure share and $\varepsilon_{iY} = \partial \ln c_i / \partial \ln Y$ is good i's income elasticity. Using (60) and (61) yields:

$$\varepsilon_{ii} = -\sigma_{ij} + \varepsilon_{ji} + (\varepsilon_{jY} - \varepsilon_{iY})\,\theta_i, \qquad i,j = 1,...n; i \neq j. \tag{63}$$

Thus, the demand elasticity for good i can be written as a function of the elasticity of substitution and the cross-price elasticity with respect to another good j and the difference between the corresponding income effects, $(\varepsilon_{jY} - \varepsilon_{iY})\,\theta_i$. This latter term disappears if $U(\cdot)$ is homothetic (as in this case $\varepsilon_{jY} = \varepsilon_{iY} = 1$) or quasi-linear with respect to the numeraire (as in this case $\varepsilon_{jY} = \varepsilon_{iY} = 0$), or if $p_i = p_j$ (as in this case $\varepsilon_{jY} = \varepsilon_{iY}$ by symmetry of the preferences over $\widetilde{\mathbf{c}}_{-0}$). More generally, unless either c_i or c_j is disproportionate, $(\varepsilon_{jY} - \varepsilon_{iY})\,\theta_i$ is an order of magnitude smaller than $1/n$.

The expression in (63) drastically simplifies when there is no numeraire (as in Krugman (1979) and in our baseline setting) and $p_i = p$ for any i, which implies a symmetric consumption pattern (i.e., $c_i = c$ for any i). In this case, $\widetilde{\varepsilon}_{ii} = -(n-1)\widetilde{\varepsilon}_{ij}$, $\widetilde{\varepsilon}_{ij} = \sigma_{ij}/n$, $\varepsilon_{ji} = (\sigma_{ij} - 1)/n$ and

$$\varepsilon_{ii} = -\frac{n-1}{n}\sigma_{ij} - \frac{1}{n}. \tag{64}$$

Note that $-1/n$ in (64) accounts for the income effect, which vanishes for n large. The same simple expression holds in the presence of a numeraire, provided that preferences are Cobb-Douglas with respect to the numeraire, i.e., $U(c_0, u(\mathbf{c}_{-0})) = c_0^{\alpha} u(\mathbf{c}_{-0})^{1-\alpha}$, where $0 < \alpha < 1$ and $u(\cdot)$ is homogeneous.[24]

Another simple case arises when preferences are quasi-linear, i.e., if $U(c_0, u(\mathbf{c}_{-0}) = c_0 + u(\mathbf{c}_{-0})$, a specification discussed in Section 3.1. In this case there are no income effects in the demand for non-numeraire goods, hence $\varepsilon_{ji} = \widetilde{\varepsilon}_{ji} = -(\sigma_{ij} + \widetilde{\varepsilon}_{i0})/n$ and

$$\varepsilon_{ii} = -\frac{n-1}{n}\sigma_{ij} - \frac{\widetilde{\varepsilon}_{i0}}{n}. \tag{65}$$

Note, from (64) and (65), that n directly affects both the income and the substitution effect, and that $\varepsilon_{ii} \simeq -\sigma_{ij}$ for n large.[25]

Turning to the elasticity of substitution, a general result is that, if $u(\cdot)$ is additive, i.e.,

[23] For expositional purposes, in this Appendix it prove convenient to denote the demand elasticity by $\varepsilon_{ii} = \frac{\partial \ln c_i}{\partial \ln p_i}$, rather than by $\varepsilon_i = \left|\frac{\partial \ln c_i}{\partial \ln p_i}\right|$ as in the main text.

[24] The reason is that in this case the income and substitution effects partially cancel out (i.e., $\sigma_{0i} = \varepsilon_{0Y} = 1$, see the footnote immediately below).

[25] It is possible to show that, for a symmetric consumption pattern over $\mathbf{c}_{-0}$, (64) and (65) are special

if $u(\mathbf{c}_{-0}) = \sum_{i=1}^{n} u(c_i)$, then $c_i = c_j$ $(i, j = 1, ..., n,\ i \neq j)$ implies:[26]

$$\sigma_{ij} = -\frac{u'(c_i)}{u''(c_i)c_i}. \tag{66}$$

To see this, note that differentiating the first-order conditions

$$p_i = \mu U_u(c_0, \sum_i u(c_i))u'(c_i),$$

where μ is the relevant Lagrangian multiplier and $U_u = \partial U/\partial u$, yields:

$$\begin{aligned} 1 &= \mu_i U_u u'(c_i) + \mu U_u u''(c_i)\widetilde{c}_{ii} + \mu u'(c_i)\left[U_{u0}\widetilde{c}_{0i} + U_{uu}\sum_{h=1}^{n} u'(c_h)\widetilde{c}_{hi}\right], \\ 0 &= \mu_j U_u u'(c_i) + \mu U_u u''(c_i)\widetilde{c}_{ij} + \mu u'(c_i)\left[U_{u0}\widetilde{c}_{0j} + U_{uu}\sum_{h=1}^{n} u'(c_h)\widetilde{c}_{hj}\right], \end{aligned}$$

where $U_{u0} = \partial^2 U/\partial u \partial c_0$, $U_{uu} = \partial^2 U/\partial u^2$ and $\mu_i = \partial \mu/\partial p_i$. Subtracting the second expression from the first, exploiting the symmetry of price effects implied by the compensated demand functions (i.e., $\widetilde{c}_{ij} = \widetilde{c}_{ji}$) and manipulating yields:

$$1 = p_i\left[\frac{\mu_i - \mu_j}{\mu} + \frac{U_{u0}}{U_u}(\widetilde{c}_{0i} - \widetilde{c}_{0j}) + \frac{U_{uu}}{U_u}\sum_{h=1}^{n} u'(c_h)(\widetilde{c}_{hi} - \widetilde{c}_{hj})\right] + \frac{u''(c_i)c_i}{u'(c_i)}(\widetilde{\varepsilon}_{ii} - \frac{c_j}{c_i}\widetilde{\varepsilon}_{ji}). \tag{67}$$

Note that, for $p_i = p_j$, then, due to symmetry of preferences over $\mathbf{c}_{-0}$, $c_i = c_j$, $\mu_i = \mu_j$, $\widetilde{c}_{hi} = \widetilde{c}_{hj}$ $(h = 0, ..., n,\ i \neq h \neq j)$ and $\widetilde{c}_{ii} = \widetilde{c}_{jj}$. Thus (67) boils down to (66).

6.2 Demand in Lancaster's Ideal Variety Approach

We now derive the aggregate demand function, $q(\omega)$, for a firm selling variety ω at the price $p(\omega)$, with contiguous competitors ω_l and ω_r charging prices $p(\omega_l)$ and $p(\omega_r)$. The clientele of firm ω is a compact set ranging from $\underline{\omega}$ to $\overline{\omega}$, where $\underline{\omega}$ and $\overline{\omega}$ are the locations of consumers just indifferent between ω and ω_l, and between ω and ω_r. The values of $\underline{\omega}$

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cases of the general expression:

$$\varepsilon_{ii} = -\frac{n-1}{n}\sigma_{ij} - \frac{1}{n}\left[1 + \frac{c_0}{Y}(\sigma_{0i} - \varepsilon_{0Y})\right].$$

Thus, the result that $\varepsilon_{ii} \simeq -\sigma_{ij}$ for n large holds for a wide class of utility functions.

[26] As is well known, if in addition preferences over varieties are CES, i.e., if $u(c_i) = \frac{c_i^{\rho}}{\rho}$, $(i = 1, ..., n,$ $0 \leq \rho \leq 1)$, then the elasticity of substitution does not even depend on the consumption level c_i, as it is constant and equal to $\frac{1}{1-\rho}$.

and $\overline{\omega}$ are therefore implicitly defined by:

$$\begin{aligned} p(\omega_l)h(\delta(\omega_l,\underline{\omega})) &= p(\omega)h(\delta(\omega,\underline{\omega})), \\ p(\omega_r)h(\delta(\omega_r,\overline{\omega})) &= p(\omega)h(\delta(\omega,\overline{\omega})). \end{aligned} \tag{68}$$

Denote by $d^* = \delta(\omega_l, \omega_r)$ the distance between firm ω's competitors, by $d = \delta(\omega_l, \omega)$ the distance between ω_l and ω, and by $\underline{d} = \delta(\underline{\omega}, \omega)$ and $\overline{d} = \delta(\overline{\omega}, \omega)$ firm ω's distance from its marginal consumers. It follows that $\delta(\omega_l, \underline{\omega}) = d - \underline{d}$, and $\delta(\omega_r, \overline{\omega}) = d^* - d - \overline{d}$. Substituting into (68) yields:

$$\begin{aligned} p(\omega_l)h(d-\underline{d}) &= p(\omega)h(\underline{d}), \\ p(\omega_r)h(d^*-d-\overline{d}) &= p(\omega)h(\overline{d}). \end{aligned} \tag{69}$$

A firm's market width is obtained by inverting the above implicit conditions:

$$\begin{aligned} \underline{d} &= \underline{\delta}(p(\omega), p(\omega_l), p(\omega_r), d^*, d), \\ \overline{d} &= \overline{\delta}(p(\omega), p(\omega_l), p(\omega_r), d^*, d). \end{aligned} \tag{70}$$

The aggregate demand for firm ω is therefore given by:

$$q(\omega) = \left[\underline{\delta}(\cdot) + \overline{\delta}\,(\cdot)\right] c(\omega) L = \left[\underline{\delta}(\cdot) + \overline{\delta}\,(\cdot)\right] \frac{L}{p(\omega)}. \tag{71}$$

Implicit differentiation of the two-equation system in (69) yields:

$$\begin{aligned} \frac{\partial \underline{\delta}(\cdot)}{\partial p(\omega)} &= -\frac{h(\underline{d})}{p(\omega_l)h'(d-\underline{d}) + p(\omega)h'(\underline{d})}, \\ \frac{\partial \overline{\delta}(\cdot)}{\partial p(\omega)} &= -\frac{h(\overline{d})}{p(\omega_r)h'(d^*-d-\underline{d}) + p(\omega)h'(\overline{d})}. \end{aligned} \tag{72}$$

Note that, in a symmetric equilibrium, $p(\omega_l) = p(\omega_r) = p(\omega)$, $\underline{d} = \overline{d} = d - \underline{d} = d^* - d - \underline{d} = \frac{d}{2}$, and $n = 1/d$. Substituting into (71) and (72) yields:

$$\begin{aligned} \frac{\partial \underline{\delta}(\cdot)}{\partial p(\omega)} &= \frac{\partial \overline{\delta}(\cdot)}{\partial p(\omega)} = -\frac{h(\frac{d}{2})}{2p(\omega)h'(\frac{d}{2})}, \\ \frac{\partial q(\omega)}{\partial p(\omega)} &= \left[\frac{\partial \underline{\delta}(\cdot)}{\partial p(\omega)} + \frac{\partial \overline{\delta}(\cdot)}{\partial p(\omega)}\right] \frac{L}{p(\omega)} - \left[\underline{\delta}(\cdot) + \overline{\delta}\,(\cdot)\right] \frac{L}{p(\omega)^2} \\ &= -\frac{h(\frac{d}{2})}{p(\omega)h'(\frac{d}{2})} \frac{L}{p(\omega)} - \left[\underline{\delta}(\cdot) + \overline{\delta}\,(\cdot)\right] \frac{L}{p(\omega)^2}, \end{aligned}$$

thus leading to the expression for the demand elasticity $\varepsilon(q(\omega))$ reported in the main text.

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