Extensive and Intensive Margins of Export and FDI: the Role of Wages *

Pamela Bombarda†
THEMA - Université de Cergy-Pontoise

15 March 2013

Abstract

Multinational and exporting firms play a key role in trade patterns. To account for it, we propose a theoretical model of trade and intra-firm trade with firm heterogeneity. The model delivers gravity equations highlighting the complementarity existing between trade and FDI strategies. In the model, trade and FDI differently depend on trade and labour costs. The purpose of the paper is to explore how trade liberalization affects the margins of economic activities while considering the role of wage differential.

JEL classification: F12; F23;

Keywords: intra-firm trade, multinational firms, export, wages.

*Acknowledgements: I would like to thank Costas Arkolakis, Richard Baldwin, Stephen Redding and Peter Schott for encouragement and discussions on the topic. All errors are mine.

†Corresponding author: 33 Boulevard du Port, F-95011 Cergy-Pontoise Cedex, France Tel.: +33 1 34 25 67 59; fax: +33 1 34 25 62 33; E-mail address: pamela.bombarda@u-cergy.fr, pamela.bombarda@gmail.com.

1 Introduction

Intra-firm trade and arm's length trade play an important role in trade arena.[1] Dunning (1994) shows that a large part of international trade is conducted by MNFs. He estimated that MNFs together with their subsidiaries are responsible for 75 percent of the world's trade commodity. UNCTAD (2000) reports that one-third of world trade is intra-firm trade (trade between MNFs' headquarters and subsidiaries, or simply among subsidiaries). More recently, Bernard, Jensen and Schott (2007) document that 90 percent of U.S. exports and imports occurs through multinational firms. Recent studies try to analyze the different behavior of related-party versus arm's-length trade (Irarrazabal, Moxnes and Opromolla (2009), Corcos et al. (2009), Bernard et al. (2010) among others). The notion of openness should therefore include trade as well as multinational production.

This paper develops a model of trade that features heterogeneous firms, multinational firms, exporters and intra-firm trade in a general equilibrium framework. Its main contribution is to explain the different impact of geographical distance on related-party versus arm's-length trade. It also provides stylized facts to support the model's main predictions using 1999-2004 data from Bureau of Economic Analysis (henceforth BEA) and from the Center for International Data (henceforth CID) at UC Davis.[2]

The theoretical framework offers a possible explanation of the puzzling larger effect of distance on trade rather than on intra-firm trade and on complementarity *versus* substitutability debate. Globalization boosts both export and affiliate revenues. However, certain intermediate goods are more complements than others. In these sectors, foreign affiliates take advantage from globalization and thus the effect of distance should be important.

In this paper a trade policy intervention affects the trading activity of firms which occurs both within and outside the boundaries of the firm and across different sectors. Globalization should increase the volume of trade. Measuring the response of arm's length and related-party trade to globalization pressures is important to define specific policy intervention. For example, a reduction in trade barriers will increase multinational activity the higher is the share of intra-firm trade between the headquarter and the affiliate.

To provide a more appealing explanation for the coexistence of national and multinational firms, we extend the Melitz model to allow for intra-industry firm heterogeneity in productivity, which avoid the coexistence of different type of firms only as knife-edge case. This extended model can explain the within-industry variation across firms in their decisions about export and FDI.

To accounts for intra-firm trade, we claim each foreign affiliate has to import an intermediate input from the home headquarter. Thus, differently from Helpman, Melitz and Yeaple (2004, henceforth HMY), trade costs apply to both exports and multinational production because both involve transportation (the first of a finished good, the second of an intermediate good). The model suggests that the more productive firms enter a larger number of markets, undertake a large part of intra-firm trade and sell more in each market that they enter than

[1] In the literature, intra-firm trade refers to trade between U.S. companies and their foreign subsidiaries as well as trade between U.S. subsidiaries of foreign companies and their foreign affiliates. In this paper, the word intra-firm trade is sometimes used interchangeably with related-party trade. Notice that, for exports, the term related-party trade is far less stringent than intra-firm: firms are considered related if either party owns, directly or indirectly, 10 percent or more of the other party.

[2] Data are disaggregated at NAICS 3 digits. Further details are provided in section 7.5.1.

less productive firms. In a similar way, countries with characteristics that are more attractive to U.S. multinationals should attract relatively less productive firms.

In this model, heightening of trade barriers affects in two opposite ways the FDI mode of supply. First, it increases the threshold productivity cutoff: the need to import intermediate goods from the headquarter makes more difficult to enter as a foreign affiliate when trade costs increase. This result is opposite to HMY, where an increase in trade costs makes FDI strategy easier. Second, sales of the existing foreign affiliates decrease (new margin of adjustment for MNFs). By contrast, in absence of traded intermediates a change in trade costs translate into a change in the number of MNFs entering the market, while the profit of the already existing foreign affiliates is left unaffected.

Trade and FDI literature has grown over time. Hanson, Mataloni, and Slaughter (2001), using detailed data on U.S. multinationals, find that vertical FDI is common, and that affiliates respond to policies and foreign countries' characteristics in different ways. Keller and Yeaple (2008) embody in a trade model two crucial elements: product's technological complexity and distance between the buyer and the seller. In their model, the interaction of these elements determines the size of the costs of reaching foreign markets. Their empirical results confirm the existence of gravity for weightless goods (complex technology products). Irarrazabal, Moxnes and Opromolla (2009) structurally estimate a model of trade and multinational production with firms heterogeneity. Their results reject the proximity versus concentration hypothesis which did not consider intra-firm trade. Corcos et al. (2009), using French firm-level data, investigate the main determinants of the internalization choice. Their findings highlight the role of capital, skill and productivity in explaining the choice of intra-firm.

Although we will not consider the choice between internalizing or not the production process, it is important to remember that intra-firm *versus* arm's length trade strategies are at the heart of the classical "make or buy" decision literature. This literature combines elements from international trade theory and from the theory of the firm. The issues of where locate the different stages of the value chain as well as the control exerted on these process have being studied, among others, by Either (1986), Grossman and Helpman (2002), Antràs (2003, 2005) and Antras and Helpman (2004).

The attempt of this paper is to shed new light on firms' global sourcing strategies focusing more on the role of distance and trade costs while omitting the issue of incomplete contracts. In the present framework, geographical distances will be crucial in explaining how firms reshape their global sourcing strategies.

The rest of the paper is organized as follows. Section 2 provides a description of facts on U.S. multinational firms. Section 3 describes the theoretical framework and Section 4 characterizes its general equilibrium. Section 5 and 6 examine the intensive and extensive margins. The welfare analysis is presented in section 7, and conclusions follow.

2 Facts on U.S. Multinational Firms

Data are obtained from the direct investment data set accessible from the Bureau of Economic Analysis (henceforth BEA) website. These data are separated into two groups. The first group includes the number of US foreign affiliates in 170 destination countries in 2004 over 20 industries. The second group considers local affiliate sales, which is reduced to a cross section

of 54 destination countries due to data limitations as well as U.S. intra firm trade, i.e. U.S. Exports of Goods Shipped to Affiliates by U.S. Parents, by Country of Affiliate.

Number of Firms

After controlling for size, a regression between number of foreign affiliates and the destination countries suggests a non linear relationship with a slope coefficient of -0.85.

Market Entry

Figure 3 plots the number of US affiliates selling to a market against total absorption in that market across 102 markets. Since I have to match the data with production data, I restrict attention to 65 countries. The number of firms selling to a market tends clearly to increase with the size of the market.

The relationship is more neat when the number of US affiliates is normalized by the share of US in a market. Following Eaton, Kortum and Kramarz (2008), the x axis of Figure 4 reports market size across the 55 destinations. While the y axis replaces the number of US affiliates in a market with that number divided by US market share. US market share is defined as total US affiliate sales to that market, $X^M_{us,j}$, divided by the market's total absorption, X_j,

$$\pi_{us,j} = \frac{X^M_{us,j}}{X_j}$$

This relationship is tight. As in Eaton, Kortum and Kramarz (2008), correcting for market share pulls Canada from the position of a positive outlier to a negative one. A regression line as a slope of 0.83.

We can interpret Figure 4 as how the number of US affiliates varies with market size. From this plot it seems clear that the number of sellers in a market varies with market size.[3]

Affiliate Sales

This part examins affiliate sales in 54 destination countries. Figures 5a and 5b plot US affiliate sales against distance and absorption in the destination countries. Total affiliate sales are negatively related with distance, Figures 5a, while they are increasing in destination country size, Figures 5b. The relationship is more neat in log. A regression line has a slope of 0.77.

3 Theoretical Framework

In what follows we propose a model of export and FDI as well as intra firm trade. Following Chaney, we do not assume free entry. In a previous version we assumed intra firm trade as well as distant dependent fixed cost. However, a model with only intra firm trade is sufficient for the purpose of our study.[4] This set up allows to study the supply mode decision between FDI and export in a multi-country framework.

[3]This also confirm the use of a model of firms heterogeneity, where the number of firms depends on country size.

[4]This model is isomorphic to a model with distant dependent fixed cost.

3.1 Preferences

Consumers in each country share the same preferences over the final good. The preferences of a representative consumer are given by C.E.S. utility function over a continuum of goods indexed by v,

$$U = \left[\int_{v \in V} c(v)^{(\sigma-1)/\sigma} dv \right]^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ represents the elasticity of substitution between any two products within the group and V is the set of available varieties.

3.2 Supply

In the following set up we have one final good, two intermediate goods and one factor. Each country is endowed with labour, L, which is supplied inelastically. There are N potentially asymmetric countries that produce goods using only labor. Country n has a population L_n.

There is one differentiated sector which produces a continuum of horizontally differentiated varieties, $q(v)$, from two intermediate goods (or tasks), y_1 and y_2. Both y_1 and y_2 are produced with one unit of labour, but y_1 can only be made at home, due to technological appropriability issues. Each variety is supplied by a Dixit-Stiglitz monopolistically competitive firm which produces under increasing returns to scale which arise from a fixed cost. We assume the fixed cost is paid in units of labor in the country where the good is produced.

We consider three modes of supply in the differentiated sector; firms which sell only domestically (D-mode); firms who export (X-mode), and firms who supply the foreign market via FDI (M-mode). Hence, when a firm decides to serve the foreign market, it chooses whether to export domestically produced goods or to produce in foreign country via affiliate production. In making those decisions, they consider the net profits from selling in a given market, and they compare the profits from exports and from FDI.

As in Helpman, Melitz and Yeaple (2004), this choice is affected by the classical scale versus proximity trade-off. Nevertheless, in our model, the introduction of intra firm trade makes the M-model of supply sensitive to geographical distance between countries. The fact that y_1 can only be made at home plays an important role. If a firm chooses to supply the foreign market via local sales of its affiliates, the affiliate must import the intermediate good y_1 from the home nation. This implies that the M-mode does not entirely avoid trade costs. The trade link between the home parent and the affiliate captures the complementary relationship between trade and FDI. In this model, the existence of asymmetric countries implies that there is not a one for one mapping between the productivity of a firm and the scale of its production.

Upon drawing its own parameter a from a cumulative density function $G(a)$ that is common to every country, each firm decides to exit (this happens if it has a low productivity draw), or to produce. In this case, the firm must face additional fixed costs linked to the mode of supply chosen. If it chooses to produce for its own domestic market, it pays the additional fixed market entry cost, f_{ii}. If the firm chooses to export, it bears the additional costs f_{ij} of meeting different market specific standards (for example, the cost of creating a distribution network in a new country). Finally, if the firm chooses to serve foreign markets through FDI, there would be two types of fixed costs: a fixed cost of creating a distribution network as well

as building up new capacities in the foreign country.[5] We call these fixed costs $f_{M,ij}$. In the following analysis we allow for the fixed costs to differ across coutries.

3.3 Intermediate Results

Demand

Given preferences across varieties have the standard C.E.S. form, the demand of a representative consumer from country i for a type a good is given by

$$c_i(a) = A_i p_i(a)^{-\sigma} \quad \text{where } A_i \equiv \frac{Y_i}{P_i^{1-\sigma}}$$

where the subscript i indicates the country, a the unit labor coefficient, A_i is the demand shifter and $p_i(a)$ is the consumer price index paid to a firm with marginal cost a. A_i is exogenous from the perspective of the firm and composed by the aggregate level of spending on the differentiated good, Y_i divided by the CES price index, $P_i^{1-\sigma}$.

Organization and Product Variety

We assume the production of the final good combines the two intermediates, y_1 and y_2, in the following Cobb-Douglas function,

$$q_i(a) = \frac{1}{a}\left(\frac{y_1}{\eta}\right)^{\eta}\left(\frac{y_2}{1-\eta}\right)^{1-\eta}, 0 < \eta < 1 \tag{1}$$

where $1/a$ represents the firm specific productivity parameter and η is the Cobb-Douglas cost share of y_1, common across all nations. When trade is possible, firms that produce decide whether to sell to a particular market and how, i.e. via export or FDI strategies. This will depend on their own productivity, on trade costs (distance) between the origin and the destination country and on the fixed costs.

The marginal costs in the exporting sector will be higher than the one in the FDI sector. Since y_1 and y_2 are produced with L, the marginal cost for domestic as well as export production is linear in τ,

$$mc_{ij} = aw_i\tau_{ij}$$

where when $i = j$ then $\tau_{ij} = 1$. The marginal cost for supplying the foreign market j via local sales of foreign affiliates is concave in τ,

$$mc_{M,ij} = aw_j^{1-\eta}(w_i\tau_{ij})^{\eta}$$

This last marginal cost combines inputs from home and host country. More precisely, $w_j^{1-\eta}$ is the labor cost for input produced in country j, while w_i^{η} is the labor cost for input imported in country j from the home country i.[6] Note that in this last marginal cost trade costs matter but only in relation to cost share, η, of the intermediate good y_1 used in the production of the

[5] In our model when a firm chooses to serve foreign markets via FDI it means local production of the intermediate good, y_2, only.

[6] When $\eta = 0$ the model delivers the HMY framework.

final good. Using the mark up, $\sigma/(\sigma-1)$, we can easily derive the price for each particular mode of supply decisions.

Mode of Supply Decisions

The mode of supply decision choice will involve the comparison of profit levels taking into account the various fixed and variable trade costs. A firm can decide to: (i) not supply a market, (ii) supply it via exports, or (iii) supply it via local sales of foreign affiliates.[7]

The optimal mode of supply depends on a firm's productivity. As described above, three cases are relevant.

Case (i). If the firm decides not supply a market and exits, the operating profits are zero.

Case (ii). If the firm in country i decides to supply market j via exports, the profits from exporting to market j are linearly decreasing in τ_{ij},

$$\pi_{ij} = \left[p_{ij}(a) - aw_i\tau_{ij}\right] q(a)_{ij} - w_j f_{ij}$$

where $q(a)_{ij}$ represents the quantity exported. Substituting the equilibrium price and quantity we have,

$$\pi_{ij} = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)} Y_j \left(w_i a\tau_{ij}\right)^{1-\sigma} / P_j^{1-\sigma} - w_j f_{ij} \tag{2}$$

where the fixed cost of exporting, f_{ij}, is evaluated at the foreign wage rate, w_j.[8]

Case (iii). If the firm in country i decides to supply market j via FDI, the profits realized by a subsidiary located in the j country depend on τ_{ij},

$$\pi_{M,ij} = \left[p_M(a) - aw_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right] q(a)_{M,ij} - w_j f_{M,ij} \tag{3}$$

where $q(a)_{M,ij}$ represents the quantity supplied by the foreign affiliate. Substituting the equilibrium price and quantity we have,

$$\pi_{M,ij} = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)} Y_j \left(aw_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma} / P_j^{1-\sigma} - w_j f_{M,ij}$$

where τ_{ij}^{η} is the trade costs associated with the intermediate good, y_1, imported from the home country. The foreign affiliate has to face both the fixed cost $f_{M,ij}$, evaluated at the foreign wage rate, and the trade costs that hit the imported intermediate.

To focus on the central case, we set parameters so that we get the same ranking as in HMY when there are only two nations. Namely, firms with sufficiently high productivity will supply the foreign market at all, with the most productive supplying it via FDI rather than exports. Hence, the regularity condition we need is,

$$\left(w_i\tau_{ij}\right)^{(\sigma-1)} w_j f_{ij} < \left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{\sigma-1} w_j f_{M,ij}$$

[7] The export cutoff also includes the situation in which the local market is supplied by domestic firm sales.

[8] Note that this model of supply collapses to domestic production when $i = j$, since $\tau_{ii} = 1$.

Rearranging terms we get:

$$f_{ij} < f_{M,ij} \frac{\left(w_j^{1-\eta} \left(w_i \tau_{ij}\right)^{\eta}\right)^{\sigma-1}}{\left(w_i \tau_{ij}\right)^{(\sigma-1)}} \tag{4}$$

The fact that the price index depends on the probability distribution implies that in order to have explicit solutions for this model, we need to assume a particular functional form for $G(a)$. Following the empirical literature on firm size distribution (see Axtell 2001 and Chaney 2008), we assume unit labor requirement are drawn from a Pareto distribution. The cumulative distribution function of a Pareto random variable a is:

$$G(a) = \left(\frac{a}{a_0}\right)^k \tag{5}$$

where k and a_0 are the shape and scale parameter, respectively. The shape parameter k represents the dispersion of cost draws. An increase in k would imply a reduction in the dispersion of firm productivity-draws. Hence, the higher is k the smaller is the amount of heterogeneity.

The support of the distribution $[0, ..., a_0]$, is identical for every country, where a_0 represents the upper bound of this distribution. The productivity distribution of surviving firms will also be Pareto with shape k. More precisely, since a firm will start producing only if it has at least a productivity of $1/a_{ij}$, the probability distribution of supplying as an exporter, or as a foreign affiliate, is conditioned on the probability of successful entry in each market,

$$G(a/a_{ii}) = \left(\frac{a}{a_{ii}}\right)^k$$

The above truncated cost distribution exploites the fractal nature of the Pareto. Here the support is $[0, ..., a_{ii}]$. Given the assumed parameterization, we can explicitly solve for the price index.

Following Chaney 2008, I assume that the total mass of potential entrants in country i is proportional to its labour income, $w_i L_i$. Hence, larger and wealthier countries have more entrants. The absence of free entry implies that firms generate net profits, which have to be redistributed. In line with Chaney (2008), I assume that each worker owns w_i shares of the global fund. This fund collects profits from the firms and redistributes them to its shareholders.

Demand for Differentiated Goods

Total income in country j, Y_j, is the sum of workers' labor income in country j, $w_j L_j$, and of the dividends they get from their portfolio, $\pi w_j L_j$, where π is the dividend per share.

Given the optimal pricing of firms and the demand by consumers, we can find the export value from country i to country j by a firm with unit labor requirement a,

$$x_{ij}^X = p_{ij}^X q_{ij}^X = Y_j \left(p_{ij}^X\right)^{1-\sigma} / P_j^{1-\sigma}$$

where $p_{ij}^X = \left[\sigma/\left(\sigma-1\right)\right] a w_i \tau_{ij}$ and $q_{ij}^X = \left(p_{ij}^X\right)^{-\sigma} \beta Y_i / P_j^{1-\sigma}$. While affiliate sales by a firm located in j are

$$x_{ij}^M = p_{ij}^M q_{ij}^M = Y_j \left(p_{ij}^M\right)^{1-\sigma} / P_j^{1-\sigma}$$

where P_j represents the price index of good q in country j. The value of export and of total production in j's foreign affiliates are therefore similar to the one derived from homogeneous firms set up. They provide basis for gravity equations of export and of affiliate sales.[9]

Since only firms with $a \leq a_{kj}$ can start producing, the ideal price index in country j is[10]

$$P_j^{1-\sigma} = \sum_{k=1}^{N} w_k L_k \left[{}_{0}^{a_{M,kj}} \left(w_j^{1-\eta} \left(w_k \tau_{kj} \right)^{\eta} \right)^{1-\sigma} a^{1-\sigma} dG(a) + {}_{a_{M,kj}}^{a_{kj}} \left(w_k \tau_{kj} \right)^{1-\sigma} a^{1-\sigma} dG(a) \right]$$

The dividend per share, π, are defined as

$$\pi = \frac{\sum_{k,l=1}^{N} w_k L_k \left[{}_{0}^{a_{M,kl}} \pi_{M,kl} dG(a) + {}_{a_{M,kl}}^{a_{kl}} \pi_{kl} dG(a) \right]}{\sum_{n=1}^{N} w_n L_n}$$

where in the square parenthesis we have the profits that a firm with a specific threshold level in country k earns from a specific mode of supply in country l.[11] A similar analysis can be extended to H sectors. In Appendix A.3. we derive solutions for the profits. Refer to profit sw file to see how I changed the dividend per share.

4 Equilibrium with Heterogeneous Firms

To compute the equilibrium of the overall economy, we solve for the selection of firms into different modes of supply. We generate predictions for aggregate bilateral trade and FDI flows.

Productivity Threshold

From the profit a firm earns from exporting we can derive the productivity threshold of the least productive firm in country i able to export to country j,

$$a_{ij}^{1-\sigma} = \lambda_1 \frac{w_j f_{ij}}{Y_j} \frac{P_j^{1-\sigma}}{\left(w_i \tau_{ij} \right)^{1-\sigma}} \tag{6}$$

where $\lambda_1 = \sigma \left(\frac{\sigma-1}{\sigma} \right)^{(1-\sigma)}$.[12] While the productivity threshold of the least productive firm in country i able to open a foreign affiliate to country j is obtained by equating the operating profits from doing FDI, (3), with the operating profit from doing export (2),

$$a_{M,ij}^{1-\sigma} = \lambda_1 \frac{w_j \left(f_{M,ij} - f_{ij} \right)}{Y_j} \frac{P_j^{1-\sigma}}{\left(w_j^{1-\eta} \left(w_i \tau_{ij} \right)^{\eta} \right)^{1-\sigma} - \left(w_i \tau_{ij} \right)^{1-\sigma}} \tag{7}$$

[9] See Redding and Venables (2004) and Kleiner and Toubal (2005).

[10] Since we are not conditioning, $G(a/a_{ij})$ the number of firms will be the number of entrants and not the number of active firms. Moreover, we consider a_{ij} to be the unit labor requirement for exporting. Note that when i=j, $\tau_{ii} = 1$ and so $a_{ij} = a_{ii}$, which corresponds to the cutoff of domestic firms.

[11] Note that when $i = j$, $\tau_{ij} = 1$ and so $\pi_{M,kl}(a_M) = \pi_{kl}(a_M)$. When $i = j$ we are considering the domestic firms.

[12] We interpret $a^{1-\sigma}$ as a measure of productivity.

Equilibrium Price Indices

Since price index adjusts depending on country characteristics, it is possible to find tractable solutions for it. Thanks to the fact that the number of potential entrants, n_E, is exogenously given, the price index will depend only on country j's characteristics,

$$P_j^{1-\sigma} = (\sigma/(\sigma-1))^{1-\sigma} \times k/(k-\sigma+1)\times$$

$$\sum_{k=1}^{N} w_k L_k \left[a_{M,kj}^{k-\sigma+1} \left(w_j^{1-\eta} \left(w_k \tau_{kj} \right)^{\eta} \right)^{1-\sigma} + \left(a_{kj}^{k-\sigma+1} - a_{M,kj}^{k-\sigma+1} \right) \left(w_k \tau_{kj} \right)^{1-\sigma} \right]$$

Plugging the productivity thresholds from (6) and (7) we can solve for the price index in the destination country j as follows,

$$P_j = \lambda_2 Y_j^{\frac{b-1}{b(1-\sigma)}} \theta_j \left(\frac{Y}{1+\pi} \right)^{\frac{1}{b(1-\sigma)}} \tag{8}$$

where $b = k/(\sigma-1)$, w_k is the wage paid to workers in country k for firms which are exporting the good, while w_j is the wage paid to the workers in country j which are producing the domestic varieties or the foreign affiliate varieties. In the expression above θ_j collects the following terms

$$\theta_j^{b(1-\sigma)} = \sum_{k=1}^{N} \frac{Y_K}{Y} \left[\left(w_j \left(f_{M,kj} - f_{kj} \right) \right)^{1-b} \left[\left(w_j^{1-\eta} \left(w_k \tau_{kj} \right)^{\eta} \right)^{1-\sigma} - \left(w_k \tau_{kj} \right)^{1-\sigma} \right]^{b} + \left[w_j f_{kj} \right]^{1-b} \left(\left(w_k \tau_{kj} \right)^{1-\sigma} \right)^{b} \right]$$

where Y is the world output, and λ_2 a constant.[13] θ_j is an aggregate index of j's remoteness from the rest of the world. It can be thought as the "multilateral trade resistance" introduced by Anderson and van Wincoop (2003). It takes into consideration the role of the fixed cost as well as trade costs and intermediate input traded.

Since total income Y will depend on the dividends received from the global fund, in equilibrium it turns out that dividend per share is a constant.

Equilibrium variables

The mode of supply choice depends on each firm productivity, the trade costs it has to face, aggregate demand, the amount of intermediates it needs, the set of competitors. Using the general equilibrium price index from (25) into (6) and (7) we can solve for the productivity threshold.

$$\overline{a}_{ij}^{1-\sigma} = \lambda_4 \frac{w_j f_{ij}}{\left(w_i \tau_{ij} \right)^{1-\sigma}} \theta_j^{1-\sigma} \left(\frac{Y}{Y_j} \right)^{\frac{1}{b}} (1+\pi)^{-\frac{1}{b}} \tag{9}$$

$$\overline{a}_{M,ij}^{1-\sigma} = \lambda_4 \frac{w_j \left(f_{M,kj} - f_{ij} \right)}{\left(w_j^{1-\eta} \left(w_i \tau_{ij} \right)^{\eta} \right)^{1-\sigma} - \left(w_i \tau_{ij} \right)^{1-\sigma}} \theta_j^{1-\sigma} \left(\frac{Y}{Y_j} \right)^{\frac{1}{b}} (1+\pi)^{-\frac{1}{b}} \tag{10}$$

where λ_4 is a constant.[14] The productivity threshold in (9) is unambiguously positively affected by the wage rate in the origin country, and distance trade costs. On the other side, the

[13] $\lambda_2^{b(\sigma-1)} = (\sigma/(\sigma-1))^{\sigma-1} \times (k-\sigma+1)/k \times \lambda_1^{b-1}$.
[14] $\lambda_4 = \lambda_1/\lambda_2^{\sigma-1}$.

productivity threshold in (10) is ambiguously affected by the wage rate in i, η and distance trade costs. A large w_j increases the productivity to be a MNFs.

The share of imported intermediates plays an important role in determining the substitutability or the complementarity between trade and FDI strategies. A low amount of imported intermediates, η, makes the FDI strategy better off when distance increases; while a high η fades out the source of ambiguity.[15] The lower is the η the more destination countries a firm can reach via HFDI when trade cost increases.

Then using the demand function, the equilibrium price as well as (25), we can find the firm level exports and the firm level affiliate sales, aggregate output and dividends per share π.

$$x_{ij}^X = p_{ij}^X q_{ij}^X = \lambda_3 \times \theta_j^{\sigma-1} \times \left(\frac{Y_j}{Y}\right)^{\frac{1}{b}} \times (1+\pi)^{\frac{1}{b}} \times (w_i \tau_{ij})^{1-\sigma} \times a^{1-\sigma} \tag{11}$$

$$x_{ij}^M = p_{ij}^M q_{ij}^M = \lambda_3 \times \theta_j^{\sigma-1} \times \left(\frac{Y_j}{Y}\right)^{\frac{1}{b}} \times (1+\pi)^{\frac{1}{b}} \times \left(w_j^{1-\eta} (w_i \tau_{ij})^{\eta}\right)^{(1-\sigma)} \times a^{1-\sigma} \tag{12}$$

$$\pi = \lambda_5 \tag{13}$$

$$Y_j = (1+\pi) w_j L_j = (1+\lambda_5) w_j L_j \tag{14}$$

where λ_3 and λ_5 are constants.[16] The equations above are functions of fundamentals only: the size L_j, the wages, the trade barriers τ_{ij}, the fixed costs $f_{M,ij}$ and f_{ij}, the proportion of intermediate imported, η, and the measure of the j's location with respect to the rest of the world, θ_j.

Similarly to Chaney (2008) exports by individual firms depend on the transportation cost τ_{ij} with an elasticity $1-\sigma$. Here we also have the sales by a foreign affiliate, which depend on the share of intermediate produced in the foreign location, y_2, and imported from the home country, y_1. Firm level FDI, (12), are unambiguously linked to trade costs: an increase in trade costs reduces the firm level FDI.

Firm level trade is the same as in Chaney (2008). Firm level affiliate sales depend on the interaction between imported and locally produced inputs. The behaviour of single firm is similar to what a traditional model of trade and FDI with representative firms would predict for aggregate bilateral trade flows and affiliate sales.

Similarly to Chaney (2008) and Irarrazabal et al. (2008), we can derive gravity equations using equations (11) and (12). In the present model aggregate bilateral trade and overseas affiliate sales will be different from traditional models.

Proposition 1 (aggregate trade) Using the firm level exports we can derive the total export (f.o.b.), X_{ij}^X, from country i to country j,

$$X_{ij}^X = \frac{Y_i Y_j}{Y} \theta_j^{b(\sigma-1)} (w_i \tau_{ij})^{1-\sigma} \times$$

[15] Low η makes FDI and Export act as substitutes. For certain parameter restrictions, the productivity threshold in (10) is decreasing in distance when η is low. For high η the productivity threshold in (10) is increasing with distance. Therefore, FDI and export become complements for sufficiently high η: both strategies require a higher productivity level when distance increases.

[16] $\lambda_3 = \lambda_2^{\sigma-1}(\sigma/(\sigma-1))^{1-\sigma}$, $\lambda_5 = \left((1-\lambda_4^{-b}\sigma)/\sigma\right) / \left(1-(1-\lambda_4^{-b}\sigma)/\sigma\right)$.

$$\left[\left(\frac{w_j f_{ij}}{(w_i\tau_{ij})^{1-\sigma}}\right)^{1-b} - \left(\frac{w_j\left(f_{M,ij}-f_{ij}\right)}{\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma} - (w_i\tau_{ij})^{1-\sigma}}\right)^{1-b}\right] \tag{15}$$

Proof. See Appendix A.1. ■

The gravity equation for export suggests that exports are a function of country sizes, Y_i and Y_j, wages, bilateral trade costs and fixed costs, and the measure of j's remoteness from the rest of the world.[17] In equation (15) wages are endogenous, so they will respond to changes in trade policy.

Using partial equilibrium analysis, we can say that under certain parameter restrictions the aggregate level of export is negatively affected by the wage rate in the origin country and trade costs. Moreover, we can analyse under which conditions trade and FDI are complements or substitutes. To highlight the role of the share of imported intermediates, η, we make the simplifying assumption of $w_i = w_j$. When η increases the marginal cost of doing FDI increases. Hence, the second element in the square bracket in (15) decreases with η. This implies that aggregate exports in (15) are increasing with η.

Remark 1 *For certain parameter restrictions, aggregate export sales decrease with trade costs. They decrease faster the larger is σ. They become negative for very high trade cost or wages. This last effect is slightly reduced when η is large.*

To conclude, under certain parameter restrictions, aggregate exports in (15) are increasing with η and w_j, while are decreasing in w_i and τ. Differently from Chaney (2008), this aggregate trade expression take into consideration the interaction between trade and FDI. This interaction makes the gravity for export non linear in logarithm.

Proposition 2 (aggregate affiliate sales) Using the firm level affiliate sales we can derive the total affiliate sales, X_{ij}^M, in country j,

$$X_{ij}^M = \frac{Y_i Y_j}{Y}\theta_j^{b(\sigma-1)}\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma} \times \left(\frac{w_j\left(f_{Mij}-f_{ij}\right)}{\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma} - (w_i\tau_{ij})^{1-\sigma}}\right)^{1-b} \tag{16}$$

Proof. See Appendix A.2. ■

The gravity equation for affiliate sales suggests that affiliate sales are a function of country sizes, Y_i and Y_j, wages, bilateral trade costs and fixed costs, intra firm trade between affiliates and the measure of j's remoteness from the ROW.[18] The last term of (16) represents the source of ambiguity.

[17] Note that if both the intermediates are produced at home, $\eta = 1$, the FDI will be too costly, and every firm will end up being an exporter, since it is more profitable. The gravity in this case will be like in Chaney (2008): $X_{ij}^X = \beta\frac{Y_i Y_j}{Y}\theta_j^{b(\sigma-1)} f_{ij}^{1-b}\left(w_i\tau_{ij}\right)^{-k}$. When all the intermediates are produced in the foreign location, $\eta = 0$, we are back in the HMY framework. Hence the gravity equation for export in HMY setup is: $X_{ij}^X = \beta\frac{Y_i Y_j}{Y}\theta_j^{b(\sigma-1)}\left(w_i\tau_{ij}\right)^{1-\sigma} \times \left[\left(\frac{f_{ij}}{\left(w_i\tau_{ij}\right)^{1-\sigma}}\right)^{1-b} - \left(\frac{f_{M,j}-f_{ij}}{w_j^{1-\sigma}-\left(w_i\tau_{ij}\right)^{1-\sigma}}\right)^{1-b}\right]$.

[18] Note that if both the intermediates are produced at home, $\eta = 1$, the FDI will be too costly, because it will incur in trade costs plus greater fixed cost, $f_M > f_{ij}$. In this case, there will be no firm supplying via FDI because the cost will be prohibitive, i.e. $\overline{a}_{M,ij} \longrightarrow 0$, or $\overline{a}_{M,ij}^{1-\sigma} \longrightarrow \infty$. Hence the gravity for FDI, X_{ij}^M, will be

Increase in trade costs reduces both total trade and intra firm trade, but the magnitude differs in relation to the amount of intermediate imported.[19] In general equilibrium, the increase in trade costs will also affect wages. The final effect of trade policy on affiliate sales depends on how wages respond to τ. Different wage responses will generate different affiliate sales reactions.

Changes in trade barriers affect aggregate affiliate sales in different ways depending on how wages respond to trade liberalization. Increase in trade barriers might create an incentive to ship production to the foreign market to avoid a part of the trade costs.[20] This will increase the demand for labor in the destination country relatively to the home country. When trade costs are sufficiently small and the difference between the wages is not too big, antiglobalization forces lead to an increase in aggregate local sales. This effect is stronger the lower is the share of intra firm trade.

Differently from Chaney I find that the elasticity of affiliate sales, as well as exports, with respect to the variable costs does depend on the elasticity of substitution between goods, σ. This result suggests that in order to understand how variable costs affect bilateral flows, it is important to consider a multi supply framework. The interaction between the two modes of supply decisions reaffirm the importance of σ even allowing for firm heterogeneity. Moreover, in these gravity equations trade between affiliates plays a role.

Remark 2 *Aggregate affiliate sales in (16) are non monotonically related to distance. Different level of trade costs, elasticity of substitution, and share of intermediate inputs traded change the way distance affects (16). Moreover, the response of wages to trade policy will be relevant to determine the final effect.*

Proposition 3 (Number of Affiliates) The aggregate number of foreign affiliates is given by

$$\begin{aligned} n_{M,ij} &= w_i L_{i0}^{\overline{a}_{M,ij}} dG(a) \\ &= \frac{Y_i Y_j}{Y} \theta_j^{b(\sigma-1)} \lambda_4^{-b} \left(\frac{\left(w_j^{1-\eta} \left(w_i \tau_{ij} \right)^{\eta} \right)^{1-\sigma} - \left(w_i \tau_{ij} \right)^{1-\sigma}}{w_j \left(f_{M,ij} - f_{ij} \right)} \right)^{b} \end{aligned} \tag{17}$$

where we used the productivity threshold in (10).

If trade cost are sufficiently low, a change in distance initially increases the number of affiliates. Neverthless, when distance becomes important the number of firms decreases. This non monotonicity is lost if the trade cost and or the elasticity of substitution are particularly high. Otherwise, if σ and or τ are sufficiently low we could observe a more persistent increase in the number of affiliates.[21]

0. When all the intermediates are produced in the foreign location, $\eta = 0$, we are back in the HMY framework. Hence the gravity equation for FDI in HMY set up is: $X_{ij}^{M} = \beta \frac{Y_i Y_j}{Y} \theta_j^{b(\sigma-1)} w_j^{1-\sigma} \times \left(\frac{f_{M,j} - f_{ij}}{w_j^{1-\sigma} - \left(w_i \tau_{ij} \right)^{1-\sigma}} \right)^{1-b}$. In this set up there is no role for complementarity between trade and FDI.

[19] Higher level of η make total trade and intra firm trade look similar. In this circumstances, the existence of wage differential will be the key element.

[20] This incentive is greater the lower is the amount of intra firm trade.

[21] If in the data I do not observe this concave relationship between number of affiliate and distance, but

Remark 3 *The aggregate number of foreign affiliates has a non monotonic behavior with respect to distance. Low levels of η exacerbate this non monotonicity. The reverse is true for high σ and or τ.*

In this model a decreasing number of firms can continue to supply via FDI when τ_{ij} increases. Only the more productive firms can continue to supply via FDI to the remote location. This result is in sharp contrast with the literature on proximity versus concentration, where the number of affiliates is increasing with distance. The introduction of intermediate input, makes the FDI strategy sensitive to trade issues.

Proposition 4 (Number of Exporters) The aggregate number of exporters is

$$\begin{aligned} n_{X,ij} &= w_i L_i \int_{\overline{a}_{M,ij}}^{\overline{a}_{ij}} dG(a) = w_i L_i \left(\overline{a}_{ij}^k - \overline{a}_{M,ij}^k \right) \\ &= \frac{Y_i Y_j}{Y} \theta_j^{b(\sigma-1)} \lambda_4^{-b} \left[\left(\frac{(w_i \tau_{ij})^{1-\sigma}}{w_j f_{ij}} \right)^b - \left(\frac{\left(w_j^{1-\eta} (w_i \tau_{ij})^{\eta} \right)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}}{w_j (f_M - f_{ij})} \right)^b \right] \quad (18) \end{aligned}$$

where we used the productivity thresholds in (9) and (10). It is difficult to interpret the role of different variables in this expression.

Remark 4 *For certain parameter restrictions the aggregate number of exporting firms is decreasing with trade costs.*

Trade and HFDI

The value of η characterizes the cost of doing HFDI, and it can be considered the parameter which captures the interaction between FDI and trade. A reduction in η generates an increase in affiliate sales and a decrease in trade. However, a decrease in η may imply larger or smaller *intra firm trade*. A smaller η shifts production so that more host country national input, y_2, is used. Nevertheless, since the decrease in η increases affiliate sales, the use of home as well as host input increases. The Hicksian factor demand for the intermediate good, y_1, imported in j from i, is

$$y_1^* = q_{Mij}(a)\, a\eta \left(\frac{w_j}{w_i \tau_{ij}} \right)^{1-\eta}$$

The demand for the intermediate good depends on the overall quantity produced in the foreign affiliate, $q_{Mij}(a)$, as well as in the share of intermediate good, η, used in the overseas affiliate final good production. The final effect of a decrease in η on the Hicksian factor demand and so on *intra firm trade* depends on which of these two effects dominate.

For certain parameter restrictions, η small, the increase in distance increases aggregate affiliate sales, (16), and decreases aggregate exports, (15) (trade and FDI are substitute). While, when η is large, the increase in distance generates a decrease in both affiliate and export sales (trade and FDI are complement).

only a monotic decreasing relationship, it can be because trade costs between nearby location are still too high. Hence, in term of policy implication, if the goverment wants to increase the number of foreign affiliates it should decrease trade costs. This reduction should have a stronger impact on the number of affiliates the larger is the amount of intra firm trade, η.

5 Intensive and Extensive Margins of Affiliate Sales

In this section we examine the intensive and extensive margins of FDI. We analyse how the elasticity of substitution as well as the share of intermediate inputs affects the sensitivity of these margins. Differentiating the total affiliate sales in country j, $X_{ij}^M = w_i L_{i0}^{\overline{a}_{M,ij}} x_{ij}^M dG(a)$, with respect to τ_{ij}, we can derive the intensive and extensive margins of FDI,

$$\frac{\partial X_{ij}^M}{\partial \tau_{ij}} = \underbrace{w_i L_{i0}^{a_{M,ij}} \frac{\partial x_{ij}^M}{\partial \tau_{ij}} dG(a)}_{\text{Intensive Margin}} + \underbrace{w_i L_i x_{ij}^M G'\left(\overline{a}_{M,ij}\right) \frac{\partial \overline{a}_{M,ij}}{\partial \tau_{ij}}}_{\text{Extensive Margin}} \tag{19}$$

where we apply the Leibniz rule to separate the intensive and intensive margins.

Defining $\psi \equiv -\partial \ln X_{ij}^M / \partial \ln \tau_{ij}$, a change in the variable costs, τ_{ij}, makes the margins of FDI react in the following way

$$\psi = \underbrace{\eta\left(\sigma - 1\right)}_{\text{Intensive Margin Elasticity}} + \underbrace{\left(k - \sigma + 1\right) \frac{\left(\eta \left(w_j^{1-\eta} \left(w_i \tau_{ij}\right)^{\eta}\right)^{1-\sigma} - \left(w_i \tau_{ij}\right)^{1-\sigma}\right)}{\left(w_j^{1-\eta} \left(w_i \tau_{ij}\right)^{\eta}\right)^{1-\sigma} - \left(w_i \tau_{ij}\right)^{1-\sigma})}}_{\text{Extensive Margin Elasticity}} \tag{20}$$

Proof. See Appendix A.5.1. ■

Intensive Margin. The sensitivity of the intensive margin of FDI to changes in trade costs, is linked to the existence of intermediate input trade. This is straightforward since the M mode of supply is affected by trade costs proportionally to the amount of intermediate being imported (which is hit by trade barriers).

What happens in the extreme case $\eta = 1$ and $\eta = 0$? When $\eta = 1$, no firm supplies via FDI. In fact, $\eta = 1$ means that the foreign affiliate is importing both inputs from the home country. This strategy is extremely costly, since it incurs in trade costs as well as in higher fixed costs. When $\eta = 1$ there will be only export as a market access strategy.

When $\eta = 0$, the foreign affiliate is producing using only foreign inputs (HMY case).[22] This means that since there is no trade in intermediate between affiliates, the volume of sales should not be affected by changes in trade costs. Therefore the intensive margin elasticity equals 0.

For intermediate level of η, the affiliate sales are affected by the intermediate trade. The behaviour of the intensive margin is unambiguous: σ magnifies the sensitivity of the intensive margin. When σ is high the change in X_{ij}^M due to a change in τ is mostly captured by the intensive margin. This happens because when σ is high and τ decreases new affiliates enter the market, but since σ is high this leads to a high level of competition. In this context a low productivity is an even bigger disadvantage, in fact the low productivity firms capture only a small market share. Meaning that their impact on the overall affiliate sales is small. The change in X_{ij}^M is mainly captured by the intensive margin (the already existing firms).

Extensive Margin. The sensitivity of the extensive margin of FDI to changes in trade costs (or distance), is more complex. Let's examine first what happens in the extreme cases $\eta = 1$

[22] For a more precise expression of the gravity in HMY framework refer to footnote (15).

and $\eta = 0$.

When $\eta = 1$ there will not be FDI as a market strategy, since it is too costly. While when $\eta = 0$, FDI and trade are substitutes (we are back in HMY where there is no interaction between trade and FDI). In the HMY setup the elasticity of the extensive margin of FDI to changes in τ is positive. This is consistent with the fact that a decrease in trade costs (or distance) should encourage export, dampening FDI.

For intermediate level of η, affiliate sales are affected by the intermediate trade and the behaviour of the extensive margin is ambiguous. For sufficiently low trade costs, the extensive margin elasticity of FDI is increasing for low level of σ and then decreasing. This behaviour is magnified the lower are the trade costs. When the substitutability across varieties is low, an increase in σ makes entrance of new affiliates more sensitive to changes in τ. On one side, it is easier to import the intermediate (since τ is decreasing), on the other side the degree of substitution is sufficiently low to keep lower the level of competition. Hence, more firms can survive as new affiliates after entry.

However, further increase in the degree of substitutability among varieties makes entrance of new affiliate less sensitive to change in τ. Indeed, despite the reduction in the intermediate trade costs, the level of competition is now too high (as a consequence of the higher level of substitutability and the freer trade), thus new entrants will capture only a small fraction of market share. Meaning that the impact of new affiliates entering the market is small.

From the results above we can highlight some propositions for the extensive and intensive margin of FDI.

Proposition 5 In a highly differentiated sector (low σ) the demand for each variety is less sensitive to changes in trade costs. Hence, when σ is low, trade barriers have a little effect on the sensitivity intensive margin of FDI. The interaction between σ and the extensive margin is more complex since it depends on other parameters.

The share of imported intermediate, η, magnifies the sensitivity of the intensive margin to changes in trade barriers. Its effect on the extensive margins is more difficult to interpret. The existence of intermediate goods trade makes these margins more sensitive to changes in trade policy.

6 Intensive and Extensive Margins of Trade

Differentiating the expression of total export in country j, $X_{ij}^X = w_i L_i \overline{a}_{M,ij}^{\overline{a}_{ij}} x_{ij}^X dG(a)$, with respect to trade costs, we derive the intensive and extensive margins of trade,

$$\frac{\partial X_{ij}^X}{\partial \tau_{ij}} = \underbrace{w_i L_i \overline{a}_{M,ij}^{\overline{a}_{ij}} \frac{\partial x_{ij}^X}{\partial \tau_{ij}} dG(a)}_{\text{Intensive Margin}} + \underbrace{w_i L_i \left[x_{ij}^X G'\left(\overline{a}_{ij}\right) \frac{\partial \overline{a}_{ij}}{\partial \tau_{ij}} - x_{ij}^M G'\left(\overline{a}_{M,ij}\right) \frac{\partial \overline{a}_{M,ij}}{\partial \tau_{ij}} \right]}_{\text{Extensive Margin}}$$

where again we applied the Leibniz rule to separate the intensive and intensive margins.

Defining $\Omega \equiv -\partial \ln X_{ij}^X / \partial \ln \tau_{ij}$, a change in the variable costs, τ_{ij}, makes the margins of

trade react differently,

$$\Omega = \underbrace{(\sigma - 1)}_{\text{Intensive Margin Elasticity}} + \underbrace{(k - \sigma + 1)\left[1 - \frac{X_{ij}^{M}}{X_{ij}^{X}}(\Gamma - \omega)\right]}_{\text{Extensive Margin Elasticity}} \tag{21}$$

Proof. See Appendix A.5.2. ■

The intensive margin of trade in (21) is identical to the one in Chaney (2008). While the extensive margin of export, depends on the trade off between affiliate sales and exports.[23]

The volume of trade is not affected by the existence of another type of firm in the economy. While the sensitivity of the extensive margin of trade to trade policy depends on the interaction between aggregate affiliate and export sales. This because the change in the number of varieties supplied via export depends on the level of profits generated by export and FDI strategies, which in turns affect the overall affiliate sales. Changes in τ should decrease more the marginal cost of exporting than the marginal cost of FDI. Nevertheless, the final effect on sales depends on the level of trade costs, on the share of intermediate input in production and on the wages.

If $X_{ij}^{M} > X_{ij}^{X}$, a decrease in trade cost decreases the extensive margin elasticity. The opposite is true if $X_{ij}^{M} < X_{ij}^{X}$. To summarize, while the elasticity of the intensive margin is always positive (a decrease in trade costs increases the volume of trade), the behaviour of the extensive margin depends on how export and affiliate sales change. If affiliate sales are bigger than export sales, $X_{ij}^{M} < X_{ij}^{X}$, the extensive margin decreases with trade costs. When $X_{ij}^{M} > X_{ij}^{X}$ the opposite is true.

Differently from Chaney (2008), the interaction between different type of firms generates a new scenario for the extensive margin elasticity of trade. If $X_{ij}^{M} > X_{ij}^{X}$, the extensive margin elasticity always decreases with σ but now the sign of the extensive margin is opposite. In fact, the extensive margin is strongly and positively affected by trade barriers: an increase in trade costs induces an increase in the number of varieties exported. Thus the number of exported varieties is increasing with trade costs.

7 Simulation and FPE

TBC...

8 Welfare Analysis

Following Arkolakis et al. (2008), we use the expression for the country i-export market share in j, (**??**), as well as country i-FDI market share in j, (**??**), to express wages in j as follows,

$$w_j^{\sigma b - 1} = \frac{1}{\lambda_{jj}} \frac{Y_j}{Y} \theta_j^{b(\sigma-1)} \times f_{jj}^{1-b}$$

where we used the fact that trade balance, $X_j = Y_j$.[24]. A reduction in domestic trade, λ_{jj}, acts as a trade liberalization effect.

[23] Note that $\Gamma > \omega$ always.

[24] Note that $Y_j = w_j L_j (1 + \pi)$, where Y_j is country j income. See Appendix for more details.

Using the price index in (25), we can have an expression for welfare,

$$\frac{w_j}{P_j} = \frac{\left(\frac{1}{\lambda_{jj}} \frac{Y_j}{Y} \theta_j^{b(\sigma-1)} \times f_{jj}^{1-b}\right)^{\frac{1}{\sigma b-1}}}{c_2 Y_j^{\frac{b-1}{b(1-\sigma)}} \theta_j \left(\frac{Y}{1+\pi}\right)^{\frac{1}{b(1-\sigma)}}} \tag{22}$$

From this equation we see that λ_{jj} influences welfare with an elasticity of $-1/(\sigma b-1)$. However here a larger country size to total welfare of a country is affected by b. Consumers in a larger country are forced to consume a higher share of varieties produced with lower productivities.

TBC...

References

[1] Alfaro, L., Charlton, A., 2007. Intra-Industry Foreign Direct Investment. CEP Discussion Paper No 825.

[2] Anderson, J.E., 1979. A Theoretical Foundation for the Gravity Equation. American Economic Review, 69 (1), 106-16.

[3] Antras, P., Helpman, E., 2004. Global Sourcing. Journal of Political Economy, 112 (3), 552-580

[4] Arkolakis, C., 2008. Market Penetration Costs and the New Consumers Margin in International Trade. NBER Working Papers 14214, National Bureau of Economic Research, Inc.

[5] Arkolakis, C., Demidova, S., Klenow, P.J., Rodriguez-Clare, A., 2008. Endogenous Variety and the Gains from Trade. American Economic Review, American Economic Association, vol. 98(2), pages 444-50, May.

[6] Axtell, R.L., 2001. Zipf Distribution of U.S. Firm Sizes. Science 293, September, 1818–20.

[7] Baldwin, R.E., Robert-Nicoud, F., 2007. Trade and growth with heterogeneous firms. Forthcoming in Journal of International Economics.

[8] Baldwin, R.E., 2005. Heterogeneous Firms and Trade: Testable and Untestable Properties of the Melitz Model. NBER Working Paper No. 11471, National Bureau of Economic Research, Inc.

[9] Barba Navaretti, G., et al., 2004. Multinational Firms in the World Economy. Princeton, NJ: Princeton University Press.

[10] Bernard, A., Jensen, B., Schott., P.K., 2005. Importers, Exporters, and Multinationals: A Portrait of Firms in the U.S. that Trade Goods. NBER Working Papers 11404, National Bureau of Economic Research, Inc. Working Paper no. 11404 (September), NBER, Cambridge, MA.

[11] Bernard, A., Jensen, J. B., Redding, S., Schott, P., 2007. Firms in International Trade. Journal of Economic Perspective, 21 (3), 105-130.

[12] Bernard, A., Jensen, J. B., Redding, S., Schott, P., 2009. The Margin of US Trade. American Economic Review Papers and Proceedings (Forthcoming).

[13] Blonigen, B.A., Davies, R.B., Naughton, H.T., Waddell, G.R., 2007. FDI in Space: Spatial Autoregressive Relationships in Foreign Direct Investment. European Economic Review, 51 (5), 1303-1325.

[14] Brainard, S.L., 1993. A Simple Theory of Multinational Corporations and Trade with a Trade-Off Between Proximity and Concentration. NBER Working Papers 4269, National Bureau of Economic Research, Inc.

[15] Brainard, S.L., 1993. An Empirical Assessment of the Factor Proportions Explanation of Multi-National Sales. NBER Working Papers 4583, National Bureau of Economic Research, Inc.

[16] Brainard, S.L., 1997. An Empirical Assessment of the Proximity-Concentration Trade-off between Multinational Sales and Trade. American Economic Review, 87 (4), 520-44.

[17] Buch, C.M., Kleinert, J., Lipponer, A., Toubal, F., 2005. "Determinants and effects of foreign direct investment: evidence from German firm-level data," Economic Policy, CEPR, CES, MSH, 20 (41), 52-110.

[18] Carr, D., Markusen, J.R., Maskus, K.E., 2001. Estimating the Knowledge-Capital Model of the Multinational Enterprise. American Economic Review, 91 (3), 693-708.

[19] Chaney, T., 2008. Distorted Gravity: The Intensive and Extensive Margins of International Trade. American Economic Review, 98(4), 1707-21.

[20] Coase, R. (1937), The Nature of the Firm. Economica, 4 (16), 386-405.

[21] Eaton, J., Kortum, S., 2002. Technology, Geography, and Trade. Econometrica, 70(5), 1741-1779.

[22] Eaton, J., Kortum, S., Kramarz, F., 2008. An Anatomy of International Trade: Evidence from French Firms. NBER Working Papers 14610, National Bureau of Economic Research, Inc.

[23] Ekholm, K., Markusen, J., Forslid, R., 2005. Export-Platform Foreign Direct Investment. Journal of European Economic Association, 5 (4), 776-795.

[24] Ethier, W.J, 1986. The Multinational Firm. The Quarterly Journal of Economics, 101 (4), 805-33.

[25] Feenstra, R.C., 1998. Integration of Trade and Disintegration of Production in the Global Economy. Journal of Economic Perspectives, 12 (4), pages 31-50.

[26] Grossman, G.M., Helpman, E., Szeidl, A., 2006. Optimal integration strategies for the multinational firm. Journal of International Economics, 70 (1), 216-238.

[27] Hanson, G.H., Mataloni, R.J., Slaughter, M.J., 2001. Expansion Strategies of U.S. Multinational Firms. NBER Working Papers 8433, National Bureau of Economic Research, Inc.

[28] Hanson, G.H., Mataloni, R.J., Slaughter, M.J., 2004. Vertical Production Networks in Multinational Firms. NBER Working Papers 9723, National Bureau of Economic Research, Inc.

[29] Head, K.,Ries, J., 2003. Heterogeneity and the FDI versus export decision of Japanese manufacturers. Journal of the Japanese and International Economies, 17 (4), 448-467.

[30] Head, K.,Ries, J., 2004. Exporting and FDI as Alternative Strategies. Oxford Review of Economic Policy, Oxford University Press, vol. 20(3), pages 409-423, Autumn.

[31] Head, K., Mayer, T., 2004. Market Potential and the Location of Japanese Investment in the European Union. The Review of Economics and Statistics, 86 (4), 959-972.

[32] Helpman, E., 1984. A Simple Theory of International Trade with Multinational Corporations. Journal of Political Economy, 92 (3), 451-71.

[33] Helpman, E., Melitz, M.J., Yeaple, S.R., 2004. Export versus FDI with Heterogeneous Firms. American Economic Review, 94 (1), 300-316.

[34] Horstmann, I.J., Markusen, J.R., 1992. Endogenous market structures in international trade (natura facit saltum). Journal of International Economics, 32 (1-2), 109-129.

[35] Hummels, D., Ishii, Jun, Yi, K., 2001. The nature and growth of vertical specialization in world trade. Journal of International Economics, 54 (1), 75-96.

[36] Irarrazabal, A., Moxnes, A., Opromolla, L.D., 2008. The Margins of Multinational Production and the Role of Intra-Firm Trade.

[37] Kleinert, J., Toubal, F., 2006. Distance costs and Multinationals' foreign activities. CEI Working Paper.

[38] Krugman, P., 1980. Scale Economies, Product Differentiation, and the Pattern of Trade. American Economic Review, 70 (5), 950-59.

[39] Markusen, J.R., 1984. Multinationals, multi-plant economies, and the gains from trade. Journal of International Economics, 16 (3-4), 205-226.

[40] Melitz, M. J., 2003. The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity. Econometrica, 71 (6), 1695-1725.

[41] Melitz, J.M., Ottaviano, G.I.P., 2005. Market Size, Trade, and Productivity. NBER Working Papers 11393, National Bureau of Economic Research, Inc.

[42] Neary, Peter J., 2007. Trade Costs and Foreign Direct Investment. CEPR Discussion Papers 5933, C.E.P.R. Discussion Papers.

[43] Ramondo, N. 2012. A Quantitative Approach to Multinational Production. (Previously circulated as "Size, Geography, and Multinational Production". First version: November 2005).

[44] Yeaple, S. R. , 2003. The Role of Skill Endowments in the Structure of U.S. Outward Foreign Direct Investment. The Review of Economics and Statistics, MIT Press, vol. 85(3), pages 726-734, 07.

9 Appendix

A.1. Proposition 1

Proof. Total exports from i to j are given by:

$$X_{ij}^{X} = w_i L_i \overline{a}_{ij}^{\overline{a}_{M,ij}} x_{ij}^{X} dG(a)$$

a firm will be exporting if $a(v) \leq \overline{a}_{ij}$. Using (11), (12), (9) and (10) and the specific assumption about the distribution of the labor unit requirement, a, we obtain:

$$X_{ij}^{X} = w_i L_i \overline{a}_{ij}^{\overline{a}_{M,ij}} \lambda_3 \times \theta_j^{\sigma-1} \times \left(\frac{Y_j}{Y}\right)^{\frac{1}{b}} \times (1+\pi)^{\frac{1}{b}} \times (w_i \tau_{ij})^{1-\sigma} \times a^{1-\sigma} dG(a)$$

$$\text{with } \overline{a}_{ij}^{1-\sigma} = \lambda_4 \frac{w_j f_{ij}}{(w_i \tau_{ij})^{1-\sigma}} \theta_j^{1-\sigma} \left(\frac{Y}{Y_j}\right)^{\frac{1}{b}} (1+\pi)^{-\frac{1}{b}}$$

$$\text{and } \overline{a}_{M,ij}^{1-\sigma} = \lambda_4 \frac{w_j f_M - w_j f_{ij}}{\left(w_j^{1-\eta} (w_i \tau_{kj})^{\eta}\right)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}} \theta_j^{1-\sigma} \left(\frac{Y}{Y_j}\right)^{\frac{1}{b}} (1+\pi)^{-\frac{1}{b}}$$

Using the assumption of the Pareto distribution and the productivity thresholds, we can then solve the integral and find (15). ■

A.2. Proposition 2

Proof. Total affiliate sale in country j are given by:

$$X_{ij}^{M} = w_i L_i \overline{a}_{M,ij}^{\overline{a}_{M,ij}} x_{ij}^{M} dG(a)$$

a firm will open a subsidiary in country j if $a(v) \leq \overline{a}_{M,ij}$. Using (12) and (7) and the specific assumption about the distribution of the labor unit requirement, a, we obtain:

$$X_{ij}^{M} = w_i L_i \overline{a}_{M,ij}^{a_{M,ij}} \lambda_3 \times \theta_j^{\sigma-1} \times \left(\frac{Y_j}{Y}\right)^{\frac{1}{b}} \times (1+\pi)^{\frac{1}{b}} \times \left(w_j^{1-\eta} (w_i \tau_{ij})^{\eta}\right)^{(1-\sigma)} \times a^{1-\sigma} dG(a)$$

$$\text{with } \overline{a}_{M,ij}^{1-\sigma} = \lambda_4 \frac{w_j f_M - w_j f_{ij}}{\left(w_j^{1-\eta} (w_i \tau_{kj})^{\eta}\right)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}} \theta_j^{1-\sigma} \left(\frac{Y}{Y_j}\right)^{\frac{1}{b}} (1+\pi)^{-\frac{1}{b}}$$

then solving the integral we get (16). ■

A.3. Profits

In what follows we determine the dividend per share in the economy. In order to do this we use the total profits from exporting from i to j (inlcuding also trade within a country):

$$\begin{aligned} \Pi_{ij} &= w_i L_i \left[\frac{1}{\sigma} x_{ij} dG(a) - w_j f_{ij} dG(a)\right] \\ &= \frac{X_{ij}}{\sigma} - w_j f_{ij} w_i L_i dG(a) \end{aligned}$$

Note that when $i = j$, this expression represents domestic profit.[25] Since $n_{ij} = w_i L_i a_{M,ij}^{a_{ij}} dG(a)$, the expression above can be rewritten as

$$\Pi_{ij} = \frac{X_{ij}}{\sigma} - n_{ij} w_j f_{ij} \tag{23}$$

The total profits for country j's affiliates are:

$$\begin{aligned} \Pi_{ij}^M &= w_i L_i \frac{1}{\sigma} x_{ij}^M dG(a) - w_j f_{Mj} dG(a) \\ &= \frac{X_{ij}^M}{\sigma} - n_M w_j f_{Mj} \end{aligned} \tag{24}$$

since $n_M = w_i L_i {}_0^{a_{M,ij}} dG(a)$.

Total profits in this economy are

$$\begin{aligned} \Pi &= \sum_i \sum_j \left(\Pi_{ij} + \Pi_{ij}^M\right) \\ &= \sum_i \sum_j \left[\left(\frac{X_{ij}}{\sigma} + \frac{X_{ij}^M}{\sigma} \right) - \left(n_{ij} w_j f_{ij} + n_M w_j f_{Mj}\right) \right] \end{aligned}$$

this expression is the sum of the overall profits produced by domestic, exporting and FDI firms in every country. Remember that country j is receiving varieties from N-1. More specifically, total sales in country j are determined by varieties sold by domestic firms, varieties exported to j, and varieties produced locally by foreign affiliates. Hence, total import in country j are $\sum_i \left(X_{ij} + X_{ij}^M\right) = Y_j$, where we used the fact that trade is balanced. Substituting the equilibrium number of exporters and affiliates we can rewrite the worldwide profits as:

$$\Pi = \sum_j \left[\frac{Y_j}{\sigma} - c_4^{-b} Y_j \right] = Y \frac{1 - c_4^{-b}}{\sigma}$$

Hence dividends per share are:

$$\begin{aligned} \pi &= \frac{\Pi}{\sum_i w_i L_i} = \frac{\Pi}{Y}(1+\pi) = \frac{1 - c_4^{-b}}{\sigma}(1+\pi) \\ &= \frac{\frac{1 - c_4^{-b}\sigma}{\sigma}}{\left(1 - \frac{1 - c_4^{-b}\sigma}{\sigma}\right)} \end{aligned}$$

9.0.1 A.4. Price Index

The price index is

$$P_j^{1-\sigma} = (\sigma/(\sigma-1))^{1-\sigma} \times k/(k-\sigma+1) \times$$

[25] If we are interested in the domestic profits from serving market i we should compute: $\Pi_{ii} = w_i L_i {}_0^{a_{ii}} \frac{1}{\sigma} x_{ii} dG(a) - {}_0^{a_{ii}} f_{ii} dG(a)$. We should procede in the same way for computing the number of firms entering a particular market i: $N_{ii} = w_i L_i {}_0^{a_{ii}} dG(a)$. This expression delivers the overall number of firms existing in i.

$$\sum_{k=1}^{N} w_k L_k \left[a_{M,kj}^{k-\sigma+1} \left[\left(w_j^{1-\eta} \left(w_k \tau_{kj} \right)^{\eta} \right)^{1-\sigma} - \left(w_k \tau_{kj} \right)^{1-\sigma} \right] + a_{kj}^{k-\sigma+1} \left(w_k \tau_{kj} \right)^{1-\sigma} \right]$$

Plugging the productivity thresholds from (6) and (7) we can solve for the price index in the destination country j,

$$P_j^{1-\sigma} = (\sigma/(\sigma-1))^{1-\sigma} \times k/(k-\sigma+1) \times \sum_{k=1}^{N} w_k L_k \times$$

$$\left\{ \left[\lambda_1 \frac{w_j f_{M,kj} - w_j f_{kj}}{Y_j} \frac{P_j^{1-\sigma}}{\left(w_j^{1-\eta} \left(w_k \tau_{kj} \right)^{\eta} \right)^{1-\sigma} - \left(w_k \tau_{kj} \right)^{1-\sigma)}} \right]^{1-b} \left[\left(w_j^{1-\eta} \left(w_k \tau_{kj} \right)^{\eta} \right)^{(1-\sigma)} - \left(w_k \tau_{kj} \right)^{1-\sigma} \right] + \right.$$

$$\left. \left[\lambda_1 \frac{w_j f_{kj}}{Y_j} \frac{P_j^{1-\sigma}}{\left(w_k \tau_{kj} \right)^{1-\sigma}} \right]^{1-b} \left(w_k \tau_{kj} \right)^{1-\sigma} \right\}$$

where $b = k/(\sigma-1)$, w_k is the wage paid to workers in country k for firms which are exporting the good, while w_j is the wage paid to the workers in country j which are producing the domestic varieties or the foreign affiliate varieties. Then solving for $P_j^{1-\sigma}$

$$P_j^{b(1-\sigma)} = (\sigma/(\sigma-1))^{1-\sigma} \times k/(k-\sigma+1) \times \lambda_1^{1-b} \times (Y_j)^{b-1} \times$$

$$\sum_{k=1}^{N} w_k L_k \left[\left(w_j f_{M,kj} - w_j f_{kj} \right)^{1-b} \left[\left(w_j^{1-\eta} \right)^{1-\sigma} {w_k}^{\eta(1-\sigma)} \phi_{kj}^{\eta} - \left(w_k \right)^{1-\sigma} \phi_{kj} \right]^{b} + \left[w_j f_{kj} \right]^{1-b} \left(\left(w_k \right)^{1-\sigma} \phi_{kj} \right)^{b} \right]$$

where $\phi_{kj} = \tau_{kj}^{1-\sigma}$.

$$P_j = \left[(\sigma/(\sigma-1))^{1-\sigma} \times (k/(k-\sigma+1)) \times \lambda_1^{1-b} \right]^{\frac{1}{b(1-\sigma)}} \times (Y_j)^{\frac{b-1}{b(1-\sigma)}} \times$$

$$\left[\sum_{k=1}^{N} \frac{Y_K}{Y} \frac{Y}{1+\pi} \left[\left(w_j f_{M,kj} - w_j f_{kj} \right)^{1-b} \left[\left(w_j^{1-\eta} \right)^{1-\sigma} {w_k}^{\eta(1-\sigma)} \phi_{kj}^{\eta} - \left(w_k \right)^{1-\sigma} \phi_{kj} \right]^{b} + \left[w_j f_{kj} \right]^{1-b} \left(\left(w_k \right)^{1-\sigma} \phi_{kj} \right)^{b} \right] \right]$$

which after rearrangements becomes:

$$P_j = \lambda_2 Y_j^{\frac{b-1}{b(1-\sigma)}} \theta_j \left(\frac{Y}{1+\pi} \right)^{\frac{1}{b(1-\sigma)}} \tag{25}$$

A.5. Intensive and Extensive Margin Elasticities

In what follows we derive in details the Intensive and Extensive Margins of FDI and trade.

A.5.1 Intensive and Extensive Margins of FDI

1) Rearranging the definition of intensive and extensive margins of FDI we get

$$-\frac{\partial X_{ij}^M}{\partial \tau_{ij}} \frac{\tau_{ij}}{X_{ij}^M} = \underbrace{-\frac{\tau_{ij}}{X_{ij}^M} \left(w_i L_{i0}^{\overline{a}_{M,ij}} \frac{\partial x_{ij}^M}{\partial \tau_{ij}} dG(a) \right)}_{\text{Intensive Margin Elasticity}} \underbrace{-\frac{\tau_{ij}}{X_{ij}^M} \left(w_i L_i x_{ij}^M G' \left(\overline{a}_{M,ij} \right) \frac{\partial \overline{a}_{M,ij}}{\partial \tau_{ij}} \right)}_{\text{Extensive Margin Elasticity}}$$

Using the definition of equilibrium individual affiliate sales, (12), and assuming that country i is small enough so that $\partial\theta_j^{\sigma-1}/\partial\tau_{ij} \approx 0$, we get:

$$\begin{aligned}\frac{\partial x_{ij}^M}{\partial \tau_{ij}} &= \eta(1-\sigma)\tau_{ij}^{\eta(1-\sigma)-1}\left(w_j^{1-\eta}(w_i d_{ij})^{\eta}\right)^{1-\sigma}\lambda_3\theta_j^{\sigma-1}\times\left(\frac{Y_j}{Y}\right)^{\frac{1}{b}}\times(1+\pi)^{\frac{1}{b}}\times a^{1-\sigma}\\ &= \eta(1-\sigma)\frac{x_{ij}^M}{\tau_{ij}}\end{aligned}$$

Hence it is easy to find the elasticity of the intensive margin of FDI with respect to the variable costs:

$$\begin{aligned}\varepsilon_{I,\tau_{ij}}^M &= -\frac{\tau_{ij}}{X_{ij}^M}\left(w_i L_{i0}^{\overline{a}_{M,ij}}\frac{\partial x_{ij}^M}{\partial\tau_{ij}}dG(a)\right)\\ &= -\eta(1-\sigma)\frac{\tau_{ij}}{X_{ij}^M}\frac{w_i L_{i0}^{\overline{a}_{M,ij}}x_{ij}^M dG(a)}{\tau_{ij}}\\ &= \eta(\sigma-1)\end{aligned}$$

2) Using the definition of the equilibrium productivity threshold from (10), we find:

$$\begin{aligned}\frac{\partial\overline{a}_{M,ij}}{\partial\tau_{ij}} &= -\overline{a}_{M,ij}\frac{\left(\eta\frac{\left(w_j^{1-\eta}(w_i\tau_{ij})^{\eta}\right)^{1-\sigma}}{\tau_{ij}}-\frac{(w_i\tau_{ij})^{1-\sigma}}{\tau_{ij}}\right)}{\left(w_j^{1-\eta}(w_i\tau_{ij})^{\eta}\right)^{1-\sigma}-(w_i\tau_{ij})^{1-\sigma})}\\ &= -\frac{\overline{a}_{M,ij}}{\tau_{ij}}\underbrace{\frac{\left(\eta\left(w_j^{1-\eta}(w_i\tau_{ij})^{\eta}\right)^{1-\sigma}-(w_i\tau_{ij})^{1-\sigma}\right)}{\left(w_j^{1-\eta}(w_i\tau_{ij})^{\eta}\right)^{1-\sigma}-(w_i\tau_{ij})^{1-\sigma})}}_{\Gamma}\end{aligned}\tag{26}$$

The sign of this derivative is ambiguous. It is positive for low level of τ, but than when τ increases it becomes negative. If the elasticity of substitution is high, the ambiguity is preserved only if τ or/and η are sufficiently low.

Using the definition of firm level affiliate sales, (12), we can write $x_{ij}^M = \lambda_{ij}^M a^{1-\sigma}$. Then since the Pareto distribution assumption implies that $G'(a) = ka^{k-1}$, we can rewrite the aggregate affiliate sales in the following way:

$$\begin{aligned}X_{ij}^M &= w_i L_{i0}^{\overline{a}_{M,ij}}x_{ij}^M dG(a)\\ &= w_i L_{i0}^{\overline{a}_{M,ij}}\lambda_{ij}^M a^{1-\sigma}ka^{k-1}da\\ &= w_i L_i\lambda_{ij}^M\overline{a}_{M,ij}^{1-\sigma}\overline{a}_{M,ij}^k\left(k/(k-\sigma+1)\right)\\ &= w_i L_i x_{ij}^M G'(\overline{a}_{M,ij})\frac{\overline{a}_{M,ij}}{k-\sigma+1}\end{aligned}\tag{27}$$

where we used the fact that $\overline{a}_{M,ij}G'(\overline{a}_{M,ij}) = k\overline{a}_{M,ij}^k$. Using (27) we can find a simple solution

for the elasticity of the extensive margin:

$$\begin{aligned}
\varepsilon^{M}_{E,\tau_{ij}} &= -\frac{\tau_{ij}}{X^{M}_{ij}}\left(w_i L_i x^{M}_{ij} G'\left(\overline{a}_{M,ij}\right)\frac{\partial \overline{a}_{M,ij}}{\partial \tau_{ij}}\right) \\
&= -\frac{\tau_{ij}}{X^{M}_{ij}} w_i L_i x^{M}_{ij} G'\left(\overline{a}_{M,ij}\right)\left(-\overline{a}_{M,ij}\frac{\eta\left(\frac{\left(w_j^{1-\eta}(w_i\tau_{ij})^{\eta}\right)^{1-\sigma}}{\tau_{ij}} - \frac{(w_i\tau_{ij})^{1-\sigma}}{\tau_{ij}}\right)}{\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma} - (w_i\tau_{ij})^{1-\sigma})}\right) \\
&= -\frac{\tau_{ij}}{X^{M}_{ij}}\frac{X^{M}_{ij}}{\tau_{ij}}\left(k-\sigma+1\right)\frac{-\left(\eta\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma} - (w_i\tau_{ij})^{1-\sigma}\right)}{\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma} - (w_i\tau_{ij})^{1-\sigma})} \\
&= \left(k-\sigma+1\right)\frac{\left(\eta\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma} - (w_i\tau_{ij})^{1-\sigma}\right)}{\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma} - (w_i\tau_{ij})^{1-\sigma})}
\end{aligned}$$

A.5.2 Intensive and Extensive Margins of FDI Trade.

1) Rearranging the definition of intensive and extensive margins of trade we get

$$-\frac{\partial X^{X}_{ij}}{\partial \tau_{ij}}\frac{\tau_{ij}}{X^{X}_{ij}} = \underbrace{-\frac{\tau_{ij}}{X^{X}_{ij}}\left(w_i L_i \frac{\overline{a}_{ij}}{\overline{a}_{M,ij}}\frac{\partial x^{X}_{ij}}{\partial \tau_{ij}} dG(a)\right)}_{\text{Intensive Margin Elasticity}} \underbrace{-\frac{\tau_{ij}}{X^{X}_{ij}} w_i L_i\left[x^{X}_{ij} G'\left(\overline{a}_{ij}\right)\frac{\partial \overline{a}_{ij}}{\partial \tau_{ij}} - x^{M}_{ij} G'\left(\overline{a}_{M,ij}\right)\frac{\partial \overline{a}_{M,ij}}{\partial \tau_{ij}}\right]}_{\text{Extensive Margin Elasticity}}$$

Using the definition of equilibrium individual affiliate sales, (11), and assuming that country i is small enough so that $\partial \theta_j^{\sigma-1}/\partial \tau_{ij} \approx 0$, we get:

$$\begin{aligned}
\frac{\partial x^{X}_{ij}}{\partial \tau_{ij}} &= \left(1-\sigma\right)\tau_{ij}^{-\sigma}(w_i)^{1-\sigma}\lambda_3\theta_j^{\sigma-1} \times \left(\frac{Y_j}{Y}\right)^{\frac{1}{b}} \times (1+\pi)^{\frac{1}{b}} \times a^{1-\sigma} \\
&= \left(1-\sigma\right)\frac{x^{X}_{ij}}{\tau_{ij}}
\end{aligned}$$

Hence, the elasticity of the intensive margin of trade with respect to the variable costs is:

$$\begin{aligned}
\varepsilon^{X}_{I,\tau_{ij}} &= -\frac{\tau_{ij}}{X^{X}_{ij}}\left(w_i L_i \frac{\overline{a}_{ij}}{\overline{a}_{M,ij}}\frac{\partial x^{X}_{ij}}{\partial \tau_{ij}} dG(a)\right) \\
&= -\left(1-\sigma\right)\frac{\tau_{ij}}{X^{M}_{ij}}\frac{w_i L_i \frac{\overline{a}_{ij}}{\overline{a}_{M,ij}} x^{X}_{ij} dG(a)}{\tau_{ij}} \\
&= \left(\sigma-1\right)
\end{aligned}$$

which is identical to the elasticity in Chaney (2008).

2) In order to derive the extensive margin of trade we need to use the equilibrium productivity thresholds from (9)and (10). Deriving these thresholds with respect to τ_{ij} we find:

$$\frac{\partial \overline{a}_{M,ij}}{\partial \tau_{ij}} = -\overline{a}_{M,ij}\Gamma/\tau_{ij}$$

$$\frac{\partial \overline{a}_{ij}}{\partial \tau_{ij}} = -\frac{\overline{a}_{ij}}{\tau_{ij}}$$

Using the definition of firm exports, $x_{ij}^X = \lambda_{ij}^X \overline{a}_{ij}^{1-\sigma}$, we can rewrite affiliate sales as:

$$x_{ij}^M = \underbrace{\lambda_{ij}^X \times \frac{\left(w_j^{1-\eta}\right)^{1-\sigma}}{\left((w_i\tau_{ij})^{1-\eta}\right)^{1-\sigma}}}_{\lambda_{ij}^M} \times \overline{a}_{M,ij}^{1-\sigma}$$

then since the Pareto distribution assumption implies that $G'(a) = ka^{k-1}$, we can rewrite the aggregate export and affiliate sales in the following way:

$$\begin{aligned}
X_{ij}^X &= w_i L_i \overline{a}_{M,ij}^{\overline{a}_{ij}} x_{ij}^X dG(a) \\
&= w_i L_i \overline{a}_{M,ij}^{\overline{a}_{ij}} \lambda_{ij}^X a^{1-\sigma} k a^{k-1} da \\
&= w_i L_i \lambda_{ij}^X \left(k/(k-\sigma+1)\right) \left[\lambda_{ij}^X \overline{a}_{ij}^{1-\sigma} - \lambda_{ij}^X \overline{a}_{M,ij}^{1-\sigma} \overline{a}_{M,ij}^{k}\right] \\
&= w_i L_i \left(1/(k-\sigma+1)\right) \left[x_{ij}^X G'(\overline{a}_{ij})\, \overline{a}_{ij} - x_{ij}^M \underbrace{\left[\left((w_i\tau_{ij})^{1-\eta}\right)^{1-\sigma} / \left(w_j^{1-\eta}\right)^{1-\sigma}\right]}_{\omega} G'(\overline{a}_{M,ij})\, \overline{a}_{M,ij}\right] \\
&= w_i L_i \left(1/(k-\sigma+1)\right) x_{ij}^X G'(\overline{a}_{ij})\, \overline{a}_{ij} - w_i L_i \left(\omega/(k-\sigma+1)\right) x_{ij}^M G'(\overline{a}_{M,ij})\, \overline{a}_{M,ij} \qquad (28)
\end{aligned}$$

where using the relationship between λ_{ij}^X and λ_{ij}^M, we define $\lambda_{ij}^X \overline{a}_{M,ij}^{1-\sigma} = x_{ij}^M \left[\left((w_i\tau_{ij})^{1-\eta}\right)^{1-\sigma} / \left(w_j^{1-\eta}\right)^{1-\sigma}\right]$.

Using (28) we can find a simple solution for the elasticity of the extensive margin:

$$\begin{aligned}
\varepsilon_{E,\tau_{ij}}^X &= -\frac{\tau_{ij}}{X_{ij}^X} w_i L_i \left[x_{ij}^X G'(\overline{a}_{ij}) \frac{\partial \overline{a}_{ij}}{\partial \tau_{ij}} - x_{ij}^M G'(\overline{a}_{M,ij}) \frac{\partial \overline{a}_{M,ij}}{\partial \tau_{ij}}\right] \\
&= -\frac{\tau_{ij}}{X_{ij}^X} w_i L_i \left[x_{ij}^X G'(\overline{a}_{ij}) \left(-\frac{\overline{a}_{ij}}{\tau_{ij}}\right) - x_{ij}^M G'(\overline{a}_{M,ij}) \frac{\Gamma}{\tau_{ij}}\right]
\end{aligned}$$

from (27) we know that:

$$w_i L_i x_{ij}^M G'(\overline{a}_{M,ij})\, \overline{a}_{M,ij} = (k-\sigma+1)\, X_{ij}^M \qquad (29)$$

while from (28) we have:

$$\begin{aligned}
w_i L_i x_{ij}^X G'(\overline{a}_{ij})\, \overline{a}_{ij} &= (k-\sigma+1)\left[X_{ij}^X + w_i L_i \left(\omega/(k-\sigma+1)\right) x_{ij}^M G'(\overline{a}_{M,ij})\, \overline{a}_{M,ij}\right] \\
&= (k-\sigma+1)\left[X_{ij}^X + \omega X_{ij}^M\right] \qquad (30)
\end{aligned}$$

Using the expressions (29) and (30) inside $\varepsilon^X_{E,\tau_{ij}}$ we get:

$$\begin{aligned}
\varepsilon^X_{E,\tau_{ij}} &= -\frac{\tau_{ij}}{X^X_{ij}}\left[(k-\sigma+1)\left[X^X_{ij}+\omega X^M_{ij}\right]\left(-\frac{1}{\tau_{ij}}\right)-(k-\sigma+1)X^M_{ij}\left(-\frac{\Gamma}{\tau_{ij}}\right)\right] \\
&= -\frac{\tau_{ij}}{X^X_{ij}}(k-\sigma+1)\frac{1}{\tau_{ij}}\left[-\left(X^X_{ij}+\omega X^M_{ij}\right)+X^M_{ij}\Gamma\right] \\
&= -\frac{1}{X^X_{ij}}(k-\sigma+1)\left[X^M_{ij}(\Gamma-\omega)-X^X_{ij}\right] \\
&= -(k-\sigma+1)\left[\frac{X^M_{ij}}{X^X_{ij}}(\Gamma-\omega)-1\right]
\end{aligned}$$

where $\Gamma > \omega$ always[26]. Therefore we can conclude the following:

$$\begin{aligned}
\text{if } X^M_{ij} &> X^X_{ij} \longrightarrow \varepsilon^X_{E,\tau_{ij}} < 0 \\
\text{if } X^M_{ij} &< X^X_{ij} \longrightarrow \varepsilon^X_{E,\tau_{ij}} > 0
\end{aligned}$$

A.6. Market Share and Revenues

The value of export is

$$\begin{aligned}
p_{ij} &= \frac{\sigma}{\sigma-1}aw_i\tau_{ij} \\
q_{M,ij} &= \frac{Y_j}{P_J^{1-\sigma}}p_{ij}^{-\sigma}
\end{aligned}$$

So the value of affiliate sales is:

$$p_{ij}\times q_{ij} = a^{1-\sigma}(w_i\tau_{ij})^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\frac{Y_j}{P_J^{1-\sigma}} \tag{31}$$

Usign the threshold profit:

$$\pi_{ij} = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)}\frac{Y_j}{P_j^{1-\sigma}}a_{ij}^{1-\sigma}(w_i\tau_{ij})^{1-\sigma} - w_jf_{ij}$$

rearranging the above expression we find:

$$\left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)}\frac{Y_j}{P_j^{1-\sigma}} = \frac{\sigma}{a_{ij}^{1-\sigma}}\frac{w_jf_{ij}}{(w_i\tau_{ij})^{1-\sigma}} \tag{32}$$

Substituting (32) into (31) we find the revenues of the exporting firm:

$$\begin{aligned}
r_{ij} &= \sigma\frac{a^{1-\sigma}}{a_{ij}^{1-\sigma}}(w_i\tau_{ij})^{1-\sigma}\frac{w_jf_{ij}}{(w_i\tau_{ij})^{1-\sigma}} &(33)\\
&= \sigma\frac{a^{1-\sigma}}{a_{ij}^{1-\sigma}}w_jf_{ij} &(34)
\end{aligned}$$

[26]Remember that $\Gamma = \frac{\left(\eta\frac{\left(w_j^{1-\eta}\left(w_id_{ij}\tau_{ij}\right)^{\eta}\right)^{1-\sigma}}{\tau_{ij}} - \frac{\left(w_id_{ij}\tau_{ij}\right)^{1-\sigma}}{\tau_{ij}}\right)}{\left(w_j^{1-\eta}\left(w_id_{ij}\tau_{ij}\right)^{\eta}\right)^{1-\sigma} - \left(w_id_{ij}\tau_{ij}\right)^{1-\sigma}}$ and $\omega = \left[\left(\left(w_id_{ij}\tau_{ij}\right)^{1-\eta}\right)^{1-\sigma} / \left(w_j^{1-\eta}\right)^{1-\sigma}\right]$

Trade liberalization reduces the productivity threshold to become an exporter, $a_{ij}^{(1-\sigma)'} < a_{ij}^{1-\sigma}$, hence the revenue from exporting increases with trade liberalization, $r'_{ij} > r_{ij}$.

9.0.2 A.7. Figures

Figure 1:

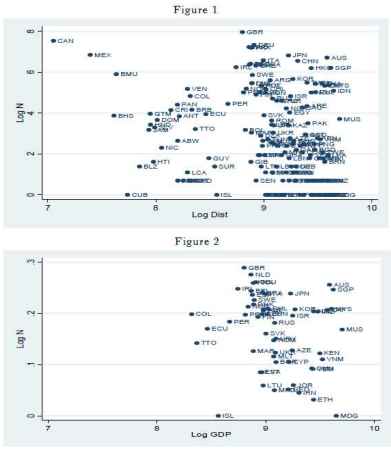


Figure 2:

Figure 3

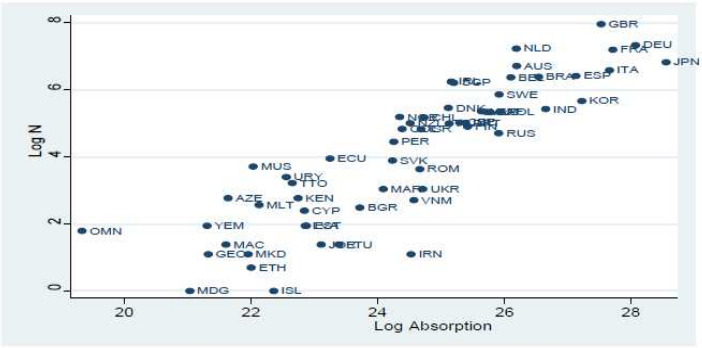


Figure 4

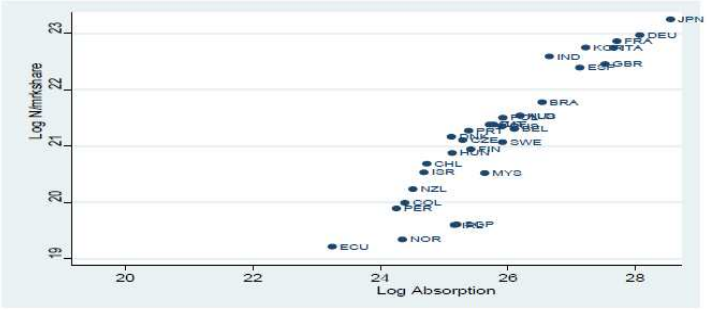


Figure 3:

Figure 5a

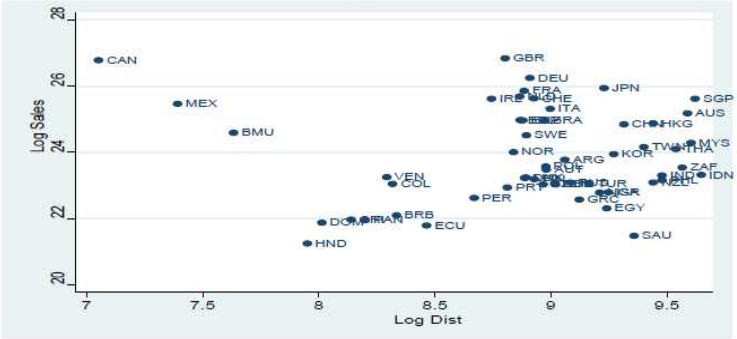
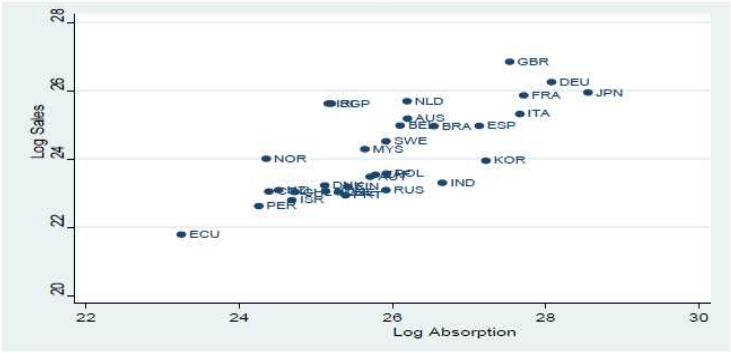
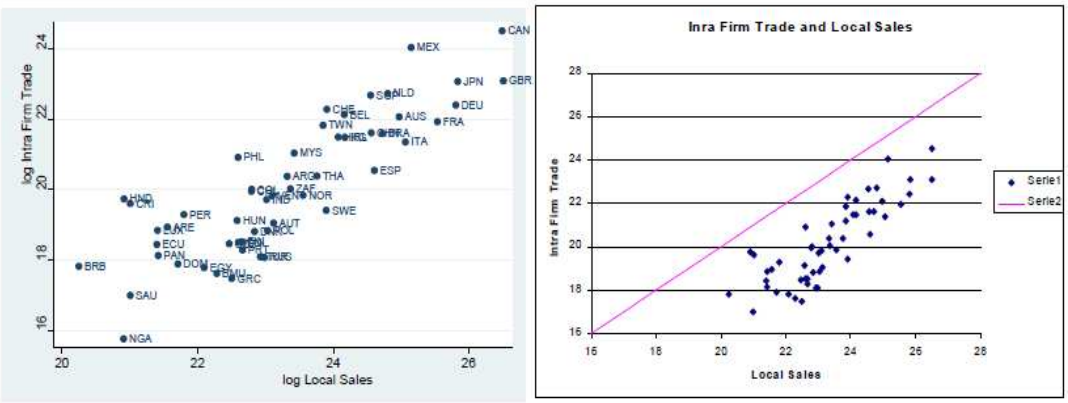


Figure 5b



Relationship between intra firm trade and local affiliate sales.

Figure 4:



All points lie below the 45 degree line indicating an increase in the value of the good sold by the affiliate in j.