# Multi-product firms in monopolistic competition: consequences of opening trade

S. Kokovin, Ph. Ushchev, E. Zhelobodko

National Research University Higher School of Economics

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**Abstract**

We develop a monopolistic competition model of an industry with multi-product firms. Both outputs and product ranges are endogenous. The technological side of the model allows for positive scale-scope spillovers: firms with broader product ranges have lower marginal production costs. We focus on how an increase in the market size (which can occur due to opening trade) affects the market outcome. Opening trade always leads to a reduction in prices, an increase in outputs (intensive margins) and a the industry-level product variety always increases. However, firm-level product ranges can increase or decrease depending on the intensity of scale-scope spillovers. We find that sufficiently high spillovers are necessary and sufficient for firm-level product ranges to increase as the market size increases. We also point out that under high (low) spillovers the number of firms in the industry increases less (more) than proportionally to an increase in market size.

## 1 Introduction

Single-product monopolistic competition models come from Spence (1976) and Dixit and Stiglitz (1977). Research based on these models provided numerous insights in international trade theory, spatial economics and endogenous growth theory. However, empiricists find a lot of multi-product firms (hereafter, MPFs). As indicated by

Bernard, Redding and Schott (2010), multi-product firms account for 91 percent of sales in the US. Accordingly, there are a number of empirical findings to be explained.

One group of stylized facts on multi-product firms is related to simultaneous choice of scope and output by firms. For example, Bernard, Redding and Schott (2010) report a strong positive correlation between the extensive margins (product ranges) and intensive margins (average output per variety) for US manufacturing firm-level panel data. Goldberg et al. (2008) find the same regularity in the Indian data.

Another group of empirical findings has to do with the interaction between economies of scale and economies of scope. For instance, Henderson and Cockburn (1996) and Cockburn and Henderson (2001) report positive correlation between firm sizes and research projects efficiency in farmaceutical industry.

In this paper we claim that stylized facts from the second group **explain** (at least partly) those from the first one. In order to justify our message, we propose a model of monopolistic competition with multi-product firms. The technological side of the model allows for positive scale-scope spillovers: firms which choose wider product ranges tend to be more efficient, i.e. they have lower marginal production costs.

As already mentioned above, empiricists normally find positive correlation between outputs and scopes of multi-product firms. However, theoretical papers where firms are assumed to be homogenous often reveal the reverse correlation: the expansion of the product line scope leads to a shrinking in demand for existing varieties. This regularity is often due to the so called "cannibalization" effect. A popular way to reconcile theory with data is to take heterogeneity of firms into consideration. We argue that economies of scope in production combined with well-behaved (but not CES) utility functions can generate empirically justified outcomes even in a homogenous world.

Our goal in this paper is to disentangle the impact of market size variations on the conduct of multi-product firms. We consider an economy endowed by identical consumers with additively separable preferences of general type. The production side of the economy is formed by a continuum of identical multi-product firms, each of them being negligibly small in comparison with the rest of the industry. Like in most models of monopolistically competitive markets, entry is assumed to be endogenous. Our setting displays a one-sector, one-factor economy. The horizontally differentiated good involves a continuum of varieties. The total mass of varieties is given by the (endogenous) mass of firms multiplied by the (endogenous) firm-level product range. The latter is chosen by firms together with their output.

On the demand side, each consumer is endowed with additively separable preferences across varieties. The love for variety is measured by the elasticity of the marginal utility derived from consuming a variety. Our approach to modeling preferences is close to the two-tier nested CES adopted by Allanson and Montagna (2005). However, following Zhelobodko et al. (2011), we drop the CES assumption conventional in monopolistic competition theory. The major reason why we allow for variable elasticity of substitution across varieties is that the model exhibits both nontrivial and tractable comparative statics with respect to the market size. Criticism the CES preferences are subject to is considerably due to their inability of catching the dependence of markups and firm sizes on the market size.

On the supply side, we consider a technology modeled by means of a cost function. Variable costs depend on total output (firm's size) and firm's product line scope. In contrast with most of the papers cited above, we do not impose any restrictions on cost function specification. The cost function is assumed to be increasing and convex with respect to both the firm's size and the scope. The design of new varieties requires costs irrespective of the output. These costs may be interpreted either as R&D costs or as monitoring costs. At the same time marginal production costs are non-increasing in scope. We refer to this effect as **positive scale-scope spillover**. Contrast to an approach typical in the literature, we do not invoke constant marginal production costs under a given number of varieties produced. Such a generalization seems to make sence, for it provides a simple way to explicitly introduce scale-scope spillovers.

As benchmark cases, we consider cost functions which are **additively separable**, either with respect to total output and firm's product line scope or across varieties. The first case corresponds to no scale-scope spillovers.

Our findings refer to comparative statics with respect to the market size. In response to an increase in market size, intensive margins (i.e. quantities supplied) always increase, prices decrease and total mass of varieties increases. As to the product line scope, firms' sizes (i.e. firm-level total outputs) and the number of firms, the set of possible patterns is broader. The key-factor is the strength of positive scale-scope spillovers, measured by the elasticity of marginal production costs with respect to the scope, or otherwise by the elasticity of marginal scope costs with respect to total output. When scale-scope spillovers are not sufficiently intensive, an increase in market size makes intensive and extensive margins move in the opposite directions. On the contrary, when technological linkages between scale and scope are close enough, the scope and output move in the same direction, which has substantial empirical support.

We also show that the association between outputs and scopes is always negative provided there are either negative or no scale-scope spillovers. The latter case is equivalent to additive separability of variable costs with respect to firm's total output and the scope. Additive separability of costs allows to obtain slightly stronger results compared to the general case. Namely, under this assumption we describe fully the patterns of the industry size (total output) behavior.

These results allow to explain cross-industries differences in scopes and outputs as well as to suggest a program of empirical work. We know that some industries include a big mass of small firms, while other have smaller number of firms but their product lines are very big (say, world tobacco industry produces hundreds of varieties with dozen of main firms). Do these differences depend mainly on characteristics of demand, on technological parameters or something else? Similar questions arise for cross-countries comparisons.

The rest of the paper is organized as follows. Section 2 outlines the place of our contribution in the literature on multi-product firms. Section 3 describes the layout of our model. In Section 4 we formulate the equilibrium conditions and report comparative statics of equilibrium with respect to the market size, i.e. study the consequences of agglomeration and/or opening trade. Section 5 provides more insight about the role of spillovers, dealing with some benchmark special cases of the general model. Section 6 concludes. Appendices contain non-intuitive proofs and technical details.

## 2 Contribution to the literature

In the literature on MPFs in industrial organization and trade, the early survey by Bailey and Friedlaender (1982) emphasized the role of scope economies. Later on, Brander and Eaton (1984), Shaked and Sutton (1990), Eaton and Schmidt (1994) and Johnson and Myatt (2003) focused on the demand-side and strategic interactions, whereas Klette and Kortum (2004) and Lentz and Mortensen (2005) stressed the role of innovation.

Ottaviano and Thisse (1999) consider the simultaneous choice of scope and output by multi-product firms in an oligopolistic industry under linear-quadratic preferences. They show that industry-level product variety is increasing with the market size (measured by the choke-off price). We find the same effect in our model, but use a disserent measure for the market size, namely the number of consumers. Firm-level product varieties in Ottaviano and Thisse are independent of the market size, which is not the case in our model: scopes can increase, remain unchanged or decrease

Allanson and Montagna (2005) develop a monopolistic competition model with homogeneous multi-product firms and two-tier nested CES preferences. This model is maybe the closest one to ours in the literature, but is different in two respects. First, Allanson and Montagna do not allow for scale-scope spillovers. Second, they use nested CES preferences. This leads to independence of market prices, outputs and firm-level product ranges of the market size, which is counterfactual. Contrast to this, our analysis does not rely on any parametric specification of preferences, which allows to obtain non-trivial and realistic comparative statics with respect to the market size.

Anderson and de Palma (2006) propose a logit model of oligopolistic competition between multi-product firms and are concerned mainly with welfare issues. One of their results is that prices are negatively associated with the number of firms, . However, we also find that scopes can be associated with prices Anderson and de Palma find that prices are independent of firms' scopes. This is not the case in our paper: we

Feenstra and Ma (2007) propose a model which is similar to Ottaviano and Thisse (1999), but they assume CES preferences and focus on free entry equilibria. Firm-level product range are increasing with respect to the market size, while quantities supplied remain unchanged (which seems to be a by-product of CES). The industry-level product range also increases, but less than proportionally, which is interpreted as a decrease in the number of firms in each country in the symmetric two-country world. This situation could be called an increase in concentration: opening trade leads to less firms with higher product ranges. In our model, we find that a decrease in concentration – scopes decrease whereas the number of firms increases more than proportionally – is also possible. The key-factor is the intensity of positive scale-scope spillovers.

Eckel and Neary (2010) also stress the role of heterogeneity of multi-product firms for the market outcome, but their approach to modeling heterogeneity differs substantially from the standard Melitz approach. Namely, each firm has a core competence in production of a particular variety. The population of a country is taken as a measure of market size. It is shown that an increase in the market size does not affect firm-level product ranges, though other globalization effects do. Alternatively, in our model market size affects equilibrium scopes: high (low) scale-scope spillovers exacerbate (suppress) firms' incentives to expand product ranges as market becomes larger.

Nocke and Yeaple (2012) consider an economy where firms face an unspecified partial equilibrium demand curve. Firms are assumed to be heterogenous in their organizational capabilities. The marginal production costs are constant provided the product line scope is given, whereas they decrease both in the scope and in organizational capabilities. Trade liberalization (defined as a reduction in tariffs) always pushes outputs and firm-level product diversities in the opposite directions. We, on the contrary, show that under sufficiently high scale-scope spillovers scopes and quantities supplied both increase in response to an increase in market size, which seems more in the line with the empirical findings, in particular those by Bernard, Redding and Schott (2010).

# 3 The model

We consider an economy involving one sector and one production factor – labour. The set of varieties is a two-dimensional continuum: each variety is identified by its producer $j \in [0, N]$ and the number $i \in [0, n_j]$ of variety within the product line of firm $j$. Here $N \in [0, \infty)$ is the *endogenous* mass of active firms and $n_j$ is the *endogenous* mass of varieties produced by firm $j$, or the *scope* of firm $j$. Each firm chooses its scope $n_j \in [0, \infty)$ and its production vector (measurable mapping)

$$\mathbf{q}_j = (q_{ij})_{i \in [0, n_j]}.$$

The commodity is horizontally differentiated across firms as well as within the product lines of the firms.

## 3.1 Consumers

The economy is endowed by $L$ identical consumers. Given a price vector $\mathbf{p} = (p_{ij})_{i \le n_j, j \le N}$, each consumer chooses her individual demand vector $\mathbf{x} = (x_{ij})_{i \le n_j, j \le N}$ in order to maximize her utility function subject to the budget constraint:

$$\max_{x_{ij}} \mathcal{U} = \int_0^N \int_0^{n_j} u(x_{ij})\, di\, dj \quad \text{s.t.} \quad \int_0^N \int_0^{n_j} p_{ij}\, x_{ij}\, di\, dj \leq 1,$$

where wage is normalized to one because labour is the numerarie.[1]

Utility $u$ is assumed strictly increasing, strictly concave and thrice continuously differentiable. To express the inverse elasticity of substitution among varieties, we exploit Arrow-Pratt measure of concavity:

$$\eta(x) = -\frac{x\, u''(x)}{u'(x)}.$$

We call $r_u$ the *relative love for variety* (RLV), for the curvature of $u$ measures relative desirability of a mixture of two bundles of goods.

To make monopolistic pricing possible, we assume

$$0 < \eta(x) < 1, \ \forall\, x \geq 0. \tag{1}$$

We also impose a technical restriction on preferences which ensures strict global concavity of profit:

$$-\frac{xu'''(x)}{u''(x)} < 2, \ \forall\, x \geq 0. \tag{2}$$

The inverse demand is found in a standard way from the first order condition (further on FOC). Due to the symmetry of preferences, the inverse demand functions are the same across varieties and are given by:

$$p_{ij} = \frac{u'(x_{ij})}{\lambda}, \tag{3}$$

where $\lambda$ is the marginal utility of income. The role of $\lambda$ in what follows is that it is a market aggregator. Because firms are non-atomic, neither of them can affect $\lambda$ by changing behavior individually.

[1]Note that all varieties here are assumed equally substitutable. By contrast, Alanson and Montagna (2005) assume within-firm varieties to be closer substitutes then varieties of different firms. However, we are not sure that big jeans of one firm can be a close substitute with small jeans of the same firm, and prefer a simpler assumption.

### 3.2 Producers

Each of $N$ identical firms chooses its product line scope $n_j$ and a production plan $\mathbf{q}_j : [0, n_j] \to \mathbb{R}_+$, which says how much of each product is produced.

Firms incur costs of two types:

- fixed costs $F > 0$;
- variable costs function $V(\mathbf{q}_j, n_j)$.

Both fixed costs $F$ and variable costs $V$ are assumed to be the same across firms. As firms are homogenous, it is plausible that symmetric equilibrium is the only possible outcome (see Appendix 1 for the proof). So, further on we drop the firm's index $j$, which is of no use.

Variable costs $V$ are assumed to be increasing and convex in each argument. To capture the idea that variable costs are symmetric across varieties, we also assume that $V$ satisfies the following *symmetry condition*:

$$V(\mathbf{q_1}, n) = V(\mathbf{q_2}, n). \tag{4}$$

where $\mathbf{q_2}$ is obtained from $\mathbf{q_1}$ by any renumbering of varieties.

An example of variable costs of type (4) is given by[2]:

$$V(\mathbf{q}, n) = v\,(y, n)\,, \tag{5}$$

where $y = \int_0^n q_i di$ is firm's total output, $v$ is an increasing and convex function.

Variable costs given by (5) are widely used in the literature. For example, Allanson and Montagna (2005) assume that $v(y, n) = \phi n$, whereas in Eckel and Neary (2010) and in Nocke and Yeaple (2012) the symmetrized cost function appears to be

$$v\,(y, n) = \phi n + c(n)y. \tag{6}$$

[2]We show in Appendix 1 that restriction to such cost functions does not lead to any loss of generality

Here $\phi$ stands for fixed costs per product line, whereas $c(n)$ stands for marginal production costs, which are the same for all varieties.

So long as we focus only on symmetric equilibria (in fact, asymmetric equilibria do not exist), we can restrict our attention only to symmetric configurations in which the quantity $q$ is the same across varieties. Setting

$$y = nq,$$

the variable costs may be rewritten as follows:

$$v(y, n) = V(\mathbf{q}, n)|_{\mathbf{q} \equiv y/n}.$$

We assume that $v$ is increasing and convex.

Each firm is a price-maker on the market of each variety it chooses to produce. The marginal utility of income $\lambda$ is taken as given because each producer is negligibly small in comparison with the rest of the industry. The value of $\lambda$ can be viewed as an aggregate market statistic which measures competitive pressure.

Firms maximize profits

$$\Pi(\mathbf{q}, n) = \int_0^n p_i q_i di - F - V(\mathbf{q}, n) \tag{7}$$

with respect to both production plan and the scope subject to the inverse demand functions $p_i = \frac{u'(x_i)}{\lambda}$.

Plugging the inverse demands into (7), we obtain the producer's problem:

$$\max_{\mathbf{q}, n} \Pi(\mathbf{q}, n) = \frac{1}{\lambda} \int_0^n u' \left( \frac{q_i}{L} \right) q_i di - F - V(\mathbf{q}, n). \tag{8}$$

It follows from (2) and (4) that:

- profit function $\Pi$ satisfies the symmetry condition (4);

- $\Pi$ is strictly concave with respect to $\mathbf{q}$.

These two considerations imply that each firm always chooses a symmetric production plan characterized by $q_i = q$ for all varieties[3] (see Appendix 2). So, what really matters is properties of $v(y, n)$.

We are now able to reformulate the producer's problem (8) as follows:

$$\max \pi(y,\, n) = \frac{1}{\lambda}\, y\, u'\left(\frac{y}{nL}\right) - v(y,\, n) - F. \tag{9}$$

The first term in (9) is the firm's total revenue:

$$R(y,\, n) = py = \frac{1}{\lambda}\, y\, u'\left(\frac{y}{Ln}\right). \tag{10}$$

Note that $R$ is homogenous of degree one with respect to $y$ and $n$. This observation will prove to be useful below.

The first order conditions for the producer's problem are:

$$R_y = v_y, \tag{11}$$

$$R_n = v_n, \tag{12}$$

where subscripts mean partial derivatives.

The second order condition reduces to $-\dfrac{xu'''(x)}{u''(x)} < 2$. See Appendix 3 for details.

### Scale-scope spillovers

It remains to describe the interaction between scale and scope economies. Most theoretical papers assume that these two variables are independent. However, as discussed in the introduction, empirical evidence strongly supports the existence of

[3] Formally, the equality should hold up to a set of varieties of zero Lebesgue measure.

positive spillovers between scale and scope economies. In our context, this means that the marginal production costs decrease with the size of the product range:

$$v_{yn} < 0.$$

To cover specifications which are in the literature, we require that weak inequality holds.

Note that externalities of this type are formally equivalent to those considered in Henderson, Cockburn (1996) and Cockburn, Henderson (2001), who report positive correlation between firm sizes and research projects efficiency in farmaceutical industry.

Note also that for variable costs given by (6) the assumption of scale-scope spillovers means $c'(n) \leq 0$[4].

# 4 Market outcome

## 4.1 Equilibrium conditions

To show that the problem we are dealing with is not vacuous, we start by establishing existence and uniqueness.

**Proposition 1.** *Assume that there exists some $\varepsilon > 0$ such that $\varepsilon < \eta(x) < 1 - \varepsilon$ $\forall x \geq 0$. Then:*

- *no asymmetric equilibria exist;*
- *a unique symmetric equilibrium exists.*

**Proof**. See Appendix 3.

[4] This is just the opposite to what Nocke and Yeaple assume. This might be the reason why their predictions run against empirical findings by Bernard et al. (2010).

Now we are ready to formulate the symmetric equilibrium conditions. Because of free entry, profits should vanish in equilibrium, i.e. total revenue should equal total costs:

$$R(y,\, n) \equiv py = v(y,\, n) + F. \tag{13}$$

Combining producer's FOCs (11) – (12) with zero profit condition (13), we get:

$$\frac{R_y y}{R} + \frac{R_n n}{R} = \frac{v_y y}{v+F} + \frac{v_n n}{v+F},$$

where the left-hand side is identically one due to homogeneity of the revenue function $R$. So we get:

$$\frac{v_y y}{v+F} + \frac{v_n n}{v+F} = 1. \tag{14}$$

The economic intuition behind (14) can be explained as follows: if a firm increases both total output and the scope by 1 percent (which implies that average output $q = y/n$ remains unchanged), there sould be a one-percent increase in total costs. Put another way, in equilibrium firms operate precisely at the boundary of economies and diseconomies of scope.

Now transform producer's FOC (11) in the following way:

$$\frac{1}{\lambda}\left(xu''(x) + u'(x)\right) = v_y,$$

where $x$ is the individual consumption level, $x = \frac{y}{nL}$.

From the inverse demand (3) we have:

$$\lambda = \frac{u(x)}{p}. \tag{15}$$

Plugging (15) into (11) yields:

$$p = \frac{v_y}{1-\eta}. \tag{16}$$

Relationship (16) is nothing but monopoly pricing formula.

To close the model, we require that labour balance. Because labour is the only production factor, labour balance identity reduces to:

$$N\ (v(y,\, n) + F) = L. \tag{17}$$

Equations (13) – (17) form a closed system of symmetric equilibrium conditions.

## 4.2 Comparative statics: prices, average outputs and total product variety

In this subsection we study the behavior of equilibrium price $p^*$, average equilibrium output $q^*$ and total mass of varieties $n^*N^*$ under an increase in market size.

Differentiation of zero-profit condition (13) with respect to the market size $L$ yields after simplification:

$$\frac{d\ln p^*}{d\ln L} = -\eta\,\frac{d\ln q^*}{d\ln L}. \tag{18}$$

See Appendix 4 for a derivation of (18).

As $\eta > 0$, formula (18) says that an increase in market size always drives prices and average quantities in the opposite directions. Also, as $\eta < 1$, percentage change of quantities caused by a market expansion is always higher than that of prices in absolute value.

The elasticity of average equilibrium output $q^*$ with respect to the market size is given by:

$$\frac{d\ln q^*}{d\ln L} = \frac{x\eta}{1-\eta}\frac{v_{nn}\frac{n}{y} + 2v_{yn} + v_{yy}\frac{y}{n}}{\frac{n}{v_y}(v_{yy}v_{nn} - v_{yn}^2) + \frac{r_u}{1-r_u}\left(2 + \frac{xu'''}{u''}\right)\left(v_{yy}\frac{y}{n} + 2v_{yn} + v_{nn}\frac{n}{y}\right)}. \tag{19}$$

The proof of (19) is given in Appendix 4.

From (19) we can see that:

$$L \uparrow \Rightarrow q^* \uparrow$$

The reasons for this are profit concavity condition $-\frac{xu'''}{u''} < 2$ and convexity of variable costs $v$, which implies positive definiteness of its Hessian matrix.

Also, as shown in Appendix 4,

$$\frac{d\ln q^*}{d\ln L} < 1. \tag{20}$$

Inequality (20) means that quantities supplied increase with respect to $L$, though less than proportionally. Using (18) we establish that the behavior of prices is the opposite. The intuition behind this result is as follows. An increase in the market size leads to a shrink in individual consumption of each variety, for

$$\frac{d\ln x^*}{d\ln L} = \frac{d\ln q^*}{d\ln L} - 1 < 0.$$

As $\eta'(x) > 0$, an increase in $L$ makes varieties to be **closer substitutes**. Thus, competitive pressure increases and drives prices down. On the other hand, higher $L$ means an increase in total market demand. Thus, to cover new demand, firms begin to produce a larger quantity of each variety.

We now turn to the analysis of total product range behavior. Combining zero profit condition (13) with labour balance (17), we see that $nN = \frac{nL}{py}$, or, in terms of elasticities,

$$\frac{d\ln(n^*N^*)}{d\ln L} = 1 - \frac{d\ln p^*}{d\ln L} - \frac{d\ln q^*}{d\ln L}.$$

Using (18), we get:

$$\frac{d\ln(n^*N^*)}{d\ln L} = 1 - (1-\eta)\frac{d\ln q^*}{d\ln L}. \tag{21}$$

As

$$0 < \frac{d\ln q^*}{d\ln L} < 1,$$

(21) implies

$$L \uparrow \Rightarrow n^*N^* \uparrow$$

and

$$\frac{d\ln(n^*N^*)}{d\ln L} < 1,$$

i.e. an increase in total product range is always less than proportional to an increase in $L$.

The results obtained above can be summarized as follows.

**Proposition 2.** *Under an increase in market size $L$*

- *the equilibrium price $p^*$ decreases;*
- *the equilibrium output of each variety $q^*$ increases;*
- *the industry-level product diversity $n^*N^*$ increases.*

**Remark 1**. *Changes of $p^*$, $q^*$ and $n^*N^*$ are less than proportional to an increase in $L$.*

**Remark 2**. *If preferences are of CES type, both $p^*$ and $q^*$ remain unchanged in response to an increase in $L$.*

The last remark follows immediately from (18) – (19). It shows that the impact of market expansion on the market outcome under CES prefserences is trivial, which is a source of criticism of models using CES in empirical literature.

### 4.3 Comparative statics: total output and scope

We now pass to the study of how an increase in the market size affects firm-level product range $n^*$. It is shown in Appendix 4 that

$$\frac{d\ln n^*}{d\ln y^*} = -\frac{x\eta'}{1-\eta} \frac{v_{yy}\frac{y}{n} + v_{yn}}{\frac{n(v_{yy}v_{nn}-v_{yn}^2)}{v_y} + \frac{\eta}{1-\eta}\left(2+\frac{xu'''}{u''}\right)\left(v_{yy}\frac{y}{n} + 2v_{yn} + v_{nn}\frac{n}{y}\right)}. \tag{22}$$

Convexity of variable costs $v$ implies that the denominator in (22) is positive. Thus, we come to the following proposition.

**Proposition 3.** *Under an increase in market size $L$ each firm's equilibrium scope $n^*$increases if and only if diseconomies of scale are dominated by scale-scope spillovers:*

$$n^* \uparrow \; \Leftrightarrow \;\; -n\frac{v_{yn}}{v_y} > y\frac{v_{yy}}{v_y}$$

Intuitively, Proposition 3 says that firms have no incentives to expand their product lines under relatively low spillovers, for an increase in $L$ makes varieties less differentiated and thus gains from economies of scope are low. However, if scope expansion results in a substantial reduction in marginal production costs, producers are motivated to elaborate new varieties, even if the degree of product differentiation is low.

Propositions 2 and 3 imply that our results are in accordance with what Bernard, Redding and Schott (2010). Namely, we claim that the outcome

$$L \uparrow \Rightarrow q^* \uparrow \;\; \text{and} \;\; n^* \uparrow$$

is in principle possible. To show this, we provide two examples.

**Example 1.** Consider the following cost function:

$$v(y,\, n) = n^{-\alpha} y^{\beta} + n^{\gamma},$$

where $0 < \alpha < \beta,\ \beta, \gamma > 1$. This cost function clearly displays a positive scale-scope spillover. The greater is $\alpha$, the stronger is the scope externality. Straightforward calculation yields that $q^*$and $n^*$ if and only if $\alpha < \beta - 1$. So, if the expansion of the product line generates a sufficiently strong improvement of the firm's production efficiency (say, by means of learning), there will be no cannibalization.

**Example 2.** Consider linear-quadratic variable costs with an interaction term:

$$v(y, n) = \frac{y^2}{2} + \frac{n^2}{2} - \gamma\, yn + \alpha y + \beta n,$$

where $\alpha,\ \beta \geq 0,\ 0 < \gamma < 1$. Take $\alpha = 15.89,\ \beta = 4,\ \gamma = 0.6,\ F = 9$.

Fig.1 displays the behavior of total output and scope under market size variations.

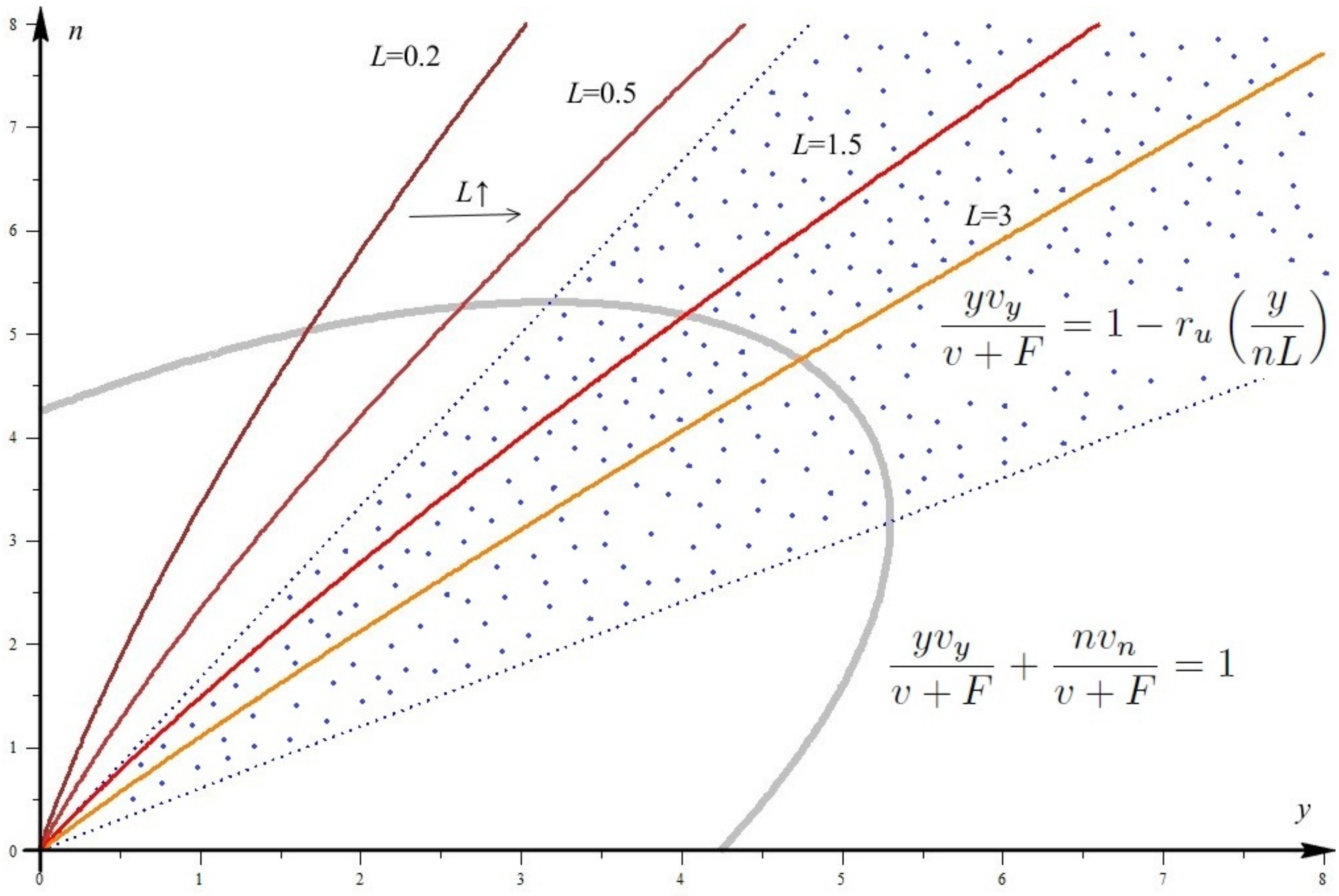


Fig.1. Total output and scope under market size variations: the case of linear-quadratic variable costs.

From fig.1 it can be seen that if $L$ is relatively small, both total output and scope increase in response to an increase in $L$. Moreover, the average output $q$, which is the inverse slope of a ray connecting the origin and the equilibrium point, also increases. So, cannibaliation is absent under small markets.

To do similar comparative statics for the firm-level total output $y^*$, we establish (see Appendix 4 for the proof) that

$$\mathcal{E}_{y/L} = \frac{x\eta'}{1-\eta} \frac{v_{nn}\frac{n}{y} + v_{yn}}{\frac{n(v_{yy}v_{nn} - v_{yn}^2)}{v_y} + \frac{\eta}{1-\eta}\left(2 + \frac{xu'''}{u''}\right)\left(v_{yy}\frac{y}{n} + 2v_{yn} + v_{nn}\frac{n}{y}\right)}. \tag{23}$$

The following result follows immediately from (23).

**Proposition 4.** *Under an increase in market size $L$ each firm's total output $y^*$ increases if and only if scale-scope spillovers are insufficient to dominate diseconomies of scope:*

$$y^* \uparrow \Leftrightarrow \; n\frac{v_{nn}}{v_n} > -y\frac{v_{ny}}{v_n}$$

**Remark**. *If preferences are of CES type, both $n^*$ and $y^*$ does not change in response to an increase in $L$.*

### 4.4 Comparative statics: the mass of firms

Finally, we examine how the mass of firms behaves if an increase in the market size occurs.

The elasticity of the mass of firms with respect to the market size is given by:

$$\frac{d\ln N^*}{d\ln L} = 1 - \frac{d\ln p^*}{d\ln L} - \frac{d\ln y^*}{d\ln L},$$

which follows from the labour balance $L = Npy$. Using (18), (19) and (23) yields:

$$\frac{d\ln N^*}{d\ln L} = 1 + \frac{v_y v_n}{v+F}\,\frac{x\eta'\left(\left(\frac{v_{yy}y}{v_y} + \frac{v_{yn}n}{v_y}\right) - \left(\frac{v_{nn}n}{v_n} + \frac{v_{ny}y}{v_n}\right)\right)}{(1-\eta)\frac{n(v_{yy}v_{nn}-v_{yn}^2)}{v_y} + \eta\left(2+\frac{xu'''}{u''}\right)\left(v_{yy}\frac{y}{n} + 2v_{yn} + v_{nn}\frac{n}{y}\right)}. \tag{24}$$

As the denominator in (21) is positive, the right-hand side can easily be compared to one.

Formula (24) implies the proposition below.

**Proposition 5.** *The mass of firms $N^*$ increases more than proportionally in response to an increase in market size $L$ if and only if in the equilibrium net diseconomies of scale dominate net diseconomies of scope:*

$$\frac{\partial \ln N^*}{\partial \ln L} > 1 \;\Leftrightarrow\; y\frac{v_{yy}}{v_y} + n\frac{v_{yn}}{v_y} > n\frac{v_{nn}}{v_n} + y\frac{v_{ny}}{v_n}$$

Proposition 5 allows to test whether the mass of firms changes more (less) than proportionally to an increase in the population $L$. The key-factor is the relative role of scale and scope, i.e. whether diseconomies of scale are higher or lower than diseconomies of. Low impact of scope expansions in MPC reduction implies that an increase in $L$ results in more firms rather than in more varieties per firm.

Note that in principle Proposition 5 does not answer a question whether whether market expansions for sure lead to additional entry rather than push active firms out of the market. However, this is a natural conjecture. So far, counterexamples are not found.

# 5 More on the role of spillovers

The above analysis provides the full characterization of equilibrium comparative statics with respect to the market size. However, the classification of possible patterns is rather cumbersome. In order to get some more intuition on the above results, we consider two special cases:

- the case of no scope externalities, $v_{yn} \equiv 0$, which is equivalent to additive separability with respect to total output and the scope;
- the case of production costs additive across varieties.

## 5.1 No spillovers

Assume that variable costs are of the form:

$$V(\mathbf{q}, n) = v\left(\int_0^n q_i di\right) + S(n). \tag{25}$$

Costs given by (25) exhibit perfect technological substitution of varieties, as well as additive separability with respect to total output and the scope.

The symmetrized form of (25) is given by $v(y) + S(n)$. Apparently, this is the most general form of variable costs which does not account for spillovers.

Using Propositions 2, 3 and 5, we obtain comparative statics in the abscence of scope externalities. For this case, we are also able to characterize the behavior of total industrial output $Y = Ny$.

**Proposition 6.** *Assume that there are no scale-scope spillovers. Then under an increase in market size $L$:*

- *each firm's total output $y^*$increases;*
- *each firm's product line $n^*$ shrinks;*
- *the mass of firms $N^*$ increases.*

Intuitively, proposition 6 says that

$$L \uparrow \Rightarrow n^* \downarrow, \ N^* \uparrow,$$

i.e. that markets always become less concentrated in response to an increase in the market size. Note that this result is exactly reverse to what Feenstra and Ma (2007) find.

Proposition 2 also says that, in the absence of scale-scope spillovers, firms' extensive margins $n^*$ and intensive margins $q^*$ are always negatively associated.

**Remark**. *Changes in $y^*$ and $n^*$ are less than proportional to an increase in $L$, whereas $y^*N^*$ increases more than proportionally. $N^*$ increases more than proportionally if and only if*

$$y\frac{v''(y)}{v'(y)} > n\frac{S''(n)}{S'(n)},$$

*i.e. if diseconomies of scale exceed diseconomies of scope.*

## 5.2 Per-variety additive costs

Consider a *per-variety additive* cost function:

$$V(\mathbf{q}, n) = \int_0^n \nu(q_i)di + S(n), \tag{26}$$

where $\nu$, $S$ are increasing and convex. Specification (26) seems plausible when all varieties are produced in different plants. In this context $\nu(q)$ is a variable cost function of a separate plant, whereas design of $n$ varieties costs $S(n)$. Here strict convexity of $\nu(q)$ may express the idea of diseconomies of scale in producing each variety, while $S(n)$ may be strictly convex to express similar diseconomies in designing more and more varieties (more varieties require a more that proportional increase in attention and effort of top managers).

Under (26), one can see that the profit-maximization problem is decomposable. When the scope $n$ is chosen by the firm's top manager, each variety's output can be chosen separately by a manager of a department responsible for this variety to fit related FOC:

$$\frac{d}{dq_i}\left(\frac{1}{\lambda}\, q_i\, u'\left(\frac{q_i}{L}\right) - \nu(q_i)\right) = 0.$$

The above decomposition makes price-quantity reactions of multi-product firms to increasing competition similar to those of single-product firms. This idea is important for understanding empirical relevance of monopolistic competition theory.

### Comparative statics with respect to the market size

We now establish comparative statics with respect to the market size. The proof is not very involved and quite intuitive, so we put it here, not in the appendices.

**Proposition 7.** *Assume that production costs are additive across varieties. Then under an increase in market size $L$:*

- *scopes $n^*$ remain unchanged*

- *the mass of firms $N^*$ increases*

**Proof**. See Appendix 5.

The most striking result is that the choice of scope in equilibrium *remains unchanged* in response to market size variations. The intuition behind this result comes from what was already noted above: in case of per-variety additive variable costs the manager of each variety maximizes individual profits regardless of what the others do. Thus, everything works as if firms were single-product.

### The irrelevance of perfect technological substitution

In this subsection we argue that some features of variable costs which appear to be of great importance in many different contexts can be in fact irrelevant for our analysis. Perfect technological substitution is one of such features. Per-variety additive costs provide a simple example.

Unlike (25), variable costs (26) do not exhibit perfect technological substitutability across varieties, except for the case when $\nu$ is linear. However, we can think of a well behaved cost function which posesses the property of perfect technological substitution but yields exactly the same equilibrium outcome. Indeed, by Proposition 1, outcomes other than symmetric equilibria are impossible. So, to generate the same outcomes, other things being equal, the two variable cost functions should only have the same symmetrized form. Thus, all we need for a suitable argument is to construct a cost function whish exhibits perfect technological substitution and has the same symmetrized form as (26).

A suitable variable cost function is:

$$V(\mathbf{q},\, n) = n\,\nu\left(\frac{1}{n}\int\limits_0^n q_i di\right) + S(n), \tag{27}$$

where $\upsilon$ is now the variable production costs per one variety which depends only on the average output, and $S$ the variable R&D (or monitoring) costs.

It may seem that (27) tells quite a different story about how production is organized then (26) does. In particular, (27) exhibits perfect technological substitution of varieties. Nevertheless, functions (26) and (27) have the same symmetrized form. Hence, they are equivalent in the sense that the market outcomes these two functions generate are identical (see Appendix 1 for more about the equivalence of variable cost functions). Thus, the differences between them are entirely irrelevant for the equilibrium outcome.

## 6 Conclusion

We have developed a model of monopolistically competitive industry with multi-product firms with unspecified technologies in a symmetric world. Our goal was to study implications of an increase in market size. We found that prices (quantities supplied) always decrease (increase) in response to a market size increase due to more severe competitive pressure. Another finding is that the patterns which firm-level product ranges and the number of firms in the industry follow are triggered by the supply sides. Namely, scopes increase (decrease) and the number of firms increases less (more) than proportionally if and only if the intensity of scale-scope spillovers is sufficiently high (low).

Our model suggests a possible explanation of findings in Bernard, Redding and Schott (2010), who report significant positive correlation between intensive and extensive margins of US manufacturing firms. According to our findings, intensive margins (quantities supplied) and extensive margins (scopes) can be pushed by an increase in market size in the same direction, provided scale-scope spillovers are high.

There are at least three possible extensions of our framework. The first one is to introduce heterogeneity among firms to examine the interactions of heterogeneity and scope externalities. The second one is to model open economy. The third one is to allow for the firm's choice between producing a *single* product and producing a variety of products within the monopolistically competitive setting. All of them are potential subjects of further research.

## Appendix 1: symmetrization of cost functions

Substituting the inverse demand functions (3) into the profit function (7), we get:

$$\Pi(\mathbf{q},\, n) = \frac{1}{\lambda}\int_0^n u'\left(\frac{q_i}{L}\right) q_i\, di - F - V(\mathbf{q},\, n). \tag{28}$$

**Lemma 1.** *If there exists a production plan maximizing the profit function* $\Pi$ *under a given scope* $n$*, then it is unique and symmetric.*

**Proof**. Let $\mathbf{q}^*$ be the production plan maximizing the profit function (28) under the given $n$. Due to convexity of $V$ and the concavity condition (2), the profit function (2) is strictly concave in $\mathbf{q}$. So, $\mathbf{q}^*$ is the unique maximum point.

Show now that $\mathbf{q}^*$ is symmetric. Assume that, on the contrary, it is not. Form production plans $\mathcal{S}_j\mathbf{q}^*$ for all $j \in [0,\, n]$ , where $\mathcal{S}_j$ is the cyclical shift operator defined as follows:

$$(\mathcal{S}_j\mathbf{q})_i \equiv \begin{cases} q_{i+j}, & i+j \leq n, \\ q_{i+j-n}, & i+j > n. \end{cases}$$

Profit $\Pi$ clearly satisfies the symmetry condition (4) with respect to $\mathbf{q}$. By Jensen's inequality

$$\Pi\left(\frac{1}{n}\int_0^n \mathcal{S}_j\mathbf{q}^* dj,\ n\right) > \frac{1}{n}\int_0^n \Pi\left(\mathcal{S}_j\mathbf{q}^*,\ n\right) dj = \Pi\left(\mathbf{q}^*,\ n\right).$$

The inequality is strict because of our assumption that $\mathbf{q}^*$ is not symmetric. Thus $\mathbf{q}^*$ is not optimal. We have come to a contradiction, hence $\mathbf{q}^*$ is symmetric. QED

In the same manner as we did for the cost function, we introduce the *symmetrized profit function*:

$$\pi(q,\, n) \equiv \Pi(\mathbf{q},\, n)|_{\mathbf{q}\equiv q}.$$

**Lemma 2.** *The maxima of* $\Pi$ *and* $\pi$ *exist or don't exist simultaneously, and if they do, the maximum points are the same.*

**Proof**. Let $(q^*,\, n^*)$ be point which maximizes $\pi$. Assume that there exist some production plan $\mathbf{q}^{**}$ and some product line scope $n^{**}$ such that $\Pi\left(\mathbf{q}^{**},\, n^{**}\right) > \pi\left(q^*,\, n^*\right)$. Then, by lemma 1, there exists a symmetric production plan $\hat{\mathbf{q}}$, such that $\Pi\left(\hat{\mathbf{q}},\, n^{**}\right) > \Pi\left(\mathbf{q}^{**},\, n^*\right)$. So we have:

$$\pi\left(\hat{q},\, n^{**}\right) = \Pi\left(\hat{\mathbf{q}},\, n^{**}\right) > \Pi\left(\mathbf{q}^{**},\, n^*\right) > \Pi\left(\mathbf{q}^*,\, n^*\right) = \pi\left(q^*,\, n^*\right),$$

which contradicts the assumption that $(q^*,\, n^*)$ maximizes $\pi$. Thus, $(q^*,\, n^*)$ maximizes $\Pi$.

We have shown that if the maximum of $\pi$ exists, the maximum of $\Pi$ also exist and the maximum points are the same. The converse follows from lemma 1. QED.

Lemma 2 allows us to replace the original producer's problem (8) with the symmetrized producer's problem (9).

We now show that

All information about technology we need to construct the symmetrized profit function is the symmetrized cost function $V$. Thus, if two different variable cost functions have the same symmetrized cost function, we should be indifferent about which of those cost functions if the "true" one. This observation motivates the following definition.

**Definition**. Call the variable cost functions $V_1$ and $V_2$ *equivalent*, if their symmetrized cost function is the same.

The content of the following proposition is that without loss of generality we can believe that variable costs depend *solely* on the output and the product line scope.

**Proposition 8.** *Let $V_1$ be some variable cost function. Then there exists an equivalent function $\mathcal{V}_2$, also satisfying assumptions 1 - 4, which depends only on the product line scope and the average output.*

**Proof**. Let $v(y, n)$ be the symmetrized form of $V_1$. Define $V_2$ as follows:

$$V_2(\mathbf{q}, n) = v\left(\int\limits_0^n q_i di, \ n\right). \tag{29}$$

The accordance of $V_2$ with all the assumptions is straightforward. QED.

Despite Proposition 8 is quite obvious, it implies a surprising conclusion: the general case of variable costs satisfying the anonimity condition *gives nothing new* in comparison with the case of perfect technological substitution, when variable costs depend solely on the total output and the product line scope.

## Appendix 2: profit concavity

Each firm chooses the product line scope $n$ and total output volume $y = \int_0^n q_i di$ in order to get the highest possible profit. Thus, the producer's problem is to maximize profit:

$$\pi(y, n) = py - v(y, n) - F,$$

subject to the inverse demand function:

$$p = \frac{u'(x)}{\lambda}.$$

Here $x$ is the individual demand for a particular variety, $x = \dfrac{y}{Ln}$.

Due to the monopolistic competition environment, each firm is negligibly small in comparison with the rest of the industry. Hence, a producer takes the aggregate market statistics $\lambda$ as given. On that ground we reformulate the firm's problem as follows:

$$\max_{y,n} \Pi(y,n) = \frac{1}{\lambda} y\, u'\left(\frac{y}{Ln}\right) - v(y,\, n) - F$$

Our task here is to establish conditions sufficient for profit concavity.

Consider the revenue function:

$$R(y,\, n) = py = \frac{1}{\lambda} y\, u'\left(\frac{y}{Ln}\right).$$

Profit is revenue net of all costs:

$$\Pi(y,n) = R(y,\, n) - v(y,\, n) - F.$$

Everything is done if we find out when revenue is concave.

Notice that $R(y,\, n)$ is positive homogenous of degree one in firm's size $y$ and firm's scope $n$. Hence, the Hessian matrix of $R(y,\, n)$ is degenerate for all $y$, $n$. Thus, at least one of its eigenvalues is always zero. To ensure that the other one is negative, it suffices to require $R_{yy} < 0$. Direct calculation yields:

$$R_{yy} = \frac{1}{\lambda}\frac{2u''(x) + xu'''(x)}{Ln} = \frac{u''(x)}{\lambda Ln}\left(2 + \frac{xu'''(x)}{u''(x)}\right). \tag{30}$$

As $u'' < 0$, (30) shows that $-\dfrac{xu'''(x)}{u''(x)} < 2$ is sufficient for profit concavity.

### Appendix 3: proof or Proposition 1

The non-existence of asymmetric equilibria follows immediately from Lemma 1. As for two other statements, we provide here the proof for the case of absence of scope externalities, i.e. for additively separable costs. The proof for the general case is based on exactly the same idea, but is more cumbersome.

If we find $(y^*,\, n^*)$, we immediately get $(p^*,\, N^*)$. So, all we need is to establish the existence and uniqueness of the following equation system:

$$yv'(y) + nS'(n) = v(y) + S(n) + F, \tag{31}$$

$$yv'(y) = \frac{1-\eta}{\eta}\, nS'(n). \tag{32}$$

**Lemma 3.** *For any fixed $y$, there exists a unique solution $n(y)$ of equation (32). Moreover,* $\lim\limits_{y\to 0} n(y) = 0,\ \lim\limits_{y\to\infty} n(y) = \infty.$

**Proof**. The first statement of the lemma implies the second one immediately. Indeed, as $\dfrac{1-\eta}{\eta} \in \left[\dfrac{\varepsilon}{1-\varepsilon}, \dfrac{1-\varepsilon}{\varepsilon}\right]$ for all $x$, the right-hand side on (32) should tend to zero (respectively, infinity) together with the left one. All we need is to establish the existence of $n(y)$.

In view of monotonicity and convexity of $S$, the right-hand side on (32) tends to infinity as $n$ grows unboudedly and tends to zero as $n$ becomes arbitrarily small. It suffices to show that the right-hand side on (32) is monotonic, for the intermediate value theorem will guarantee the existence of a unique solution.

Calculate the derivative of the right-hand side with respect to $n$:

$$\frac{\partial}{\partial n}\left(\frac{1-\eta}{\eta} nS'(n)\right) = \frac{1-\eta}{\eta}\left(S'(n) + nS''(n)\right) + \frac{x\,\eta'}{\eta^2} S'(n).$$

Applying the identity

$$x\,\eta' = \eta(1 + \eta + \frac{xu'''}{u''}) \tag{33}$$

and the second-order condition $-\dfrac{xu'''}{u''} < 2$, we get:

$$\frac{\partial}{\partial n}\left(\frac{1-\eta}{\eta} nS'(n)\right) = \frac{1-\eta}{\eta} nS''(n) + \frac{2 + \dfrac{xu'''}{u''}}{\eta} S'(n) > 0.$$

So, the right-hand side on (32) is a strictly increasing function. QED.

**Lemma 4.** *$n(y)$ is increasing.*

**Proof.**

$$n'(y) = \frac{v'(y) - nS'(n)\frac{\partial}{\partial y}\left(\frac{1-\eta}{\eta}\right)}{\frac{\partial}{\partial n}\left(\frac{1-\eta}{\eta}nS'(n)\right)}.$$

Since $\eta'(x) > 0$, we see that

$$\frac{\partial}{\partial y}\left(\frac{1-\eta}{\eta}\right) < 0 < \frac{\partial}{\partial n}\left(\frac{1-\eta}{\eta}nS'(n)\right),$$

hence $n'(y) > 0$. So, the function $n(y)$ is increasing. QED.

Plugging $n = n(y)$ into (32), we get:

$$(yv'(y) - v(y)) + (n(y)S'(n(y)) - S(n(y))) = F. \tag{34}$$

From Lemma 3 we get that the left-hand side of (34) tends to zero (respectively, infinity) together with $y$. Thus, at least one solution $y^*$ exists. Moreover, as $\eta'(x) > 0 \ \forall x$, the left-hand side of (34) is an unboundedly increasing continuos function, due to lemma 4. Applying again the intermediate value theorem, we establish the existence and uniqueness of $y^*$. Setting $n^* = n(y^*)$, we obtain a unique solution of (31) - (32). This completes the proof.

## Appendix 4: proof of formulas (18) – (20)

The equilibrium conditions are:

$$v_y = (1 - \eta)\, p, \tag{35}$$

$$v_y y + v_n n = v(y,\, n) + F, \tag{36}$$

$$py = v(y,\, n) + F. \tag{37}$$

Differentiating the zero-profit condition (37), we obtain:

$$\frac{\partial p}{\partial L}y + \frac{\partial y}{\partial L}p = v_y\frac{\partial y}{\partial L} + v_n\frac{\partial n}{\partial L}.$$

Multiplying both sides by $L/py$ and accounting for (36) yields:

$$\frac{\partial p}{\partial L}\frac{L}{p} + \frac{\partial y}{\partial L}\frac{L}{y} = \frac{v_y}{p}\frac{\partial y}{\partial L}\frac{L}{y} + \left(1 - \frac{v_y}{p}\right)\frac{\partial n}{\partial L}\frac{L}{n}.$$

Finally, using (35), we get:

$$\mathcal{E}_{p/L} = -\eta(x)(\mathcal{E}_{y/L} - \mathcal{E}_{n/L}),$$

which is (18).

Here and further on we use $\mathcal{E}_{p/L}$ for price elasticity with respect to $L$, etc.

Differentiating the markup condition (35), we get:

$$v_{yy}\frac{\partial y}{\partial L} + v_{yn}\frac{\partial n}{\partial L} = -\eta'(x)\frac{\partial x}{\partial L}p + (1 - \eta(x))\frac{\partial p}{\partial L}. \tag{38}$$

The derivative of individual consumption level $x$ with respect to the market size is:

$$\frac{\partial x}{\partial L} = \frac{x}{L}\,\mathcal{E}_{x/L} = \frac{x}{L}\,\left(\mathcal{E}_{y/L} - \mathcal{E}_{n/L} - 1\right).$$

Plugging into (38) yields:

$$yv_{yy}\frac{\partial y}{\partial L}\frac{L}{y} + nv_{yn}\frac{\partial n}{\partial L}\frac{L}{n} = -\eta'(x)\,x\left(\mathcal{E}_{y/L} - \mathcal{E}_{n/L} - 1\right)p + p(1 - \eta(x))\frac{\partial p}{\partial L}\frac{L}{p}.$$

Further simplifications yield

$$\frac{yv_{yy}}{v_y}\mathcal{E}_{y/L} + \frac{nv_{yn}}{v_y}\mathcal{E}_{n/L} = \frac{\left(2 + \frac{xu'''}{u''}\right)\mathcal{E}_{p/L} + x\eta'}{1 - \eta}.$$

Accounting for (18) and rearranging, we come to:

$$\left(\frac{yv_{yy}}{v_y}+\frac{\eta}{1-\eta}\left(2+\frac{xu'''}{u''}\right)\right)\mathcal{E}_{y/L}+\left(\frac{nv_{yn}}{v_y}-\frac{\eta}{1-\eta}\left(2+\frac{xu'''}{u''}\right)\right)\mathcal{E}_{n/L}=\frac{x\eta'}{1-\eta}. \tag{39}$$

Differentiating (36), we get:

$$(v_y+v_{yy}y+v_{yn}n)\frac{\partial y}{\partial L}+(v_n+v_{nn}n+v_{yn}y)\frac{\partial n}{\partial L}=v_y\frac{\partial y}{\partial L}+v_n\frac{\partial n}{\partial L},$$

which yields after simplifications:

$$\mathcal{E}_{y/L}=-\frac{v_{nn}\frac{n}{y}+v_{yn}}{v_{yy}\frac{y}{n}+v_{yn}}\,\mathcal{E}_{n/L}. \tag{40}$$

Solving the linear system (39) – (40) with respect to $\mathcal{E}_{y/L}$, $\mathcal{E}_{n/L}$, we find:

$$\mathcal{E}_{n/L}=-\frac{x\eta'}{1-\eta}\,\frac{v_{yy}\frac{y}{n}+v_{yn}}{\frac{n(v_{yy}v_{nn}-v_{yn}^2)}{v_y}+\frac{\eta}{1-\eta}\left(2+\frac{xu'''}{u''}\right)\left(v_{yy}\frac{y}{n}+2v_{yn}+v_{nn}\frac{n}{y}\right)},$$

$$\mathcal{E}_{y/L}=\frac{x\eta'}{1-\eta}\,\frac{v_{nn}\frac{n}{y}+v_{yn}}{\frac{n(v_{yy}v_{nn}-v_{yn}^2)}{v_y}+\frac{\eta}{1-\eta}\left(2+\frac{xu'''}{u''}\right)\left(v_{yy}\frac{y}{n}+2v_{yn}+v_{nn}\frac{n}{y}\right)}.$$

So, we have proven (23) – (22)

As $\mathcal{E}_{y/L}-\mathcal{E}_{n/L}=\mathcal{E}_{q/L}$, we can find an expression for $\mathcal{E}_{q/L}$

$$\mathcal{E}_{q/L}=\frac{x\eta'}{1-\eta}\frac{v_{nn}\frac{n}{y}+2v_{yn}+v_{yy}\frac{y}{n}}{\frac{n(v_{yy}v_{nn}-v_{yn}^2)}{v_y}+\frac{\eta}{1-\eta}\left(2+\frac{xu'''}{u''}\right)\left(v_{yy}\frac{y}{n}+2v_{yn}+v_{nn}\frac{n}{y}\right)}.$$

Thus (19) is also established

It remains to verify the inequalities:

$$v_{yy}v_{nn}-v_{yn}^2>0, \tag{41}$$

$$v_{yy}\frac{y}{n} + 2v_{yn} + v_{nn}\frac{n}{y} > 0. \tag{42}$$

To do this, we take into account that the Hessian matrix of $v$

$$H_v = \begin{pmatrix} v_{yy} & v_{yn} \\ v_{yn} & v_{nn} \end{pmatrix}$$

is positive definite, for $v$ is convex. But the left hand side of (41) is nothing but $\det(H_v)$, so (41) holds. Note also that

$$v_{yy}\frac{y}{n} + 2v_{yn} + v_{nn}\frac{n}{y} = \begin{pmatrix} \sqrt{\frac{y}{n}} & , & \sqrt{\frac{n}{y}} \end{pmatrix} \begin{pmatrix} v_{yy} & v_{yn} \\ v_{yn} & v_{nn} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{y}{n}} \\ \sqrt{\frac{n}{y}} \end{pmatrix} > 0,$$

so (42) holds as well. This completes the proof.

In order to obtain (20), we use (33) to rewrite (19) as follows:

$$\mathcal{E}_{q/L} = \frac{\eta(1+\eta+\frac{xu'''}{u''})\left(v_{nn}\frac{n}{y} + 2v_{yn} + v_{yy}\frac{y}{n}\right)}{(1-\eta)\frac{n(v_{yy}v_{nn}-v_{yn}^2)}{v_y} + \eta\left(2+\frac{xu'''}{u''}\right)\left(v_{yy}\frac{y}{n} + 2v_{yn} + v_{nn}\frac{n}{y}\right)}.$$

As $0 < \eta < 1$, we have $1+\eta+\frac{xu'''}{u''} < 2+\frac{xu'''}{u''}$. Thus, the denominator is positive, and the numerator is always less then the denominator, which implies (20).

**Appendix 5: proof of Proposition 7**

We first show that $\mathcal{E}_{n/L} = 0$ regardless of the elasticity of substitution behavior.

To establish this result, we find the symmetrized form of the variable costs (26):

$$v(y,\, n) = n\nu\left(\frac{y}{n}\right) + S(n). \tag{43}$$

The own scale effect equals identically the cross scale effect:

$$\frac{yv_{yy}}{v_y} \equiv -\frac{nv_{yn}}{v_y} = \frac{q\,\nu''(q)}{\nu'(q)},$$

where $q = \frac{y}{n}$ is the average output.

However, we already know from Proposition 3 that in this case the equilibrium scope does not change under an increase in the market size.

As $n$ remains unchanged, $\mathcal{E}_{y/L} = \mathcal{E}_{q/L}$ and $\mathcal{E}_{N/L} = \mathcal{E}_{nN/L}$. But these elasticities are known from Proposition 2. QED.

## References


[1] Allanson, P. and C. Montagna (2005). Multi-product Firms and Market Structure: an Explorative Application to the Product Life Cycle // International Journal of Industrial Organization, Vol. 23, No. 7 - 8, pp. 587 - 597.

[2] Bailey, E.E. and A.F. Friedlaender (1982). Market Structure and Multiproduct Industries // Journal of Economic Literature, Vol. 20, No. 3, pp. 1024 - 1048.

[3] Bernard, S. Redding and Schott (2010). Multiple-product Firms and Product Switching // American Economic Review, Vol. 100, No. 1, pp. 70 - 97.

[4] Brander, J.A. and J. Eaton (1984). Product Line Rivalry // American Economic Review, Vol. 74, No. 3, pp. 323 - 334.

[5] Cockburn, I. and R. Henderson (2001). Scale and scope in drug development: unpacking the advantages of size in pharmaceutical research // Journal of Health Economics, Vol. 20, No 6, pp. 1033 - 1057.

[6] Dixit, A.K. and J.E. Stiglitz (1977). Monopolistic Competition and Optimum Product Diversity // The American Economic Review Vol. 67, No. 3, pp. 297-308.

[7] Eaton, B.C. and N. Schmidt (1994). Flexible Manufacturing and Market Structure // American Economic Review, Vol. 84, No. 4, pp. 875 - 888.

[8] Eckel, C. and J.P. Neary (2010). Multi-Product Firms and Flexible Manufacturing in the Global Economy // The Review of Economic Studies, Vol. 77, pp. 188217.

[9] Feenstra, R., and H. Ma (2007). Optimal choice of product scope for multiproduct firms under monopolistic competition. NBER working paper 13703.

[10] Goldberg, P., A. Khandewal, N. Pavnik and P. Topalova (2008) Multi-product Firms and Product Turnover in the Developing World: Evidence from India. NBER Working Paper No. 14127.

[11] Henderson, R. and I. Cockburn (1996). Scale, Scope, and Spillovers: The Determinants of Research Productivity in Drug Discovery // The RAND Journal of Economics Vol. 27, No. 1 (Spring, 1996), pp. 32-59

[12] Johnson, J.P. and D.P. Myatt (2003). Multiproduct Quality Competition: Fighting Brands and Product Line Pruning // The American Economic Review, Vol. 93, No. 3, pp. 748 - 774.

[13] Klette. T.J. and S. Kortum (2004). Innovating Firms and Aggregate Innovations // Journal of Political Economy, Vol. 112, No. 5, pp. 986 - 1018.

[14] Lentz, R. and D.T. Mortensen (2005). Productivity Growth and Worker Reallocation // International Economic Review, Vol. 46, No. 3, pp. 731 - 749.

[15] Nocke, V. and S. Yeaple (2006). Globalization and Endogenous firm scope. NBER working paper 12322.

[16] Ottaviano G.I.P. and J.F. Thisse (1999). Monopolistic Competition, Multiproduct Firms and Optimum Product Diversity. CORE discussion paper 9919.

[17] Shaked, A. and J. Sutton (1990). Multiproduct Firms and Market Structure // The RAND Journal of Economics, Vol. 21, No. 1, pp. 45 - 62.

[18] Spence, M. (1976). Product Selection, Fixed Costs, and Monopolistic Competition // The Review of Economic Studies, Vol. 43, No. 2, pp. 217-235.

[19] Zhelobodko, E., S. Kokovin, M. Parenti and J.F. Thisse. Monopolistic Competition in General Equilibrium: beyond the CES. Working Paper 2011 - 08, Paris School of Economics.