When Veblen meets Krugman.*

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Comments are welcome.

Abstract

We introduce relative concerns in the form of conspicuous consumption in a standard economic geography model a la Krugman. The primary intuition is that conspicuous consumption imposes a negative externality on some agents and generates a centrifugal force. We show that this is not always the case as the relative concern also rises the demand for the sophisticated good, strengthening the standard centripetal market size effect. We show that the resulting force is very sensitive to the topology of the network of "conspicuous" links in each region and on the level of economic integration. For instance, with relatively large shares of income devoted to the consumption of the standard good, we show that when trade is moderately costly and classes of workers are segregated, relative concerns tends to stabilize the symmetric equilibrium; on the other hand, if workers of different classes interact via their relative concerns, conspicuous consumption is a centripetal force generating stable fully or partially agglomerated equilibria. Finally, when the level of integration is high, the intuition holds and even small relative concerns destabilize the full agglomeration equilibrium, which is stable in the Krugman model.

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1 Introduction

The understanding of the forces shaping economic geography has mainly focused on the production of goods and services, on their exchange via markets and on the flow of inputs, particularly labour. In this literature, social interaction typically does not play a major role, surprisingly not even in the determinant of migration. Clearly, location decisions of agents depend on the perceived well being. And a recurrent theme is that, in addition to own consumption and leisure, well being appears to depend on the consumption of others with whom we interact and compare ourselves. The aim of the present paper is to shed some light on the role of this type of social interaction on city dynamics and migration. In doing so the paper is a contribution to what Duranton (2008) considers as one of the major challenges for spatial economics: the development of a 'theory of proximity' that explains why direct interactions between economic agents matter and how.

The effect of social interaction on migration decisions has attracted some attention in recent years (see Radu (2008) for a survey). It has been acknowledged that agents do evaluate their income not only in absolute terms but also in comparison with others and that this may change the migration decisions substantially (Stark and Taylor (1991) is an influential paper in this area). More recently, Quinn (2006) shows "relative deprivation" is a significative factor in domestic migration decisions among Mexicans. We depart from this body of research as we look at the general equilibrium effects of the social comparisons, including its effect on prices and wages, and therefore are able to address the full role of social interaction on city dynamics. Secondly, our work explores the role of the finer city organization, particularly the level of segregation among its citizen.

Although proximity effects have not been much studied in economic geography, there is now a vast literature in other areas stressing the importance of relative concerns for individual well being. Surely, the early literature building on the work of Veblen's (1899) on "conspicuous consumption" was mainly qualitative and followed simple introspection. In recent years empirical evidence supporting the existence of relative concerns has been documented and the effect quantified, see for example, Blanchflower and Oswald (2004), Layard (2005) and (Kuhn, Kooreman, Soetevent, Kapten (2008)). Another interesting study is provided by Luttmer (2005) who finds that changes in the incomes of neighbors have effects on self-reported levels of individual happiness and that the magnitude of these effects depends on the frequency of interaction with the neighbours. Another influential example is provided by the "Easterlin puzzle" that happiness is positively related to incomes at any point in time, but is not related to increases in income over time. As pointed out by Easterlin (1974), relative concerns provide a simple resolution of this puzzle. Over the years, several formal models including habit formation, relative consumption or income comparisons and status concerns have been proposed. Among the many, see e.g., Abel (1990), Frey and Stutzer (2002), Hopkins and Kornienko (2004), Arrow and Dasgupta (2007) and Ghiglino and Goyal (2010).

In the first step to build a model of location with relative concerns, we are guided by the literature of new economic geography, which developed since the publication of Krugman's (1991) seminal work. This literature explains the riddle of unequal spatial development and clarifies many of the microeconomic underpinnings of both spatial economic agglomerations and regional imbalances at different spatial levels. This has been achieved by combining imperfect competition, increasing returns, and transportation costs in full-fledged general equilibrium models that describe how the interactions of centripetal and centrifugal forces determine the locational decisions of firms and workers between two or more regions involved in trade.

In this work we adopt the "footloose entrepreneur" model proposed by Forslid and Ottaviano (2003).[1] In particular, we consider an economy that consists of two regions populated by immobile unskilled workers, and by interregionally mobile skilled workers, each supplying his/her specific type of labour inelastically. Skilled workers can be thought as self-employed entrepreneurs that move freely between the two regions. Two types of goods are produced and exchanged in the model. The agricultural good is produced

[1] We choose this particular model because of its analytical tractability and because it turns out to be identical to the core-periphery model by Krugman (2001).

under perfect competition with constant returns to scale and is freely exchanged between them. There is also a continuum of varieties of a modern manufactured good: each variety is produced by a monopolistically competitive firm with increasing returns to scale employing both skilled and unskilled workers. We assume that there is a transportation cost: to import one unit of a variety in each region, $\tau > 1$ units have to be shipped from the other region.

Agents care about the consumption of both types of goods but also about the consumption of the manufactured good by other agents. Introducing formally relative concerns in this model is relatively easy. The empirical analysis has shown that relative concerns are important only within "neighbours", where the neighbourhood generating the reference group, are friends, family etc. In particular, the network effect is clearly present in the empirical analysis of Luttmer (2005). We will describe these neighborhood by a network of links representing the channels along which comparisons take place. More precisely, we assume that agents care not only about their consumption of the differentiated good but also about each of those of their neighbours, as described by the network. In other words, we assume that there is a *conspicuous effect* of consumption and that it affects agents that are both geographically and socially close as described by an exogenous network.

The primary intuition is that introducing conspicuous consumption should strengthen the centrifugal force because the comparison to other's consumption exerts a direct negative externality. In the paper we show that the presence of the conspicuous good effect also tends to strengthen the two centripetal forces at work in the original model by Forslid and Ottaviano (2003), and, as result, it may contradict the intuition. In particular, we find that when a worker moves in a region, the demand for each firm in this region (determined for a given wage and price index) increases not only because of the presence of the same worker, as in Forslid and Ottaviano (2003), but also because all other workers increase their demand according to the conspicuous good effect. This larger increase in the demand per firm strengthens the "market size" effect, produces a larger increase in operating profits of firms, and finally translates in higher real wages that attract more workers. Moreover, given that when workers move in a region the number of firms producing in that region increases, and the local price index decreases, this greater number of migrants may result in a larger reduction of the local price index. Consequently, because of the larger increase in the local real wage, the "cost-of-living" effect attracts in the region even more workers than in the traditional case.

In line with the intuition, consumption relative concerns tend also to reinforce the traditional 'market crowding' effect. Indeed, when a worker moves in a region also the number of firms increases in the same region and, as explained by Forslid and Ottaviano (2003), this tends to depress the local price index inducing a fall in the local per firm demand or, given their proportionality, per worker demand. Hence, because of the conspicuous good effect, this initial reduction in the per worker demand is strengthened by the reduction in all other workers demand that follows that of their neighbors. Lower demand leads to lower operating profits for firms and, therefore, lower skilled wages.

Which of these effects prevails depends on the level of economic integration between the two regions because this determines the degree of localization of the effects of relative concerns. Specifically, the introduction of conspicuous good effects, which increase the demand for differentiated goods, acts in a stronger way in the larger market when trade costs are high tending to destabilize the symmetric equilibrium. On the contrary, when trade costs are low the conspicuous good effects are less localized: in this case an increase in the demand for differentiated goods due to relative concerns spills over to the other region, because imports from this region become cheaper and, therefore, tends to reinforce dispersion forces.

In the analysis in Sections 2-4 and 6.1-6.2 we consider a very simple network structure. Indeed, the economy is composed of two complete "regional" regular networks. These are endogenously determined and are equal in the case of the symmetric equilibrium and different in partially or fully agglomerated equilibria. Except for the symmetric equilibrium, the global equilibrium network, composed by the two regional networks, has a core-periphery structure. Of course, in the case of the symmetric equilibrium this is a regular network with each agent having the same number of links (neighbors).

In the complete network the comparison group is the entire population in the region of residence. However, typically interactions are expected to be stronger within a smaller group. We consider economies, called segregated, in which agents are sensitive only to comparisons within their own type group. We will also consider other asymmetric cases, as the case of a directed network in which the unskilled workers compare their own consumption with that of the skilled workers but not reciprocally.

We pursue further our analysis and depart from complete regional networks, obtaining new interesting results. Indeed, it is plausible that workers in a given region don't compare themselves with all other agents in that region but rather to a subset of them. The details of the network of comparisons may play here an important role. For example, different ethnic groups or workers in different sectors of the economy, may be associated to different networks of relationships and therefore react differently to the interpersonal comparisons, and ultimately affect city dynamics. Thus, in Section 5, we consider the case of asymmetric networks with the additive specification and we find that: (i) with free trade (full integration) economic activity is dispersed (as in the case of the complete network); (ii) for intermediate trade costs, the agglomeration equilibrium is favored by relative concerns in the case of the star and complete networks, while it can be destabilized by relative concerns in the case of the segregated network; (iii) for high trade costs, the symmetric equilibrium is always stable. By investigating the economic incentives to migrate of workers with relative concerns, this paper constitutes a first step toward the explanation of the shape of the endogenous network formed in equilibrium.

We also consider a mixed case in which agents choose between a region with a complete network and a fully segregated region. An interesting pattern emerges: although for very high trade costs interior solutions may survive, there is a strong tendency to observe full agglomeration in the integrated region in specific situations.[2] At the same time full agglomeration in the segregated region can also co-hexists for intermediate trade costs.

In most of the analysis it is assumed that: 1) individuals compare their consumption with the average consumption of the reference group, 2) there is a linear effect of the size of the reference group on the strength of the effect. However, it is often assumed that the agent only cares about the deviations from his/her consumption and the *average* consumption of his/her

[2] We will show that this is true, for instance, when consumers devote a relatively high value of their income (net of the conspicuous effect) to acquire the differentiated good, and when the proportion of unskilled workers is large.

neighbours. In the last Section of the paper we investigate how our previous results are affected by this modification.

With the complete network and no size effect, the direct centrifugal effect of relative concerns is weaker than in the additive specification. When the coupling parameter that measures the strength of relative concerns α' is sufficiently large, the effect is able to stabilize the symmetric equilibrium for low levels of trade costs, provided skilled population is below some treshold. When the coupling parameter α' is relatively low, the symmetric equilibrium is stable for low levels of costs, for any value of the population. Note that the role of the population is absent in the additive formulation. With the complete network and no size effect, full agglomeration is never stable for high trade costs and it is stable for intermediate or low levels of costs. In the case of the average specification with segregated networks, increases in α' favour agglomeration. Specifically, increases in α' destabilize the symmetric equilibrium while stabilizing full agglomeration. In the case of the average specification with star networks, the numerical analysis shows that increases in α' favour agglomeration. Specifically, increases in the coupling parameter α' destabilize the symmetric equilibrium and stabilize full agglomeration with full agglomeration sustainable for low level of trade costs.

The paper is organized as follows. In Section 2 we introduce the new economic geography model modified to take into account the conspicuous effect of consumption when individuals care about the consumptions of their neighbours. In Section 3 we define our equilibrium and stability concepts. In Section 4 we present our findings and discuss the properties of the equilibria of the model when the network in each region is complete. In Section 5 we extend the analysis to asymmetric networks. In Section 6 we modify the specification chosen for the utility function assuming that each agent cares about the average of his/her neighbours. Section 7 concludes. Most of the proofs are contained in the Appendixes.

2 The general model

We consider an economy that consists of two regions indexed by $r = 1, 2$, populated by a mass $L_1 + L_2 = 2L$ of immobile unskilled workers, and a mass $H_1 + H_2 = H$ of interregionally mobile skilled workers, each supplying inelastically one unit of his/her specific type of labour. We assume that unskilled workers are evenly distributed between the two regions, with $L_r = L$. Skilled workers can be thought as self-employed entrepreneurs, as in the footloose entrepreneur model by Forslid and Ottaviano (2003), that move freely between the two regions, with their level in each region endogenously determined and given by H_r and $H_v = H - H_r$, with $v = 1, 2$ and $r \neq v$. Wages perceived by skilled and unskilled workers in region r are, respectively, given by w_{H_r} and w_{L_r}.

Two goods are produced and exchanged in the model. The agricultural good is produced under perfect competition with constant returns to scale employing one unit of unskilled labour to obtain one unit of output and it is homogeneous across the two regions and freely exchanged between them. The price p_{Ar} of the agriculture good in region r is chosen as the numeraire. Therefore, the unskilled wage, w_{L_r}, is equal to 1 in both regions ($w_{L_r} = p_{Ar} = 1$). There is also a mass N of varieties of a modern manufactured good: each variety is produced by a monopolistically competitive firm with increasing returns to scale employing both skilled and unskilled workers.

2.1 The demand side

Each individual i located in region r consumes the quantity A_{ir} of the agricultural good, and a mass N of varieties of the manufactured good, with each variety denoted by index s and consumed in quantity $X_{ir}(s)$. The differentiated varieties are aggregated by a constant elasticity of substitution function in X_{ir}. We assume that agents care not only about their consumption of the differentiated good X_{ir}, but also about those of their neighbours X_{-ir} in the same region r. We note $\Lambda_r(i)$ the set of direct neighbours/acquaintances to agent i in region r, which are also denoted by $-i$. In other words, we assume that there is a *conspicuous effect* of consumption and that it affects only agents that are both geographically and socially close.

Given these assumptions, and adapting Ghiglino and Goyal (2010) to the continuous case, we assume that workers' preferences are represented by the following utility function

$$U(A_{ir}, \Phi(X_{ir}, X_{-ir}, \Lambda_r(i))) = A_{ir}^{1-\mu} \left(\Phi(X_{ir}, X_{-ir}, \Lambda_r(i))\right)^{\mu} \quad \text{with} \quad 0 < \mu < 1 \tag{1}$$

where

$$X_{-ir} = \int_{\Lambda_r(i)} X_{jr} dj$$

and $\Phi : R \times R \times D \rightarrow R$, with X_{jr} denoting the consumption of the differentiated good by individual $j \in \Lambda_r(i)$, with the average consumption in his neighbourhood, $\Lambda_r(i) \in D$ an element of the set D of subset of the real line. The specification of $\Lambda_r(i)$ will be refined when considering specific networks and the size effect.

Note that this formulation appears to be the interesting case to consider; see Frank (2007) for a discussion on differences in social sensitiveness across goods and Kuhn et al. (2008) for evidence that relative consumption effects are prominent with some goods but not others. It is also natural to assume that if consumption externalities are symmetric for both goods, then the social comparisons will simply wash out and equilibrium will be analogous to the equilibrium in an economy with no consumption externalities (see Arrow and Dasgupta (2007)).

For fixed set of neighbours, the function Φ is increasing in X_{ir} and decreasing in X_{-ir}. Individuals care about their consumption relative to the average consumption of their neighbors, but this effect might be weighted by a term $S(\Omega(\Lambda_r(i)))$ characterizing the *size of the neighborhood*. We assume that $S()$ is an arbitrary non decreasing function taking values between 0 and $\Omega(\Lambda_r(i))$, with $\Omega(\Lambda_r(i))$ denoting the Lebesgue measure of the set of neighbours. We will focus on the two polar cases: the *additive specification* with $S(\Omega(\Lambda_r(i))) = \Omega(\Lambda_r(i))$, and the *average specification* with $S(\Omega(\Lambda_r(i))) = 1$. These considerations are reflected in the following general formulation, which is valid when $\Lambda_r(i) \neq \emptyset$:[3]

$$\Phi\left(X_{ir}, X_{-ir}, \Lambda_r(i)\right) = X_{ir} + \alpha S(\Omega(\Lambda_r(i))) \left[X_{ir} - \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\int_{\Lambda_r(i)} dj} \right]. \tag{2}$$

Some remarks are in order here. When the conspicuous effect is absent, that is when $\alpha = 0$, we fall back to the Forslid and Ottaviano (2003) model.[4] When $\alpha > 0$, individuals are negatively affected by the consumption of their neighbors; and $\alpha < 0$ corresponds to a positive externality. We will focus on the case $\alpha > 0$, because it captures the idea that individuals are negatively affected by an increase in consumption by neighbors. Importantly, we highlight the role of $S(\Omega(\Lambda_r(i)))$. If $S(\Omega(\Lambda_r(i))) = \Omega(\Lambda_r(i))$ then the effect of the size of the neighborhood is linear, whereas if $S(\Omega(\Lambda_r(i))) = 1$ then only

[3] If $\Lambda_r(i) = 0$ then obviously $\Phi(X_{ir}, X_{-ir}) = X_{ir}$.

[4] Forslid and Ottaviano (2003) assume that $L_r = L/2$ and $H_r + H_v = H = 1$.

the average consumption of neighbors matters. We focus on these two polar cases because they help us clarify the types of effects at work; however, we recognize that the intermediate specification with a small size effect is more realistic.

The total mass of produced varieties N, is the sum of the mass of varieties produced in region r, n_r, and the mass of varieties produced in region v, n_v. The horizontally differentiated manufactured good consumed by individual i in region r is given by

$$X_{ir} = \left(\int\limits_{s \in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}} ds \right)^{\frac{\sigma}{\sigma-1}} \tag{3}$$

where $\sigma > 1$ is both the elasticity of demand of any variety and the elasticity of substitution between any two varieties.

Let $p_{kr}(s)$ be the price of a manufactured good s produced in $k = 1, 2$ and sold in r. Given the price for the agricultural good $p_{Ar} = 1$ and his/her income w_{ir}, the problem of a consumer i in region r is to maximize the utility function in (1) subject to the budget constraint

$$A_{ir} + \int\limits_{s \in N} p_{kr}(s) X_{ir}(s) ds = w_{ir} \tag{4}$$

In Appendix I (I.B) it is shown that then the individual demand in region r for variety s produced in k is given by

$$X_{ikr}(s) = \frac{p_{kr}(s)^{-\sigma}}{p_{X_r}^{1-\sigma}} E_{ir} \tag{5}$$

where $E_{ir} = p_{X_r} X_{ir}$ is the individual expenditure on manufactures in region r and p_{X_r} is the local price index of manufactures in r defined as follows

$$p_{X_r} = \left(\int\limits_{s \in n_r} p_{rr}(s)^{1-\sigma} ds + \int\limits_{s \in n_v} p_{vr}(s)^{1-\sigma} ds \right)^{\frac{1}{1-\sigma}} \tag{6}$$

From the consumer's problem in Appendix I (I.A), we also find that the consumer demand for agriculture in region r is

$$A_{ir} = (1-\mu) \left(w_{ir} - p_{X_r} \frac{\alpha S(\Omega(\Lambda_r(i)))}{1 + \alpha S(\Omega(\Lambda_r(i)))} \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\int_{\Lambda_r(i)} dj} \right) \tag{7}$$

and that the individual demand of manufactures in region r is

$$X_{ir} = \frac{\mu}{p_{X_r}} \left(w_{ir} + p_{X_r} \frac{1-\mu}{\mu} \frac{\alpha S(\Omega(\Lambda_r(i)))}{1+\alpha S(\Omega(\Lambda_r(i)))} \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\int_{\Lambda_r(i)} dj} \right) \tag{8}$$

Let us define the wage net of the conspicuous effect of individual i in region r as follows

$$W_{\alpha ir} \equiv w_{ir} - p_{X_r} \frac{\alpha S(\Omega(\Lambda_r(i)))}{1+\alpha S(\Omega(\Lambda_r(i)))} \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\int_{\Lambda_r(i)} dj} \tag{9}$$

Notice that $W_{\alpha ir}$ must be positive to ensure the positivity of the demand for the agricultural good in (7). Furthermore, the individual demand for agriculture in region r can be rewritten as follows

$$A_{ir} = (1-\mu) W_{\alpha ir}, \tag{10}$$

while the individual demand of manufactures in region r (8) is

$$X_{ir} = \frac{\mu}{p_{X_r}} W_{\alpha ir} + \frac{\alpha S(\Omega(\Lambda_r(i)))}{1+\alpha S(\Omega(\Lambda_r(i)))} \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\int_{\Lambda_r(i)} dj} \tag{11}$$

Expressions (10) and (11) show that for given prices and wages, the demand in both goods depend linearly on the wage net of the conspicuous consumption effect, $W_{\alpha ir}$. This decomposition highlights the loss in disposable income as agents waste part of their income only to adjust their own consumption to the consumption of their neighbours. In particular, it shows that there is a fall in the demand of the ordinary good.

Concerning the differentiated good, we see from (11) that the demand has an additional term that increases with α, for given consumption of neighbours ($\int_{\Lambda_r(i)} X_{jr} dj$). In fact, expression (8) shows that when α increases, this second effect dominates that produced by the reduction of $W_{\alpha ir}$ and, thus, the individual demand X_{ir} increases with α.[5]

[5] Moreover, if we define $E_{\Lambda_r(i)} \equiv p_{X_r} \int_{\Lambda_r(i)} X_{jr} dj$ as the *total expenditure on the differentiated good by neighbours* of individual i in region r, we can also point out what follows: for given wages w_{ir} and total expenditures of neighbours of individual i in region r on the differentiated good $E_{\Lambda_r(i)}$, an increase in conspicuous consumption effects (that is an increase in α) reduces the share of wage used by individuals to buy the traditional good, $\frac{A_{ir}}{w_{ir}} = (1-\mu) \left(1 - \frac{\alpha S(\Omega(\Lambda_r(i)))}{1+\alpha S(\Omega(\Lambda_r(i)))} \frac{E_{\Lambda_r(i)}}{w_{ir} \int_{\Lambda_r(i)} dj} \right)$, and, consequently, increases the share of wage used to buy the manufactured good.

To simplify the analysis we restrict our attention to the case in which the network of interactions within cities/regions is *regular* in the sense that all agents of a given type in region r occupy equivalent positions and therefore can be treated as identical. This implies that the consumption of the differentiated good is equal for all skilled in region r, noted then $X_{ir} = X_{H_r}$. The same must be true for the unskilled, so that $X_{ir} = X_{L_r}$. Consider now a skilled agent i in region r. The term $\int_{\Lambda_r(i)} X_{jr} dj$ can be rewritten in this case as

$$\int_{\Lambda_{H_r}} X_{jr} dj = h_{H_r}(H_r) X_{H_r} + l_{H_r}(H_r) X_{L_r}$$

where $h_{H_r}(H_r)$ is the mass of high skilled neighbours to a high skill agent in r, $l_{H_r}(H_r)$ is the mass of low skilled neighbours to a high skill agent in r. The mass of neighbours for the skilled agent i in r is then

$$\Lambda_{H_r} = h_{H_r}(H_r) + l_{H_r}(H_r)$$

Note that these masses depend only on the total mass of skilled agents in region r, because the network is regular and because the unskilled are immobile. If, instead, i is an unskilled agent, the total consumption of his/her neighbours is

$$\int_{\Lambda_{L_r}} X_{jr} dj = h_{L_r}(H_r) X_{H_r} + l_{L_r}(H_r) X_{L_r}$$

where $h_{L_r}(H_r)$ is the mass of high skilled neighbours to a low skill agent in region r, $l_{L_r}(H_r)$ is the mass of low skilled neighbours to a low skill agent in region r, and the total mass of neighbours for the unskilled agent i in r is

$$\Lambda_{L_r} = h_{L_r}(H_r) + l_{L_r}(H_r)$$

It is important to stress that we assume that the network in each region is exogenously given, because agents choose to which region they migrate but do not chose on the type of network that prevails in these regions. Still, typically the masses $h_{H_r}(H_r)$, $h_{L_r}(H_r)$, $l_{H_r}(H_r)$ and $l_{L_r}(H_r)$ are endogenously determined and depend on H_r. In the sequel we simplify notation and drop the explicit dependence on H_r.

The individual demand of the differentiated good by a skilled consumer in region r , X_{H_r}, and that by an unskilled consumer in region r, X_{L_r}, depend on the structure of the network and the size of the neighborhood $S()$. In general, in Appendix I (I.C) we describe how we obtain the system (51) of two equations that can be solved *for a given structure of the network* and *size of the neighborhood* to find X_{H_r} and X_{L_r}. Clearly, the quantities depend on h_{H_r}, h_{L_r}, l_{H_r} and l_{H_r}. Finally, total demand for the differentiated good in region r, X_r, is given by

$$X_r = X_{H_r} H_r + X_{L_r} L, \tag{12}$$

while the total demand in region r for variety s produced in region k derived from (5) and (12) is

$$X_{kr}(s) = \frac{p_{kr}(s)^{-\sigma}}{p_{X_r}^{-\sigma}} \left(X_{H_r} H_r + X_{L_r} L \right) \tag{13}$$

2.2 The supply side

Each manufacturing variety is produced by a monopolistically competitive firm with increasing returns to scale employing both skilled and unskilled workers. Specifically, to produce $Q_r(s)$ units of variety s the firm located in region r incurs a fixed input requirement of f units of skilled workers independently of the production level, and $\beta Q_r(s)$ units of unskilled workers. The cost function for each firm producing variety s in region r is

$$TC_r(s) = f w_{H_r} + \beta Q_r(s)$$

Given the fixed input requirement, the mass of firms producing in region r is proportional to the mass of its skilled residents with

$$n_r = \frac{H_r}{f} \tag{14}$$

To import one unit of a variety in each given region, $\tau > 1$ units have to be shipped from the other region. Hence, profits for a firm producing variety s in region r are given by the sum of revenues from the domestic (r) and the foreign ($v \neq r$) region net of total cost of production

$$\Pi_r(s) = p_{rr}(s) X_{rr}(s) + p_{rv}(s) X_{rv}(s) - f w_{H_r} - \beta \left[X_{rr}(s) + \tau X_{rv}(s) \right] \tag{15}$$

From the first order condition for the maximization of profits, we obtain that the price set by the firm producing variety s for the domestic market r is

$$p_{rr}(s) = \frac{\sigma}{\sigma - 1} \beta \tag{16}$$

and the price set for the foreign market v is

$$p_{rv}(s) = \frac{\sigma}{\sigma - 1} \beta \tau \tag{17}$$

for every $r, v = 1, 2$ and $r \neq v$.

Using (16) and (17), the price index in (6) becomes

$$p_{X_r} = \frac{\sigma}{\sigma - 1} \beta \left(n_r + n_v \phi \right)^{\frac{1}{1-\sigma}} \tag{18}$$

where $\phi = \tau^{1-\sigma} \in [0, 1]$ is a direct measure of the *freeness of trade*, with its value equal to zero when trade costs are prohibitively high ($\tau \to \infty$), and equal to 1 when markets are perfectly integrated ($\tau = 1$). Then, notice that using equations (14), $H_v = H - H_r$ and (18), the price indexes p_{X_r} and p_{X_v} can be written as follows

$$p_{X_r} = \frac{\beta \sigma}{\sigma - 1} \left(\frac{1}{f} \right)^{\frac{1}{1-\sigma}} \left[H_r + (H - H_r) \phi \right]^{\frac{1}{1-\sigma}} \tag{19}$$

$$p_{X_v} = \frac{\beta \sigma}{\sigma - 1} \left(\frac{1}{f} \right)^{\frac{1}{1-\sigma}} \left[(H - H_r) + H_r \phi \right]^{\frac{1}{1-\sigma}} \tag{20}$$

Wages for skilled are derived from the free entry condition, which implies that profits in (15) are equal to zero in equilibrium with

$$w_{H_r} = \frac{p_{rr}(s) X_{rr}(s) + p_{rv}(s) X_{rv}(s) - \beta \left[X_{rr}(s) + \tau X_{rv}(s) \right]}{f} \tag{21}$$

Using (16) and (17) we can rewrite the wage in (21) as follows

$$w_{H_r} = \tfrac{\beta}{(\sigma-1) f} \left[X_{rr}(s) + \tau X_{rv}(s) \right] = \tfrac{\beta}{(\sigma-1) f} Q_r(s) \tag{22}$$

where $Q_r(s) \equiv X_{rr}(s) + \tau X_{rv}(s)$ is the total production by the firm producing variety s in region r. Finally, we can use (13), (16), (17), (19) and (20) to obtain that the production of the firm producing variety s in r, $Q_r(s)$, is

$$Q_r(s) = f^{\frac{\sigma}{\sigma-1}} \left\{ \tfrac{X_{H_r} H_r + X_{L_r} L}{\left[H_r + (H - H_r) \phi \right]^{\frac{\sigma}{\sigma-1}}} + \phi \tfrac{X_{H_v} (H - H_r) + X_{L_v} L}{\left[(H - H_r) + H_r \phi \right]^{\frac{\sigma}{\sigma-1}}} \right\}$$

3 Mobility decision and equilibrium

The indirect utility function for agent i, as obtained in Appendix I (I.A) is

$$U(A_{ir}, \Phi(X_{ir}, X_{-ir}, \Lambda_r(i))) = \eta \left[1 + \alpha S(\Omega(\Lambda_r(i)))\right]^{\mu} \frac{W_{\alpha ir}}{(p_{X_r})^{\mu}} \tag{23}$$

where $\eta \equiv (1-\mu)^{1-\mu} \mu^{\mu}$.

Taking into account that $H_v = H - H_r$ and that $W_{\alpha H_r}$ is the value that $W_{\alpha ir}$ takes for skilled workers in region r, we can use (23) to analyze the location decision of skilled workers. Formally, we need to consider the logarithm of the ratio of the current indirect utility levels in region r with respect to region v, that is[6]

$$\begin{aligned} V(H_r, \phi, \alpha) &\equiv \ln \left[\frac{\frac{\eta(1+\alpha S(\Omega(\Lambda_{H_r})))^{\mu} W_{\alpha H_r}}{p_{X_r}^{\mu}}}{\frac{\eta(1+\alpha S(\Omega(\Lambda_{H_v})))^{\mu} W_{\alpha H_v}}{p_{X_v}^{\mu}}} \right] = \\ &= \ln \left[\left(\frac{p_{X_v}}{p_{X_r}} \right)^{\mu} \left(\frac{1+\alpha S(\Omega(\Lambda_{H_r}))}{1+\alpha S(\Omega(\Lambda_{H_v}))} \right)^{\mu} \frac{W_{\alpha H_r}}{W_{\alpha H_v}} \right] \end{aligned} \tag{24}$$

where $\Lambda_{H_k} = \Omega(\Lambda_k(i))$ is the mass of neighbours to a skilled worker in region k. From (19) and (20) we can see that the value of α does not directly influence the price indexes in the two regions in (24). However, it can have an indirect effect as it brings a change in the mass of skilled workers (and of firms) in the two regions and therefore of H_r. Another indirect effect may come through the wage in (22).

In the model the notion of equilibrium is associated to the absence of migration. We then define a *spatial equilibrium* by $\dot{H}_r = 0$. In fact, the decision of a worker to migrate depends on the value taken by the function V. When $0 < H_r < H$, if $V(H_r, \phi, \alpha) > 0$ then a worker from region v will move to region r while a worker from region r would not move. On the other hand, if $V(H_r, \phi, \alpha) < 0$ a worker from region r would migrate to region v. Consequently, at an interior equilibrium we expect $V(H_r, \phi, \alpha) = 0$. When there is *full agglomeration* in region r, or in region v, that is $H_r = H$ or $H_r =$

[6] Alternatively, we could examine the location decision of skilled workers studying the current indirect utility differential, as in Forslid and Ottaviano (2003). However, we choose to work with the logarithm of the ratio of the current indirect utility levels because it gives an expression that is analytically more tractable, which, in any case, identifies the same critical values of ϕ for the sustainability of the different types of equilibria.

0, the dynamics is slightly more subtle. Formally, the process of migration is determined by an equation of motion. Expressing time with t, and following Forslid and Ottaviano (2003), the migration process is summarized as follows

$$\dot{H}_r \equiv dH_r/dt = \begin{cases} V\left(H_r, \phi, \alpha\right) \text{ if } 0 < H_r < H \\ \min\left\{0, V\left(H_r, \phi, \alpha\right)\right\} \text{ if } H_r = H \\ \max\left\{0, V\left(H_r, \phi, \alpha\right)\right\} \text{ if } H_r = 0 \end{cases} \tag{25}$$

According to the Samuelson's principle only stable equilibria deserve interests as they are the only one surviving an evolving environment. A spatial equilibrium is *stable* if a deviation from the equilibrium, that is a deviation of H_r from its equilibrium value, generates changes in H_r - described by the equation of motion in (25) - that bring the distribution of workers back to the original one. Hence, an interior equilibrium ($0 < H_r < H$) is stable only if $\frac{dV}{dH_r}\left(H_r, \phi, \alpha\right) \equiv V_{H_r}\left(H_r, \phi, \alpha\right) < 0$. A corner configuration ($H_r = 0$ or $H_r = H$) is an equilibrium as soon as $V\left(0, \phi, \alpha\right) < 0$ or $V\left(H, \phi, \alpha\right) > 0$. Therefore, a corner equilibrium is generically stable.

The function $V\left(H_r, \phi, \alpha\right)$ explicitly, but also implicitly through the wages, depends on H_r and on the conspicuous effect. Indeed, substituting (19) and (20) into (24) we obtain that

$$V\left(H_r, \phi, \alpha\right) \equiv \ln\left\{\left[\frac{(H-H_r)+H_r\phi}{H_r+(H-H_r)\phi}\right]^{\frac{\mu}{1-\sigma}}\left(\frac{1+\alpha S(\Omega(\Lambda_{H_r}))}{1+\alpha S(\Omega(\Lambda_{H_v}))}\right)^{\mu}\frac{W_{\alpha H_r}}{W_{\alpha H_v}}\right\} \tag{26}$$

4 The complete network

The benchmark economy we investigate is one in which in each given region, each worker compares his/her consumption with the average consumption of all the agents in the region weighted by the measure of the comparison group. In other words, the network in each region is *complete (or integrated)* and $S(\Omega(\Lambda_r(i))) = \Omega(\Lambda_r(i))$. This assumption implies that $h_{H_r} = h_{L_r} = H_r$ and $l_{H_r} = l_{L_r} = L$. As shown in Appendix I (I.C), equations (7) and (8) can be used to show that the individual demand of the differentiated good by a skilled workers in region r is

$$X_{H_r} = \frac{\mu}{p_{X_r}}\frac{w_{H_r} + \alpha\left(L + H_r w_{H_r}\right) + \alpha\mu\left(w_{H_r} - 1\right)L}{1 + \alpha\mu\left(L + H_r\right)} \tag{27}$$

and that by an unskilled workers living in the same region is

$$X_{L_r} = \frac{\mu}{p_{X_r}}\frac{1 + \alpha\left(L + H_r w_{H_r}\right) - \alpha\mu\left(w_{H_r} - 1\right)H_r}{1 + \alpha\mu\left(L + H_r\right)} \tag{28}$$

Moreover, from (12), (13), (16)-(18), (27) and (28) we find that for a firm producing variety s in region r, the aggregate local demand is

$$X_{rr}(s) = \mu \frac{(\sigma - 1)\left(L + H_r w_{H_r}\right)}{\sigma\beta\left(n_r + n_v \phi\right)} \frac{1 + \alpha\Lambda_r}{1 + \alpha\mu\Lambda_r} \tag{29}$$

while the aggregate foreign demand in region v is

$$X_{rv}(s) = \mu \frac{(\sigma - 1)\,\tau^{-\sigma}\left(L + H_v w_{H_v}\right)}{\sigma\beta\left(n_v + n_r \phi\right)} \frac{1 + \alpha\Lambda_v}{1 + \alpha\mu\Lambda_v} \tag{30}$$

with $r, v = 1, 2$ and $r \neq v$, and with the total number of neighbors in region r and v respectively given by $\Lambda_r = H_r + L$ and $\Lambda_v = H_v + L$.

4.1 The new forces produced by conspicuous consumption.

A first insight on the effect of relative concerns can be gained from equation (29). Indeed, taking the derivative of $X_{rr}(s)$ with respect to α, we find that for a given distribution of workers (and consequently of firms) and for given wages, the demand $X_{rr}(s)$ increases with α. Moreover, for fixed wages, price indexes and α, an increase in H_r and in Λ_r, increase the demand $X_{rr}(s)$, the strength of this effect being increasing in α. This property is due to the fact that the migration of skilled workers to a region induces the workers in the region to increase their demand, larger values of α generating larger increases.[7]

The increase in the demand implies that the operating profits of firms producing in the same region increase. Finally, this produces an increase in

[7] To show this, for given values of α, of the wages and of the price indexes - which imply a given value of $(n_r + n_v\phi)$, we evaluate the following expression

$$\frac{\partial X_{rr}(s)}{\partial H_r} - \left.\frac{\partial X_{rr}(s)}{\partial H_r}\right|_{\alpha=0} =$$

$$= \frac{\partial\left(\mu \frac{(\sigma-1)\left(L+H_r w_{H_r}\right)}{\sigma\beta(n_r+n_v\phi)} \frac{1+\alpha(H_r+L)}{1+\alpha\mu(H_r+L)}\right)}{\partial H_r} - \left.\frac{\partial\left(\mu \frac{(\sigma-1)\left(L+H_r w_{H_r}\right)}{\sigma\beta(n_r+n_v\phi)}\right)}{\partial H_r}\right|_{\alpha=0} =$$

$$= \alpha\mu\,(1-\mu)\,(\sigma-1)\,\frac{L+2w_{H_r}H_r+Lw_{H_r}+w_{H_r}\alpha\mu(L+H_r)^2}{\sigma\beta(L\alpha\mu+\alpha\mu H_r+1)^2(n_r+\phi n_v)} > 0$$

the wages received by workers employed in the region, a centripetal force. To be more specific, consider a symmetric initial situation with $H_r = H/2$, equal expenditures in the two regions (i.e., $(L + H_r w_{H_r})$ and $(L + H_v w_{H_v})$) and equal price indexes (i.e., which imply given values of $H_r + (H - H_r)\,\phi$ and $(H - H_r) + H_r\phi$). In this case, the wage w_{H_r} given by expression (53) in Appendix II (II.A) tends to increase as the number of neighbours in r increases (because H_r increases). Interestingly, the extent of the increase of w_{H_r} is larger when ϕ is low, and is reduced as ϕ increases.[8]

To summarize, relative concerns strengthen the traditional centripetal force identified as the *market-size effect* in Forslid and Ottaviano (2003). Consequently, this force intensifies the process of agglomeration of workers and firms in a region. While the modification of the two other traditional forces in Forslid and Ottaviano (2003) is discussed in Appendix I (I.D), here we can state the following:

Lemma 1 *The increase in the mass of skilled neighbors in a region induces a positive effect on demand per local firm, which rises skilled real wage. Relative concerns therefore strengthen the traditional "market size" effect in the original core-periphery model as they produce a new* ***centripetal force*** *attracting further skilled workers in the region.*

Conspicuous consumption has also a more direct effect on individual welfare. Indeed, comparing the expression for $V\,(H_r, \phi, \alpha)$ in (26) when $\alpha > 0$ with the case $\alpha = 0$, we see that the introduction of relative concerns brings in the factor $\frac{1+\alpha\Lambda_{H_r}}{1+\alpha\Lambda_{H_v}} = \frac{1+\alpha\Lambda_r}{1+\alpha\Lambda_v} = \frac{1+\alpha\Lambda_r}{1+\alpha(H-\Lambda_r+2L)}$ that is increasing in the number of neighbors Λ_r. Furthermore, relative concerns also modify the third factor, as it affects the wage net of relative concerns (see equation (9)). To analyze this effect, notice that the conspicuous effect in $W_{\alpha H_r}$ can be rewritten using (27) and (28). Indeed, we can substitute the total amount of consumption

[8] Indeed, at the symmetric equilibrium (with $H_r = H/2$ and $\frac{(L+H_r w_{H_r})}{H_r+(H-H_r)\phi} = \frac{(L+H_v w_{H_v})}{(H-H_r)+H_r\phi}$), for given total expenditures and price indexes, the sign of $\frac{\partial w_{H_r}}{\partial H_r}$ with $w_{H_r} = \frac{\mu}{\sigma}\frac{(L+H_r w_{H_r})}{H_r+(H-H_r)\phi}\left[\frac{1+\alpha(H_r+L)}{1+\alpha\mu(H_r+L)} + \phi\frac{1+\alpha(H-H_r+L)}{1+\alpha\mu(H-H_r+L)}\right]$ is equal to the sign of $\frac{\partial\left(\frac{1+\alpha(H_r+L)}{1+\alpha\mu(H_r+L)}+\phi\frac{1+\alpha(H-H_r+L)}{1+\alpha\mu(H-H_r+L)}\right)}{\partial H_r} = \alpha\,(1-\mu)\,\frac{(H\alpha\mu+2L\alpha\mu+2)^2(1-\phi)}{4(L\alpha\mu+\alpha\mu H_r+1)^2(H\alpha\mu+L\alpha\mu-\alpha\mu H_r+1)^2} > 0$ which is positive when $\alpha > 0$. Let us notice that the extent of this increase is larger when ϕ is low, and that it becomes smaller when ϕ increases.

by the neighbours of skilled workers $\int_{\Lambda_{H_r}} X_{jr}dj = X_{H_r}H_r + X_{L_r}L$ in the definition of $W_{\alpha H_r}$ in (9) and rewrite the wage of skilled workers net of the conspicuous effect. The following expression is obtained;[9]

$$W_{\alpha H_r} = w_{H_r} - \alpha\mu \tfrac{L+H_r w_{H_r}}{1+\alpha\mu(L+H_r)} \tag{31}$$

Expression (31) shows that for a given skilled wage, $W_{\alpha H_r}$ tends to decrease when H_r increases.[10] This identifies the existence of a negative effect on $W_{\alpha H_r}$ produced by the increase in the mass of skilled workers in a region (i.e., H_r) in the presence of conspicuous good consumption. The presence of this negative effect tends to push skilled workers back to the region of origin, and allows us to state the following.

Lemma 2 *The increase in the mass of skilled workers in a region increases the mass of neighbors and of their manufacturing consumption, inducing a welfare loss on both skilled and unskilled workers that generates a new* ***centrifugal force*** *for mobile skilled agents.*

We can now focus our attention on the existence and stability of equilibria in this model.

4.2 The existence and stability of equilibria

The model can generate several types of equilibria, that is, situations in which agents are not willing to migrate. As in the standard core-periphery model, it can be shown that the *symmetric equilibrium*, with $H_r = H_v = H/2$ and all variables assuming the same value in both regions, always exists as an interior equilibrium. However, its stability depends on the underlying parameters. To be stable the derivative of $V(H_r, \phi, \alpha)$ with respect to H_r evaluated at

[9] Let us observe that we can get an analogous expression for the wage of unskilled workers net of the conspicuous effect $W_{\alpha L_r}$. Specifically, we can substitute the total amount of consumptions by the neighbours of unskilled workers $\int_{\Lambda_r(i)} X_{jr}dj = X_{H_r}H_r + X_{L_r}L$ in the definition of $W_{\alpha L_r}$ in (9) and obtain that

$$W_{\alpha L_r} \equiv 1 - \alpha\mu \tfrac{L+H_r w_{H_r}}{1+\alpha\mu(L+H_r)}$$

This expression is important to check the positivity of agricultural demand by unskilled workers in (7).

[10] This is true when the skilled wage is above a minimum level smaller than 1, that is for $w_{H_r} > L\alpha\mu/(1 + L\alpha\mu)$.

the symmetric equilibrium, that is $V_{H_r}(H/2, \phi, \alpha)$, must be negative. We can state the following Proposition:

Proposition 3 *In the case of the additive specification, $S(\Omega(\Lambda_r)) = \Omega(\Lambda_r)$, with a complete network and conspicuous consumption effects, the* ***symmetric equilibrium*** *is always stable for high levels of the openness to trade ϕ.*

Proof Consider the derivative of $V(H_r, \phi, \alpha)$ at the symmetric equilibrium. It can be shown using the expressions for skilled wages derived in Appendix II (II.A) that

$$V_{H_r}(H/2, \phi, \alpha) = \frac{4\left(f_2\phi^2 + f_1\phi + f_0\right)}{H(\phi+1)(\sigma-1)[\alpha(H+2L)+2]\{2+\alpha[2L-H(\sigma-1)]\}} * \frac{1}{\{2[\sigma-\mu+\phi(\sigma+\mu)]+\mu\alpha(H+2L)[\sigma-1+\phi(\sigma+1)]\}} \tag{32}$$

where the coefficients f_0, f_1 and f_2 are functions of μ, σ, L, H and α. We know that all factors in the denominator are positive, given that $\sigma > 1 > \mu$ and that the term $\{2 + \alpha[2L - H(\sigma-1)]\}$ is positive when $W_{\alpha H_r}$ evaluated at $H_r = H/2$ is positive.[11] Thus, the sign of $V_{H_r}(H/2, \phi, \alpha)$ depends on the sign of the expression $F \equiv f_2\phi^2 + f_1\phi + f_0$ in the numerator, and the symmetric equilibrium is stable when $F < 0$ and unstable when $F > 0$. However, we know that when $\alpha = 0$, $F = a_0 = 8(1-\phi)(2\phi\sigma\mu - \mu\phi - \phi\sigma + \phi\sigma^2 + \mu^2\phi - \mu^2 + \sigma - \sigma^2 + 2\sigma\mu - \mu)$. In this case, a_0 is the relevant term in determining the sign of $V_{H_r}(H/2, \phi, \alpha)$, and, as in Forslid and Ottaviano (2003), the symmetry breaking point $\phi_b^{FO} = \frac{(\sigma-1-\mu)(\sigma-\mu)}{(\mu+\sigma)(\mu+\sigma-1)} < 1$ is such that the symmetric equilibrium is stable only for $\phi \in (0, \phi_b^{FO})$.[12] Note that for $\phi = 1$, $F = a_0 = 0$. As α becomes positive and rises, F becomes negative, that is $F = 4\mu H\alpha^2\sigma(\sigma-1)(1-\mu)(2L + H)\{\alpha[H(\sigma-1) - 2L] - 2\} < 0$,[13] and the symmetric equilibrium remains stable for $\phi = 1$. By continuity the result holds for high levels of integration, i.e., large ϕ.

We now focus our attention on equilibria in which all skilled workers move to one region. Note that, provided these *full agglomeration equilibria* exist, they are stable. We now state the following.

[11] This requires that the relative mass of skilled workers with respect to the mass of unskilled is not relatively too high, that is, $H < 2\frac{(1+L\alpha)}{\alpha(\sigma-1)}$.

[12] We assume that the no black hole condition, that is $\mu < \sigma - 1$, holds. This condition rules out the case in which the symmetric equilibrium is never stable.

[13] We recall that we assume $H < 2\frac{(1+L\alpha)}{\alpha(\sigma-1)}$ to have $W_{\alpha H_r}(H/2) > 0$.

Proposition 4 *Assume that the network in each region is complete and $S(\Omega(\Lambda_r)) = \Omega(\Lambda_r)$. Then for low trade costs, weak positive relative concerns destroy* ***full agglomeration****. On the other hand, this equilibrium exists, and is stable, for intermediate values of trade openness ϕ.*

Proof The value of $V(H_r, \phi, \alpha)$ when all skilled workers are located in region r is given by

$$V(H, \phi, \alpha) = \ln \left\{ \frac{g_0\{2+\alpha[2L-H(\sigma-1)]\}\phi^{1-\frac{\mu}{\sigma-1}}}{d_2\phi^2+d_1\phi+d_0} \right\} \tag{33}$$

where

$$\begin{aligned}
g_0 &= \sigma(1+\alpha\mu L)\left[\tfrac{\alpha(H+L)+1}{\alpha L+1}\right]^{\mu} > 1 \\
d_2 &= [1+\alpha(H+L)][\mu\alpha L(\sigma+1)+\sigma+\mu] > 0; \\
d_1 &= -H\alpha\sigma[\mu\alpha(H+L)(\sigma-1)+\sigma-\mu] < 0; \\
d_0 &= (1+\alpha L)[\mu\alpha(H+L)(\sigma-1)+\sigma-\mu] > 0
\end{aligned}$$

When $\alpha = 0$, expression (33) becomes

$$V(H, \phi) = \ln\left(\frac{2\sigma\phi^{1+\frac{\mu}{1-\sigma}}}{\sigma(1+\phi^2)-\mu(1-\phi^2)}\right) \tag{34}$$

Equation (34) generates the sustain point ϕ_s, i.e., the value of ϕ such that full agglomeration is stable for $\phi > \phi_s$, as in Forslid and Ottaviano (2003).[14] When α is positive, we know that $g_0 > 1$ and that expression $\phi^{1+\frac{\mu}{1-\sigma}}$ is increasing in $\phi \in [0,1]$ from 0 (if $\phi = 0$) to 1 (if $\phi = 1$).[15] Given the sign of the parameters d_2, d_1 and d_0, expression $d_2\phi^2 + d_1\phi + d_0$ in the denominator of (33) is an upward opening parabola in ϕ with positive value d_0 when $\phi = 0$.[16] Moreover, we know that, when workers are all agglomerated in r, expression $\{2 + \alpha[2L - H(\sigma-1)]\}$ in the numerator must be positive to have $W_{\alpha H_r} > 0$.[17] Finally, the parabola in the denominator must be positive in order to have $W_{\alpha H_v} > 0$. Figure 1 presents the three possible scenarios

[14] See expression (25) at page 236 in Forslid and Ottaviano (2003).

[15] We recall that we assume the no black hole condition corresponding to the case of $\alpha = 0$ holds, that is $\mu < \sigma - 1$.

[16] Moreover, its minimum value is attained at $\phi = -\frac{d_1}{2d_2} > 0$ when $\alpha > 0$ (and at $\phi = 0$ when $\alpha = 0$), so that the parabola has a negative slope in $\phi = 0$ only if α is positive.

[17] This requires that $H < 2\frac{(1+L\alpha)}{\alpha(\sigma-1)}$.

for this parabola. In Figure 1 the numerator is represented by the dotted line, which characterizes an increasing function in ϕ taking the value 0 when $\phi = 0$ and $g_0 \{2 + \alpha [2L - H(\sigma - 1)]\} > 0$ when $\phi = 1$.

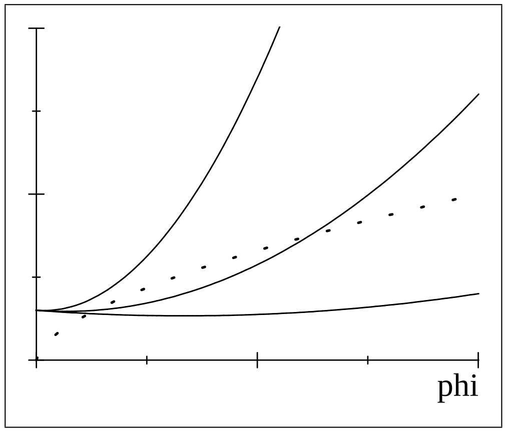

Figure 1

The higher parabola obtained for large values of α corresponds to the case in which full agglomeration is never stable. The lower parabola can be excluded as it implies that with $\phi = 1$ the full agglomeration equilibrium is stable, which contradicts direct computation.[18] The only relevant case is then

[18] Indeed, we know that when $\phi = 1$ the denominator is

$$d_2\phi^2 + d_1\phi + d_0 = \sigma (H\alpha\mu + L\alpha\mu + 1)(H\alpha + 2L\alpha - H\alpha\sigma + 2)$$

while the numerator is

$$g_0 \{2 + \alpha [2L - H(\sigma - 1)]\} \phi^{1-\frac{\mu}{\sigma-1}} = \sigma (1 + \alpha\mu L) \left[\frac{\alpha(H+L)+1}{\alpha L+1}\right]^{\mu} \{2 + \alpha [2L - H(\sigma - 1)]\}$$

Hence when $\phi = 1$, the argument of the logarithm in (33) is given by $\left[\frac{\alpha(H+L)+1}{\alpha L+1}\right]^{\mu} \frac{(1+\alpha\mu L)}{(H\alpha\mu+L\alpha\mu+1)}$, which is equal to 1 when $\alpha = 0$. With a positive value of α, we can show that $\left[\frac{\alpha(H+L)+1}{\alpha L+1}\right]^{\mu} \frac{(1+\alpha\mu L)}{(H\alpha\mu+L\alpha\mu+1)} < 1$ and, thus, agglomeration is never an equilibrium when $\phi = 1$.

Proof. $\left[\frac{\alpha(H+L)+1}{\alpha L+1}\right]^{\mu} \frac{(1+\alpha\mu L)}{(H\alpha\mu+L\alpha\mu+1)} < 1$ requires that $\left[\frac{\alpha(H+L)+1}{\alpha L+1}\right]^{\mu} < \frac{H\alpha\mu+L\alpha\mu+1}{1+\alpha\mu L}$. We know that these two expressions, defined respectively as $LHS = \left[\frac{\alpha(H+L)+1}{\alpha L+1}\right]^{\mu}$ and $RHS = \frac{H\alpha\mu+L\alpha\mu+1}{1+\alpha\mu L}$, are both equal to 1 when $\mu = 0$, and that they both increase in the range $\mu \in [0, 1]$ and assume the same value $\frac{\alpha(H+L)+1}{(1+\alpha L)}$ when $\mu = 1$. However, since $\forall\mu \in (0, 1)$ the $\frac{\partial^2 LHS}{\partial\mu^2} > 0$ and $\frac{\partial^2 RHS}{\partial\mu^2} < 0$, the LHS is convex in μ and the RHS is concave in μ, which implyies that $LHS < RHS$. Q.E.D.

the "middle" parabola. In this case, full agglomeration is stable only for intermediate values of ϕ (when the second higher parabola lies below the dotted curve).

4.3 A general overview: the bifurcation diagram

To explore further the relationship between the strength of relative concerns (α), the openness to trade (ϕ) and the existence and stability of the equilibria we need to focus on "calibrated" versions of the model. First, the numerical analysis is performed on a model in which $L = 20$, $H = 10$, $\sigma = 2$ and $\mu = 0.4$ (Figure 2.a) or $\mu = 0.11$ (Figure 2.b). The black curves represent $F = a_0$ as a function of ϕ when $\alpha = 0$. They show that in this case the symmetric equilibrium is stable only if $F < 0$, that is, only if $\phi < \phi_b^{FO}$.[19] On the other hand the full agglomeration equilibrium is stable for $\phi > \phi_s$. Importantly, $\phi_s < \phi_b$ so there are parameters values such that both types of equilibria are stable and coexists.

Insert Figures 2.a and 2.b at the end of the manuscript about here

Assume now the existence of relative concerns, $\alpha > 0$. Figure 2.a and Figure 2.b plot F for $\alpha = 0.02, 0.04, 0.06, 0.08$. They show that with relative concerns, as ϕ rises the symmetric equilibrium becomes a stable as soon as $\phi > \phi_d$ where ϕ_d is the "dispersion point". Hence, the symmetric equilibrium is stable both for small values of the freeness of trade ϕ, such that $\phi < \phi_b$, and for high levels of integration, such that ϕ is above the new critical level ϕ_d.

Consider now the equilibrium with full agglomeration. As ϕ rises and reaches the sustain point, ϕ_s, full agglomeration becomes a stable equilibrium, as in the case $\alpha = 0$. However, this equilibrium again disappears after trade costs have decreased beyond a new critical level, that we name "unsustain point", ϕ_u. Table 1 show the values of ϕ_s, ϕ_b, ϕ_u and ϕ_d obtained numerically for the model with $L = 4$, $H = 10$, $\sigma = 2$ and $\mu = 0.11$.[20]

[19] The values of the other parameters for which curves in Figure 2.$a-b$ are drawn (equal to those used to derive Figures 5.$a-b$, 6.$a-b$ and 9.$a-b$) are given by: $L = 20$, $H = 10$, $\sigma = 2$, $\mu = 0.4$ in Figure 2.a and $\mu = 0.11$ in Figure 2.b.

[20] The choice of a smaller value of L and $\mu = 0.11$ alows for the asymmetric stable equilibria when $\alpha > 0.04$.

	ϕ_s		ϕ_b	ϕ_d		ϕ_u
$\alpha = 0$	0.7167	$<$	0.7181	/		/
$\alpha = 0.02$	0.6347	$<$	0.6350	0.9891	$<$	0.9892
$\alpha = 0.04$	0.5850	$\boxed{>}$	0.5845	0.9731	$<$	0.9736
$\alpha = 0.06$	0.5504	$>$	0.5495	0.9589	$<$	0.9599
$\alpha = 0.08$	0.5245	$>$	0.5235	0.9475	$<$	0.9489
$\alpha = 0.1$	0.5043	$>$	0.5032	0.9386	$<$	0.9403
$\alpha = 0.12$	0.4877	$>$	0.4867	0.9317	$<$	0.9338
$\alpha = 0.14$	0.4739	$>$	0.4731	0.9265	$<$	0.9289
$\alpha = 0.16$	0.4622	$>$	0.4616	0.9227	$<$	0.9254

Table 1: Critical points for different values of α.

The analysis in Table 1 can be summarized by the bifurcation diagrams reported in Figure 3. In this figure the spatial equilibrium distribution of workers and firms is drawn as a function of the freeness of trade ϕ, $\phi \in [0, 1]$, and for three values of α. The bold continuous lines represent stable equilibria (as pointed out by the arrows), while the bold discontinuous lines represent unstable asymmetric equilibria. Specifically: the first diagram is drawn for $\alpha = 0$ and replicates the bifurcation diagram in Forslid and Ottaviano (2003, p. 237); the second diagram is obtained with $\alpha = 0.02$; the third graphic shows how the bifurcation diagram appears for $0.04 \leq \alpha \leq 0.016$. An an important regularity for its implications, is that when α increases, all the four critical points ϕ_s, ϕ_b, ϕ_d and ϕ_u decrease progressively. The bifurcation diagrams therefore shifts on the left as α increases.

From the bifurcation diagrams and Table 1 we deduce the four main properties that we summarize as follow.

Proposition 5 *In the calibrated model with* $L = 4$, $H = 10$, $\sigma = 2$ *and* $\mu = 0.11$ *and in the case of the complete network we find the following results.*

1. *For low trade costs, relative concerns tends to destabilize the full agglomeration equilibrium and stabilize the symmetric equilibrium. For medium and high trade costs, relative concerns tends to destabilize the symmetric equilibrium and stabilize the agglomeration equilibrium.*

2. *More specifically*

(a) When $\alpha = 0$, full agglomeration is stable for high levels of integration and the symmetric equilibrium is stable for low levels of integration, as $\phi_s < \phi_b$. Both types of equilibrium are stable for intermediate values of integration ϕ.

(b) When $\alpha = 0.02$, we find that $\phi_s < \phi_b < \phi_d < \phi_u$. Then the symmetric equilibrium is stable for low and for high levels of integration ϕ. Full agglomeration is stable for intermediate values of ϕ while no stable partial agglomeration is a stable equilibrium.

(c) When $\alpha \geq 0.04$, we find that $\phi_b < \phi_s < \phi_d < \phi_u$. Then the symmetric equilibrium is stable for low and for high levels of integration ϕ. Full agglomeration is stable for intermediate values of ϕ and stable partial agglomeration equilibria exist for $\phi \in (\phi_b, \phi_s)$.

Insert Figure 3 at the end of the manuscript about here

Figure 3 also shows an interesting property: when $\alpha \geq 0.04$, stable asymmetric equilibria are possible and there exists a *pitchfork* pattern with a continuous, and easily reversible, transition from symmetry to agglomeration. This is not a traditional property either of the seminal core-periphery model by Krugman (1991) or of the Forslid and Ottaviano (2003) footloose entrepreneur model. Indeed, these models exhibit catastrophic agglomeration and locational hysteresis with a tomahawk pattern, as can be seen from the bifurcation diagram obtained with $\alpha = 0$. In these models, once trade freeness has increased beyond the "break point", ϕ_b, all the mobile manufacturing sector catastrophically fully agglomerates in one region. If then trade costs increase again, they do not not restore the symmetric equilibrium the "sustain point", ϕ_s, which lies at a strictly lower level of trade freeness than the break point ϕ_b.

Proposition 6 *For sufficiently large values of α, as openness to trade increases, stable asymmetric equilibria appear and a pitchfork pattern can emerge with a continuous, and easily reversible, transition from symmetry to agglomeration.*

The presence of the conspicuous good effect seems responsible of a location pattern which is new compared to, and sometimes replaces, the original tomahawk diagram identified by the literature on the core-periphery model (see for e.g. Helpman (1998), Tabuchi (1998), Ludema and Wooton (1999), Ottaviano et al. (2002), Tabuchi and Thisse (2002), Murata (2003), Pflüger (2004), Nocco (2009), Berliant and Kung (2009) and Pflüger and Suedekum (2010)).

We conclude with some intuition on the forces generating the above results. When trade costs are small, relative concerns tend to stabilize the symmetric equilibrium while they tend to destabilize it when the cost is intermediate. Indeed, the effect of introducing relative concerns, which increase the conspicuous demand for the differentiated goods, are more localized when trade costs are high (low ϕ), rising more sharply the interior demand. This force, destabilize the symmetric equilibrium as it increases the new centripetal market size effect in Forslid and Ottaviano.[21] On the contrary, when trade costs are low, the increase in the demand for differentiated goods spill over to the other region and the dispersion forces again dominate.

5 Asymmetric networks

So far we assumed that the comparison group was the entire population in the region of residence. However, it might be argued that in the real world only interactions within a smaller group occur. We will start by considering that agents are sensitive to comparisons only within their own type group. Other asymmetric cases will be considered in later sections, for example the case of a directed network in which the unskilled workers compare their own consumption with a subset of skilled workers but not the other way round. At the end of the section, we will extend the analysis to a mixed case in which agents choose between two regions, each with a different network. Finally note that as in the previous section, we assume that agents compare their consumption with the average consumption of the other agents in the relevant neighbourhood weighted by the measure of the comparison group.

Figure 4 illustrates three types of networks we analyze. For these, we consider both the full agglomeration equilibrium and the symmetric equilibrium in which agents are symmetrically dispersed across the two regions; the

[21] We recall that in Forslid and Ottaviano (2003) we have two centrifugal forces, that is the market-size effect and the cost-of-living effect and one centrifugal force, that is the market-crowding effect. These forces are defined as the "traditional" or "original" forces.

orange and blue points represent, respectively, the skilled and the unskilled workers. Note that the illustration is a finite version of the formal model, which has a continuum of agents.

Insert Figure 4 at the end of the manuscript about here

5.1 The segregated network

We consider here the case in which skilled workers are affected only by the consumption of other skilled workers and unskilled workers only by that of other unskilled workers. Formally, we let $h_{H_r} = H_r$, $l_{L_r} = L$, and $h_{L_r} = l_{H_r} = 0$. We show in Appendix II (II.B) that in this case $V(H_r, \phi, \alpha)$ in (26) can be written as

$$V(H_r, \phi, \alpha) = \ln \left\{ \left[\frac{(H-H_r)+H_r\phi}{H_r+(H-H_r)\phi} \right]^{\frac{\mu}{1-\sigma}} \left[\frac{1+\alpha H_r}{1+\alpha(H-H_r)} \right]^{\mu} \frac{\frac{w_{H_r}}{1+\alpha\mu H_r}}{\frac{w_{H_v}}{1+\alpha\mu(H-H_r)}} \right\} \tag{35}$$

where the expressions for the two regional wages for skilled workers, w_{H_r} and w_{H_v}, are given by (58) and (59) in the same Appendix.

We first focus on the equilibrium with full agglomeration. Expression (35) evaluated when *all* skilled workers are located in region r (i.e. $H_r = H$ and $H_v = 0$) is

$$V^s(H, \phi, \alpha) = \ln \left\{ \frac{2\sigma(1+\alpha H)^{\mu} \phi^{1+\frac{\mu}{1-\sigma}}}{\mu H\left[\sigma-1+\phi^2(\sigma+1)\right]\alpha + \left[\sigma-\mu+\phi^2(\sigma+\mu)\right]} \right\}$$

The value of $V^s(H, \phi, \alpha)$ in this case does not depend on L. On the one hand, when $\phi \to 0$ (that is, in autarky), $V^s(H, 0, \alpha) \to -\infty$ and full agglomeration is not sustainable. On the other hand, with complete integration, $\phi = 1$, we find that $V^s(H, \phi, \alpha) = \ln \frac{(1+\alpha H)^{\mu}}{(1+H\alpha\mu)}$, which is negative, provided $\alpha > 0$, because $\frac{\partial\left(\frac{(1+\alpha H)^{\mu}}{(1+H\alpha\mu)}\right)}{\partial \alpha} = -\mu\alpha H^2 \frac{(1-\mu)}{H\alpha+1} \frac{(H\alpha+1)^{\mu}}{(H\alpha\mu+1)^2} < 0$. We can state the following proposition:

Proposition 7 *In the case of segregated networks,* ***full agglomeration*** *is sustainable only for intermediate values of trade costs ϕ.*

We now consider the symmetric equilibrium. The derivative of $V(H_r, \phi, \alpha)$ with respect to H_r evaluated at the symmetric equilibrium is given by the expression

$$V^s_{H_r}(H/2, \phi, \alpha) = \frac{4\left(k_2\phi^2 + k_1\phi + k_0\right)}{H(1+\phi)(\sigma-1)(2+\alpha H)(h_1\phi + h_0)}$$

where the coefficients k_2, k_1, k_0, h_1 and h_0 are functions of μ, σ, H and α and they do not depend on L.[22] As the denominator of $V^s_{H_r}(H/2, \phi, \alpha)$ is positive, the sign of $V^s_{H_r}(H/2, \phi, \alpha)$ depends on that of the parabola $G \equiv k_2\phi^2 + k_1\phi + k_0$, with the symmetric equilibrium stable (vs unstable) only when G is negative (vs positive). We focus our analysis on the calibrated model with $H = 10$ and $\sigma = 2$. Figure 5.a (on the left) represents the value of G as a function of ϕ in the case of $\alpha = 0$ (black curve), $\alpha = 0.02$ (red curve), $\alpha = 0.04$ (blue curve), $\alpha = 0.06$ (green curve) and $\alpha = 0.08$ (yellow curve) when $\mu = 0.4$. Figure 5.b (on the right) represents G for the same values except that now $\mu = 0.11$, where μ is proportional to the share of income, net of the conspicuous effect, devoted to acquire the differentiated good.[23]

The numerical analysis shows that, the introduction of weak relative concerns tends to stabilize the symmetric equilibrium for high levels of integration ϕ. Stability always holds for $\phi = 1$ as then $G = 4\alpha^2H^2\mu\sigma(\sigma-1)(\mu-1) < 0$. The size of the interval of ϕ on which stability holds is increasing in α and decreasing with μ. Figure 5.a also shows that for intermediate levels of integration ϕ the symmetric equilibrium is destabilized when when μ is large. Indeed, in this case the agglomerative effect they produce is stronger than the dispersion effect. On the other hand, Figure 5.b shows that when

[22] Specifically,

$$\begin{aligned}
k_2 &= \alpha^2H^2\mu(\sigma+1)\left[\sigma(\mu-2) + 2(1-\mu)\right] - 2\alpha H\left[\sigma(\sigma+\sigma\mu-1) + 3\mu(\sigma+\mu-1)\right] \\
&\quad -4(\sigma+\mu-1)(\sigma+\mu); \\
k_1 &= 2\alpha^2H^2\mu^2\left[\sigma(\sigma-1)+1\right] + 4\alpha H\left[\sigma(1+\mu)(\sigma-1) + 2\mu^2\right] + 8\left[\sigma(\sigma-1) + \mu^2\right] > 0; \\
k_0 &= \alpha^2H^2\mu(\sigma-1)\left[\sigma\mu - 2(\sigma-1)\right] - 2\alpha H\left[\mu(\mu+3) + \sigma^2(\mu+1) - \sigma(5\mu+1)\right] \\
&\quad -4(\sigma-\mu-1)(\sigma-\mu); \\
h_1 &= 2(\sigma+\mu) + \mu\alpha(\sigma+1)H > 0; \\
h_0 &= 2(\sigma-\mu) + \mu\alpha(\sigma-1)H > 0.
\end{aligned}$$

[23] Specifically, from (4), (9) and (10), we know that $\mu = \frac{p_{X_r}\left(X_{ir} - \frac{\alpha S(\Omega(\Lambda_r(i)))}{1+\alpha S(\Omega(\Lambda_r(i)))}\frac{\int_{\Lambda_r(i)} X_{jr}dj}{\int_{\Lambda_r(i)} dj}\right)}{W_{\alpha ir}}$.

μ is relatively low the symmetric equilibrium can be stabilized for low and intermediate values of economic integration ϕ provided α is sufficiently large.

Hence, we can state the following Proposition:

Proposition 8 *Consider the calibrated model of Proposition 5. For segregated networks when trade costs are small, positive values of α stabilize the **symmetric equilibrium**. For intermediate values of economic integration ϕ, relative concerns tend to destabilize the symmetric equilibrium when μ is relatively large while they tend to stabilize the symmetric equilibrium when μ is relatively small.*

Insert Figure $5.a - b$ at the end of the manuscript about here

5.2 The multiple star network

Star configurations are very popular in sociology and network economics. When agents are in discrete number, a star is a configuration in which an agent, the center, is connected to some other agents, while these are not connected among themselves. Extending the notion to a continuum of agents posses some issues. The problem we face is simplified by the fact that in the model we introduce relative concerns as a deviation from the average consumption of the reference group, weighted by the size of the group. We consider the configuration in which each unskilled worker is affected by the average "conspicuous" consumption of the skilled workers in his, or her, neighbourhood. The generalization of the discrete star to the "continuous"star is performed assuming the size effect is 1 for the effect of the center (the unskilled) on the skilled workers, while it is H_r/L for the weight of the skilled on the unskilled (as there are L stars with a populations of H_r unskilled).

Formally, we let $h_{L_r} = H_r/L,\ \ l_{H_r} = 1$ and $h_{H_r} = l_{L_r} = 0$. In Appendix II (II.C) it is shown that $V(H_r, \phi, \alpha)$ in (26) is given by

$$V^*(H_r, \phi, \alpha) = \ln \left\{ \left[\frac{(H-H_r)+H_r\phi}{H_r+(H-H_r)\phi} \right]^{\frac{\mu}{1-\sigma}} \frac{\left[w_{H_r} - \alpha\mu \frac{L+\alpha H_r + H_r \alpha w_{H_r}(1-\mu)}{L+\alpha(L+H_r)+H_r\mu\alpha^2(2-\mu)} \right]}{\left[w_{H_v} - \alpha\mu \frac{L+\alpha(H-H_r)+(H-H_r)\alpha w_{H_v}(1-\mu)}{L+\alpha(L+H-H_r)+(H-H_r)\mu\alpha^2(2-\mu)} \right]} \right\} \tag{36}$$

where the two regional wages for skilled workers, w_{H_r} and w_{H_v}, are given by the expressions (62) and (63) in the Appendix.

Consider the full agglomeration equilibrium in which *all* skilled workers are located in one region, say region r. Evaluating $V^*(H_r, \phi, \alpha)$ in (36) we obtain the following expression

$$V^*(H, \phi, \alpha) = \ln \left\{ \frac{H^2\sigma(1+\alpha)\left\{-\left[(\sigma-1)+\frac{L}{H}(\mu+1)\right]\alpha^2+\frac{L}{H}\left[2\frac{L}{H}+(3-\sigma)\right]\alpha+2\left(\frac{L}{H}\right)^2\right\}\phi^{1-\frac{\mu}{\sigma-1}}}{z_2\phi^2+z_1\phi+z_0} \right\}$$

where the coefficients z_0, z_1 and z_2 are functions of μ, σ, L, H and α, and the denominator is a parabola open above (given that $z_2 > 0$[24]). The qualitative analysis can be performed using Figure 1 in Section 4.2. As in the case of a complete network analyzed in Section 4.2, for sufficiently low values of α, the numerator in the argument of the logarithm is positive and, as we assume that $W_{\alpha H_r}$ and $W_{\alpha H_v}$ are positive, the denominator is also positive.[25] Furthermore, the numerator can be represented by an increasing function in ϕ which takes value 0 when $\phi = 0$ and is represented by the dotted curve in Figure 1. The denominator is represented by the three continuous curves, of which the higher parabola can be excluded, as it implies that full agglomeration is never stable, which contradicts direct computation. For small values of α two scenario are possible: 1) either full agglomeration is stable for intermediate values of ϕ, this happens when the second higher parabola is below the dotted curve; or 2) full agglomeration is stable for high levels of economic integration ϕ, when the lowest parabola is below the dotted curve.[26] We exclude case 2) as it contradicts numerical analysis and we state what follows.

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[24] Indeed,

$$z_2 = \left[\sigma H^2(1-\mu) + HL(\mu+\sigma) + L^2\mu(1-\mu)\right]\alpha^2 +$$
$$+\left[L^2\sigma + L(2-\mu)(H\sigma + L\mu) + HL\mu\right]\alpha + \left(L^2\sigma + L^2\mu\right) > 0$$

$$\begin{aligned} z_1 &= -\tfrac{z_0\alpha H\sigma}{L(1+\alpha)} \\ z_0 &= L(1+\alpha)\left[\left(L\mu^2 - L\mu - H\mu + 2H\sigma\mu - H\sigma\mu^2\right)\alpha^2 + \right. \\ &\quad \left. + \left(H\sigma - H\mu + L\sigma - 2L\mu + L\mu^2\right)\alpha + (L\sigma - L\mu)\right] \end{aligned}$$

[25] More precisely, Figure 1 can be applied in the case in which $z_0 > 0$ (which is true for sufficiently low values of α), with the numerator represented by the dotted curve and the denominator by one of the parabola.

[26] The minimum value of the parabola in the denominator is attained at $\phi = -\frac{z_1}{2z_2} = \frac{z_0\alpha H\sigma}{2z_2L(1+\alpha)} > 0$ if $z_0 > 0$.

Proposition 9 *In the case of the star networks with weak relative concerns,* ***full agglomeration*** *is stable only for intermediate levels of openness to trade* ϕ*.*

We focus now on the symmetric equilibrium. The derivative of $V^*(H_r, \phi, \alpha)$ in (36) with respect to H_r evaluated at the symmetric equilibrium is

$$V^*_{H_r}(H/2, \phi, \alpha) = \frac{4\left(n_2\phi^2 + n_1\phi + n_0\right)}{H(\phi+1)(\sigma-1)(\alpha H+2L)[\alpha H(\sigma-1)-2L(\alpha+1)]\left(d_1^*\phi+d_0^*\right)}$$

where the coefficients n_2, n_1, n_0, d_1^* and d_0^* are all functions of μ, σ, H, α and L.

To obtain insightful result we focus our analysis on the calibrated model with $H = 10$, $\sigma = 2$, and $L = 20$. Figure 6.a shows the value of $V^*_{H_r}(H/2, \phi, \alpha)$ as a function of ϕ in the case of $\alpha = 0$ (black curve), $\alpha = 0.02$ (red curve), $\alpha = 0.04$ (blue curve), $\alpha = 0.06$ (green curve) and $\alpha = 0.08$ (yellow curve) when $\mu = 0.4$. Figure 6.b represents $V^*_{H_r}$ for the same values but with $\mu = 0.11$. The numerical analysis shows that for high values of economic integration ϕ, the introduction of weak relative concerns stabilizes the symmetric equilibrium. On the other hand, for low values of ϕ the effect destabilizes the symmetric equilibrium. This behavior is qualitatively similar to the one observed in the complete network. However, the size of the effect is very weak for $\mu = 0.4$, much weaker than in the case of a complete network.

Insert Figure 6.$a-b$ at the end of the manuscript about here

Proposition 10 *In the case of star networks, weak relative concerns stabilise the* ***symmetric equilibrium*** *for high levels of openness to trade* ϕ *while they destabilise the symmetric equilibrium for low or intermediate levels of* ϕ*. These effects are weak when* $\mu = 0.4$ *but increase when* μ *decreases.*

5.3 Choosing between an integrated region and a fully segregated region

We now consider the case in which skilled workers can choose between two regions: one characterized by an integrated (complete) network and the other characterised by two segregated networks. Let r be the region with an integrated network and v the region with two segregated networks, one composed only of skilled workers and the other one composed only of unskilled workers. As previously, we focus our analysis on the case in which relative concerns are additive and evaluate the spatial distribution of workers for different levels of trade integration ϕ.

In Appendix II (II.D) we compute the log of the indirect utility levels in the mixed case, noted $V^{mix}(H_r, \phi, \alpha)$ from expression (64). Not surprisingly, we find that $V^{mix}(H/2, \phi, \alpha)$ is different than 0 provided $\alpha > 0$, so that the symmetric configuration is not an equilibrium with relative concerns. Interior equilibria with partial agglomeration may however exist.

The analysis focuses on the calibrated model with parameter values $\alpha = 0.08$, $L = 4$, $H = 10$, $\sigma = 2$ and $\mu = 0.11$, that is, the same used to derive the bifurcation diagram for the complete network in Figure 3. We see from Figure 7.a that for large shipping costs, there is an interior equilibrium, but this is not symmetric. As ϕ rises, the population in the segregated region v increases, until full agglomeration is reached, and becomes sustainable for $\phi \geq 0.3106$.

Insert Figure 7.a and 7.b at the end of the manuscript about here

The same analysis is performed with a higher value of the parameter μ proportional to the share of income net of the conspicuous effect devoted to acquire the differentiated good, and a larger proportion of unskilled workers. Figure 7.b has been drawn for $\mu = 0.4$ and $L = 20$ (satisfying the condition $H < 2L/(\sigma - 1)$ derived in Appendix II (II.D)). The analysis shows that for small ϕ, as ϕ rises, the population density in the integrated region r increases. Full aggregation in this region is reached for a relatively low value of ϕ. This bifurcation diagram shows that for intermediate values of ϕ full agglomeration in the segregated region is also a stable equilibrium.[27]

Proposition 11 *In an economy in which skilled workers can choose between migrating to an integrated or a segregated region when $\alpha = 0.08$, $L = 20$, $H = 10$, $\sigma = 2$ and $\mu = 0.4$, for very low ϕ there exists an **interior equilibrium** in which the integrated region has a higher density. For all other values of ϕ*

[27] If instead, we increase only one of the two parameters to the new value, and the other remain equal to that used to draw Figure 7.a, we get a bifurcation diagram that resembles that in Figure 7.a.

there exists ***full agglomeration*** *in the integrated region. For intermediate values of ϕ there also exists an equilibrium with full agglomeration in the segregated region.*

In light of the above case, we conjecture that in general, when agents can choose between migrating to regions with different networks of interpersonal comparisons, there exists interval of values of trade costs such that there exists an asymmetric interior equilibrium and intervals for which the full agglomeration equilibrium exists and is stable.

5.4 Summary of the analysis: The role of the network for different levels of economic integration

Table 3, column 1, summarizes our findings for the additive specification for the configurations we have considered so far (column 2 of the table considers the average specification that will be presented in Section 6). The analysis in this section has shown that also in the case of asymmetric networks, the interplay of agglomeration and dispersion forces is affected by the conspicuous effect. Furthermore, the resulting force is very sensitive to the topology of the network of "conspicuous" links in each region, as well as to the level of economic integration ϕ.

We have shown that the complete network and the star network exhibit similar patterns for the effects produced by conspicuous consumptions but with different strength (see Figures 2, 5, and 6). On the other hand, the effects differ in the case of the segregated network when μ is large. Specifically, we observe that:

1. With relative concerns, the symmetric equilibrium is stabilized for low trade costs (large ϕ) and this for all types of networks. With low trade costs the direct centrifugal force due to relative concerns dominates the others. The strength of the effect produced by a given value of α varies across the three networks, the weakest effect being with the star. Note that in the absence of relative concerns, the symmetric equilibrium is never stable for high levels of ϕ.

2. For low and intermediate values of ϕ

    (a) In the complete and the star networks, the conspicuous consumption effect tends to destabilize the symmetric equilibrium. Indeed, the rise in domestic demand generates a strong agglomeration force that overpowers the others.

(b) In the segregated network, conspicuous consumption tends to destabilize the symmetric equilibrium when μ, proportional to the share of income devoted to differentiated good consumption, is relatively large. However, here the centrifugal force is large in the segregated case because skilled only compare with skilled and the centripetal market size effect is less important because the unskilled are unaffected. Consequently, when the expenditures for conspicuous consumption is low (μ is relatively low), the resulting force is centrifugal, stabilizing the symmetric equilibrium.

3. Not surprisingly, the masses H and L are both relevant for the stability of equilibria in the case of the star and of the complete network, while only H is relevant for the segregated network. Note that they are not relevant in the model with no relative concerns of Forslid and Ottaviano (2003).

4. When skilled workers can choose whether to migrate to a region with a complete network or to a region with two segregated networks, for very high trade costs, there exists an asymmetric interior equilibrium, which can be in one of the two regions depending on the parameters of the model, and becomes full agglomeration once trade costs decrease to intermediate and low levels. When μ and L are relatively large, the region in which agglomeration takes place is that with a complete network, even though, for an open interval of ϕ, there also exists a stable full agglomeration equilibrium in the segregated region.

6 A different specification for the conspicuous good effect

6.1 The demand side with the pure average specification

In previous Sections the strength of the relative concern was affected by the size of the comparison group. In particular, as the number of neighbours increases, the conspicuous good effect also increases. However, in a popular specification the agent only care about the deviations from his/her consumption and the *average* consumption of his/her neighbours. In this Section we investigate how our previous results are affected when we assume that there is no size effect, that is, $S(\Omega(\Lambda_r(i))) = 1$ for the expressions in Sections 2.1 and 3. Moreover, we use α' instead of α to distinguish the particular case under scrutiny in which the agent cares about the average of his/her neighbours' consumption. Finally, the production side is as in Section 2.2.

It can be readily verified from expressions (10) and (11) that, for given levels of the wage (w_{ir}), of the number of neighbours ($\Lambda_r(i)$) and of the value of their total expenditure on the differentiated good ($E_{\Lambda_r(i)}$), the presence of relative concerns tends to produce a larger value of the wage, net of the conspicuous effect, within the average specification than within the additive specification. Nevertheless, we know from the expressions (8) that the demand X_{ir} for the differentiated good is larger in the case of the additive specification.[28]

The location decision of skilled workers is determined by the value of the logarithm of the ratio of the current indirect utility levels in (24), that with $S(\Omega(\Lambda_r(i))) = 1$ becomes

$$V\left(H_r, \phi, \alpha'\right) = \ln\left[\left(\frac{p_{X_v}}{p_{X_r}}\right)^{\mu} \frac{W_{\alpha' H_r}}{W_{\alpha' H_v}}\right] \tag{37}$$

where price indexes p_{X_r} and p_{X_v} are substituted from (19) and (20).

For any specific network, the individual demands of the differentiated good by the skilled workers and by the unskilled workers in region r, i.e., X_{H_r} and X_{L_r}, is obtained as described in Appendix III.

6.2 The complete network without group size effects

We consider first the case in which the reference group is the entire population of the residents in region r. This assumption implies that $h_{H_r} = h_{L_r} = H_r$ and $l_{H_r} = l_{L_r} = L$. Appendix III (III.A) shows how to compute the wages which are then substituted into (37) in order to evaluate the stability of the symmetric and of the fully agglomerated equilibria.

The *symmetric equilibrium*, with all variables assuming the same value in both regions, is one of the interior equilibria also in the case under scrutiny

[28] If $\int_{\Lambda_r(i)} dj = 1$, then $W_{\alpha ir} = W_{\alpha' ir}$ and, for given values of the other variables, the demand X_{ir} for the differentiated good is equal for both additive and average specifications.

without size effects. In Appendix III.A we show that the following Proposition holds.

Proposition 12 *With a complete network and no size effect, the direct centrifugal effect of relative concerns is weaker than in the additive specification. However, when α' is relatively low the symmetric equilibrium is stable for high levels of ϕ as in the additive case. On the other hand, for sufficiently large α' the symmetric equilibrium is stable for high levels of ϕ only provided $H < \frac{2L(1+\alpha')}{\frac{\sigma}{\mu}+[\alpha'(\sigma-1)-1]}$, for larger H the equilibrium is unstable.*

The comparison with the additive specification is interesting. With low levels of α', the symmetric equilibrium becomes stable for high levels of ϕ independently of the presence of group size effects. Instead, for a relatively high value of α' the symmetric equilibrium is stable for high levels of ϕ only if the mass of skilled workers is not too large. The role of H in stability is novel. It is related to the fact the size H matters in the strength of the indirect effects. With the additive specification considered earlier, the centrifugal effects produced with high values of ϕ were sufficiently strong to stabilize the symmetric equilibrium for any value of H.

Let us now turn to the case of *full agglomeration in region r*. In the final part of Appendix III.A we show that, in general, we can state what follows.

Proposition 13 *With the complete network and no group size effects, full agglomeration is never stable for low levels of ϕ and it is stable for intermediate levels of economic integration ϕ. For very low trade costs this equilibrium might or might not be stable depending on the parameter values, while in the economy with group size effects it is never stable.*

The economic intuition regarding the difference with the economy with group size effects is that here the centrifugal forces are weaker.

6.3 The segregated network without group size effects

We consider here the case in which the reference group is composed only of workers of the same type and ignore group size effects. In this specific case, we let $h_{H_r} = H_r$, $l_{L_r} = L$ and $h_{L_r} = l_{H_r} = 0$. In Appendix III (III.B) we compute the wages that are then substituted into (37) to give $V^s(H_r, \phi, \alpha')$ in (74) that allow to evaluate the stability of the symmetric and the fully agglomerated equilibria. Taking into account what shown in Appendix III (III.B), we can state the following Proposition:

Proposition 14 *In the case of the pure average specification with segregated networks, increases in relative concerns favour agglomeration. Specifically, increases in α' reduce the lower break point ϕ_b^s such that for $\phi \in [0, \phi_b^s)$ the symmetric equilibrium is stable: relative concerns tend to destabilize this equilibrium. The sustain point ϕ_s^s also decreases with α', extending therefore the range of existence and stability of full agglomeration.*

Note that in the case of an economy with group size effects, for large values of ϕ, positive values of α stabilize the symmetric equilibrium, a property that doesn't occur without size effects. For intermediate values of economic integration ϕ, relative concerns with size effects tend to stabilize the symmetric equilibrium when μ is relatively small. Again this effect is absent without size effects.

6.4 The multiple star network without group size effects

As in Section 5.2 we now assume that the unskilled agent compares his or her consumption with the average consumption of the skilled workers and that the skilled compares his or her consumption with the unskilled worker (the center of the star). As in the present analysis we neglect size effects, the size of the reference group does not matter, as soon as it is not of zero measure. Formally, the *"star average"* is characterized by $h_{L_r} = H_r/L$, $l_{H_r} = 1$ and $h_{H_r} = l_{L_r} = 0$. In Appendix III (III.C) we compute the wages that are then substituted into (37) to evaluate $V^s(H_r, \phi, \alpha')$ as given in (77) and we show what is summarized in the following Proposition.

Proposition 15 *In the case of the average specification with star networks, direct computation shows that with relative concerns full agglomeration in a region is never sustainable with low values of ϕ. Moreover, the numerical analysis of the calibrated model in Appendix III.C shows that: (i) increases in α' tend to destabilize the symmetric equilibrium for low values of ϕ (as they reduce the break point ϕ_b^s) and have no effect for high values of ϕ; (ii) on the other hand, increases in α' tend to stabilize full agglomeration for high levels of ϕ (as they reduce the sustain point ϕ_s^s).*

Hence, while with group size effects we found that weak relative concerns stabilize the symmetric equilibrium for high levels of openness to trade ϕ, we know that this effect can vanish without size effects. On the other hand, relative concerns destabilize the symmetric equilibrium for low or intermediate levels of ϕ for both formulations, i.e., with or without group size effects.

6.5 The role of the size of the reference group in migration decisions with relative concerns

In this paper we have explored the role of relative concerns in location decision and city dynamics, via two polar specifications on how these concerns enter the utility function. We summarize our findings as follows.

1. For complete networks:
   (a) for high values of ϕ and with group size effects, the symmetric equilibrium is stabilized by relative concerns. This is due to the strong centrifugal force of relative concern. Without size effects this result holds provided the mass of skilled workers is not too large. Otherwise, the symmetric equilibrium is unstable even for $\phi = 1$ because in the case the centrifugal force does not increase with (as we assume no group size effect) while the centripetal market size effect increases.
   (b) for medium values of ϕ relative concerns tend to destabilize the symmetric equilibrium and stabilize full agglomeration. Indeed, the rise in domestic demand generates a strong agglomeration force that overpowers the other forces.
2. For segregated networks
   (a) an increase in relative concerns stabilize the symmetric equilibrium for high ϕ, provided there are group size effects. Without group size effects the centrifugal force is weaker and there is full agglomeration.
   (b) for intermediate and low values of economic integration ϕ the symmetric equilibrium is destabilized for large μ independently of the size effects due to the strong market size effect. For small μ and with group size effects the symmetric equilibrium is stabilized by relative concerns because of strong centrifugal directs effect of relative concerns.

3. For multiple star networks

   (a) an increases in relative concerns stabilize the symmetric equilibrium for large values of ϕ only with group size effects while it has no effect without size effects. Then relative concerns destroy the full agglomeration at very high ϕ only with size effects.

   (b) for intermediate values of ϕ, relative concerns destabilize the symmetric equilibrium and stabilize full agglomeration.

7 Conclusion

The goal of the paper was to explore the role of relative concerns in location decision and city dynamics. In core-periphery models of economic geography, full agglomeration is typically the outcome when trade costs vanish. We have seen that relative concerns generates a powerful centrifugal force that can stop this process. However, the notion of trade costs should not be defined too narrowly and they merely represent a wide variety of obstacles to economic integration. As a result, these trade costs are difficult to evaluate and the analysis of the present model should be extended to the case with intermediate costs. Surprisingly, when trade costs are intermediate, relative concerns also generate a new centripetal force favoring full agglomeration. The driving force is the market size effect, that is, a centripetal force produced by the increase in per firm domestic demand of the conspicuous good, which rises profits and wages. The direction of the resultant force is however highly sensitive to the network structure.

In particular, for a given value of trade costs, relative concerns might be such that full agglomeration emerges when agents in the city are fully integrated while a stable symmetric equilibrium emerges when the cities are segregated in homogeneous but separated areas. In general, it appears that the centrifugal force of relative concern is strong in segregated organisations, at least when only one type of the agents is mobile. On the other extreme, this effect is weaker in star like configurations where agents are only affected by the behavior of agents of the other type.

An interesting pattern emerges when agents can choose between regions with different organisations. Although for some costs, interior solutions may survive, there is a strong tendency to observe full agglomeration in the integrated region. To sum up, for intermediate shipping costs and relative concerns, workers tends to migrate to regions in which the different types of agents affect each others behavior.

The previous analysis has interesting policy implications. Indeed, the planner can, by favoring one type of city organisation rather than another, affect the migration patterns and the type of agglomeration arising in equilibrium. Clearly, the optimal type of structure depends on the welfare function of the planner. For example, if the welfare loss of shipping goods dominates, full agglomeration appears as the goal to decentralise and favoring integrated societies appears as to be a favorable factor. On the other hand, if full agglomeration is costly from a social point of view, favoring segregated communities favors interior solutions, and at the end, less dense cities.

References

[1] Abel, A., 1990. "Asset Prices under habit formation and catching up with Jonesses", American Economic Review, 80, 2, 38-42.

[2] Arrow, K., and P. Dasgupta, 2007. "Conspicuous consumption; inconspicuous leisure". mimeo, Stanford University and Cambridge University.

[3] Berliant, M., and F.-C. Kung. 2009. "Bifurcations in Regional Migration Dynamics", Regional Science and Urban Economics, 39, 714–720.

[4] Blanchflower, D. and A. Oswald. 2004. "Well-Being Over Time in Britain and the USA", Journal of Public Economics, 88, 1359–1386.

[5] Duranton, G., 2008. "Spatial Economics", in The New Palgrave Dictionary of Economics, Second Edition, 2008, edited by Steven N. Durlauf and Lawrence E. Blume.

[6] Dussenbery, J., 1949. Income, savings and the theory of consumer behavior. Harvard University Press, Cambridge Mass.

[7] Easterlin, R. A., 1974. "Does Economic Growth Improve the Human Lot?", in Paul A. David and Melvin W. Reder, eds., Nations and House-

holds in Economic Growth: Essays in Honor of Moses Abramovitz, New York: Academic Press, Inc.

[8] Forslid, R., and G. Ottaviano. 2003. “An Analytically Solvable Core-Periphery Model”, Journal of Economic Geography, 3, 229–240.

[9] Frank, R., 1985. Choosing the right pond. Oxford University Press. Oxford.

[10] Frey, B., and A. Stutzer, 2002. “What can economists learn from happiness research?”, Journal of Economic Literature, XL, 402–435.

[11] Ghiglino, C. and S. Goyal, 2010. “Keeping up with the neighbors: social interaction in a market economy”, Journal of the European Economic Association, 8, 90–119.

[12] Helpman, E., 1998. “The Size of Regions”, in D. Pines, E. Sadka, and I. Zilcha (eds.), Topics in Public Economics. Theoretical and Empirical Analysis. Cambridge University Press, 33–54.

[13] Hopkins, E. and Kornienko, 2004. “Running to keep in the same place: consumer choice as a game of status”, American Economic Review, 94, 4, 1085-1107.

[14] Helsley, R. and Y. Zenou, 2011. “Social networks and interactions in cities”, CEPR Discussion Paper No. 8260.

[15] Kuhn, P., P. Koorman, A. Soetevent and A. Kapteyn. 2008. “The own and social effects of an unexpected income shock: evidence from the Dutch Postcode Lottery”, mimeo, Tilburg University and Rand Corporation.

[16] Krugman, P.R., 1991. “Increasing returns and economic geography”, Journal of Political Economy, 99, 483– 499.

[17] Layard, R., 2005. Happiness: lessons from a new science, Penguin Books. London.

[18] Ludema, R., and I. Wooton. 1999. “Regional Integration, Trade and Migration: Are Demand Linkages Relevant in Europe?” in R. Faini, J. De Melo, and K.F. Zimmermann (eds.), Migration. The Controversies and the Evidence. CEPR and Cambridge University Press, pp. 51–75.

[19] Luttmer, E., 2005. "Neighbors as Negatives: Relative Earnings and Well-Being", Quarterly Journal of Economics, 120, 3, 963–1002.

[20] Murata, Y., 2003. "Product Diversity, Taste Heterogeneity, and Geographic Distribution of Economic Activities: Market vs. Non-Market Interactions", Journal of Urban Economics, 53, 126–144.

[21] Nocco, A., 2009. "Preference Heterogeneity and Economic Geography", Journal of Regional Science, 49, 33–56.

[22] Ottaviano, G., T. Tabuchi, and J.-F. Thisse. 2002. "Agglomeration and Trade Revisited," International Economic Review, 43, 409–435.

[23] Pflüger, M., 2004. "A Simple, Analytically Solvable, Chamberlinian Agglomeration Model," Regional Science and Urban Economics, 34, 565–573.

[24] Pflüger, M., and J. Suedekum, 2010. "On Pitchforks and Tomahawks", Journal of Regional Science, early view.

[25] Quinn, M. A., 2006. "Relative Deprivation, Wage Differentials and Mexican Migration," Review of Development Economics, 10, 135–153.

[26] Radu, D. 2008. "Social Interactions in Economic Models of Migration: A Review and Appraisal," Journal of Ethnic and Migration Studies, 34, 531–548.

[27] Stark, O. and J.E. Taylor, 1991. "Migration Incentives, Migration Types: the Role of Relative Deprivation," Economic Journal, 101, 1163–1178.

[28] Tabuchi, T., 1998. "Urban Agglomeration and Dispersion: A Synthesis of Alonso and Krugman", Journal of Urban Economics, 44, 333–351.

[29] Tabuchi, T. and J.-F. Thisse, 2002. "Taste Heterogeneity, Labor Mobility and Economic Geography," Journal of Development Economics, 69, 155–177.

[30] Veblen, T., 1899. The Theory of the Leisure Class. Macmillan.

APPENDIX

APPENDIX I.

I.A. Consumer's demand for the two goods and indirect utility

In this part of the Appendix we compute the consumer's demand for the differentiated good and for the agricultural good in region r and the indirect utility function $U(A_{ir}, \Phi(X_{ir}, X_{-ir}, \Lambda_r(i)))$.

Each individual i in region r solves the program

$$Max_{A_{ir},X_{ir}} U(A_{ir}, \Phi(X_{ir}, X_{-ir}, \Lambda_r(i)))$$

$$s.t.\ A_{ir} + p_{X_r} X_{ir} = w_{ir}$$

with

$$\Phi(X_{ir}, X_{-ir}, \Lambda_r(i)) = X_{ir} + \alpha S(\Omega(\Lambda_r(i))) \left[X_{ir} - \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\int_{\Lambda_r(i)} dj} \right],$$

As $\int_{\Lambda_r(i)} dj = \Omega(\Lambda_r(i))$, the Lagrangean function is

$$L = A_{ir}^{1-\mu} \left[X_{ir} + \alpha S(\Omega(\Lambda_r(i))) \left(X_{ir} - \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\Omega(\Lambda_r(i))} \right) \right]^{\mu} + \lambda (w_{ir} - A_{ir} - p_{X_r} X_{ir})$$

with the first order conditions respectively given by

$$(1-\mu) A_{ir}^{-\mu} \left[X_{ir} + \alpha S(\Omega(\Lambda_r(i))) \left(X_{ir} - \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\Omega(\Lambda_r(i))} \right) \right]^{\mu} - \lambda = 0, \quad (38)$$

$$\mu A_{ir}^{1-\mu} \left[X_{ir} + \alpha S(\Omega(\Lambda_r(i))) \left(X_{ir} - \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\Omega(\Lambda_r(i))} \right) \right]^{\mu-1} [1 + \alpha S(\Omega(\Lambda_r(i)))] - \lambda p_{X_r} = 0 \quad (39)$$

and

$$w_{ir} - A_{ir} - p_{X_r} X_{ir} = 0 \quad (40)$$

From (38) and (39) we get that

$$A_{ir} = \frac{(1-\mu) p_{X_r} \left[X_{ir} + \alpha S(\Omega(\Lambda_r(i))) \left(X_{ir} - \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\Omega(\Lambda_r(i))} \right) \right]}{\mu [1 + \alpha S(\Omega(\Lambda_r(i)))]} \quad (41)$$

which can be substituted into (40) to obtain the consumer's demand for the differentiated good in region r

$$X_{ir} = \frac{\mu}{p_{X_r}} \left(w_{ir} + p_{X_r} \frac{1-\mu}{\mu} \frac{\alpha S(\Omega(\Lambda_r(i)))}{1 + \alpha S(\Omega(\Lambda_r(i)))} \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\Omega(\Lambda_r(i))} \right) \tag{42}$$

Previous expression can be substituted into (41) to obtain the consumer's demand for the agricultural good in region r, that is

$$A_{ir} = (1-\mu) \left(w_{ir} - p_{X_r} \frac{\alpha S(\Omega(\Lambda_r(i)))}{1 + \alpha S(\Omega(\Lambda_r(i)))} \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\Omega(\Lambda_r(i))} \right) \tag{43}$$

Then, substituting (42) and (43) into

$$U(A_{ir}, \Phi\left(X_{ir}, X_{-ir}, \Lambda_r(i)\right)) = A_{ir}^{1-\mu} \left(\Phi\left(X_{ir}, X_{-ir}, \Lambda_r(i)\right)\right)^{\mu}$$

we obtain the indirect utility function:

$$\begin{aligned} & U(A_{ir}, \Phi\left(X_{ir}, X_{-ir}, \Lambda_r(i)\right)) \\ = & \frac{(1-\mu)^{1-\mu} \mu^{\mu} \left(1 + \alpha S(\Omega(\Lambda_r(i)))\right)^{\mu}}{(p_{X_r})^{\mu}} \left(w_{ir} - p_{X_r} \frac{\alpha S(\Omega(\Lambda_r(i)))}{1 + \alpha S(\Omega(\Lambda_r(i)))} \frac{\int_{\Lambda_r(i)} X_{jr} dj)}{\Omega(\Lambda_r(i))} \right) \end{aligned}$$

I.B. Individual demand in region r for variety s and the price index p_{X_r}

In this second part of the Appendix we show that the individual demand in region r for variety s is given by

$$X_{ir}(s) = \frac{p_r(s)^{-\sigma}}{p_{X_r}^{1-\sigma}} E_{ir} \tag{44}$$

and define the price index p_{X_r} by $\left(\int\limits_{s \in N} (p_r(s))^{1-\sigma} ds \right)^{\frac{1}{1-\sigma}}$.

Using the definition $X_{ir} \equiv \left(\int\limits_{s \in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}} ds \right)^{\frac{\sigma}{\sigma-1}}$, the problem faced by consumer i in region r becomes

$$Max\, U(A_{ir}, \Phi\left(X_{ir}, X_{-ir}, \Lambda_r(i)\right)) = A_{ir}^{1-\mu} \left\{ X_{ir} + \alpha S(\Omega(\Lambda_r(i))) \left(X_{ir} - \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\Omega(\Lambda_r(i))} \right) \right\}^{\mu} =$$

$$= A_{ir}^{1-\mu}\{\left(\int\limits_{s\in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}}ds\right)^{\frac{\sigma}{\sigma-1}} + \alpha S(\Omega(\Lambda_r(i)))*$$

$$\left[\left(\int\limits_{s\in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}}ds\right)^{\frac{\sigma}{\sigma-1}} - \frac{1}{\Omega(\Lambda_r(i))}\int_{\Lambda_r(i)}\left(\int\limits_{s\in N} X_{jr}(s)^{\frac{\sigma-1}{\sigma}}ds\right)^{\frac{\sigma}{\sigma-1}}dj\right]\}^{\mu}$$

$$s.t.\ A_{ir} + \int\limits_{s\in N} p_r(s)X_{ir}(s)ds = w_{ir}.$$

The lagrangean function is

$$L = A_{ir}^{1-\mu}\{\left(\int\limits_{s\in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}}ds\right)^{\frac{\sigma}{\sigma-1}} + \alpha S(\Omega(\Lambda_r(i)))*$$

$$\left[\left(\int\limits_{s\in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}}ds\right)^{\frac{\sigma}{\sigma-1}} - \frac{1}{\Omega(\Lambda_r(i))}\int_{\Lambda_r(i)}\left(\int\limits_{s\in N} X_{jr}(s)^{\frac{\sigma-1}{\sigma}}ds\right)^{\frac{\sigma}{\sigma-1}}dj\right]\}^{\mu} +$$

$$+\lambda\left(w_{ir} - A_{ir} - \int\limits_{s\in N} p_r(s)X_{ir}(s)ds\right)$$

The first order condition (FOC) with respect to the consumption of variety s, $X_{ir}(s)$, is:

$$\frac{\mu A_{ir}^{1-\mu}\left[\frac{\sigma}{\sigma-1}(x)^{\frac{\sigma}{\sigma-1}-1}\frac{\sigma-1}{\sigma}X_{ir}(s)^{\frac{\sigma-1}{\sigma}-1}+\alpha\frac{\sigma}{\sigma-1}S(\Omega(\Lambda_r(i)))(x)^{\frac{\sigma}{\sigma-1}-1}\frac{\sigma-1}{\sigma}X_{ir}(s)^{\frac{\sigma-1}{\sigma}-1}\right]}{\left\{X_{ir}+\alpha S(\Omega(\Lambda_r(i)))\left(X_{ir}-\frac{\int_{\Lambda_r(i)}X_{jr}dj}{\Omega(\Lambda_r(i))}\right)\right\}^{1-\mu}} = \lambda p_r(s)$$

while that written with respect to the consumption of variety v, $X_{ir}(v)$, is:

$$\frac{\mu A_{ir}^{1-\mu}\left[\frac{\sigma}{\sigma-1}(x)^{\frac{\sigma}{\sigma-1}-1}\frac{\sigma-1}{\sigma}X_{ir}(v)^{\frac{\sigma-1}{\sigma}-1}+\alpha\frac{\sigma}{\sigma-1}S(\Omega(\Lambda_r(i)))(x)^{\frac{\sigma}{\sigma-1}-1}\frac{\sigma-1}{\sigma}X_{ir}(v)^{\frac{\sigma-1}{\sigma}-1}\right]}{\left\{X_{ir}+\alpha S(\Omega(\Lambda_r(i)))\left(X_{ir}-\frac{\int_{\Lambda_r(i)}X_{jr}dj}{\Omega(\Lambda_r(i))}\right)\right\}^{1-\mu}} = \lambda p_r(v)$$

with $x = \int\limits_{s\in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}}ds$. Considering the ratio of the two previous FOC, we obtain:

$$\frac{X_{ir}(s)^{-\frac{1}{\sigma}}}{X_{ir}(v)^{-\frac{1}{\sigma}}} = \frac{p_r(s)}{p_r(v)} \tag{45}$$

Let us substitute λ from (38) into the first order condition with respect to variety s, $X_{ir}(s)$, to get

$$\mu \frac{A_{ir}\left(\int\limits_{s\in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}}ds\right)^{\frac{\sigma}{\sigma-1}-1} X_{ir}(s)^{\frac{\sigma-1}{\sigma}-1}}{\left\{X_{ir}+\alpha S(\Omega(\Lambda_r(i)))\left(X_{ir}-\frac{\int_{\Lambda_r(i)} X_{jr}dj}{\Omega(\Lambda_r(i))}\right)\right\}}\left[1+\alpha S(\Omega(\Lambda_r(i)))\right] = (1-\mu)\, p_r(s) \quad (46)$$

Moreover, from (41) we know that

$$\frac{A_{ir}}{X_{ir}+\alpha S(\Omega(\Lambda_r(i)))\left(X_{ir}-\frac{\int_{\Lambda_r(i)} X_{jr}dj}{\Omega(\Lambda_r(i))}\right)} = \frac{(1-\mu)\, p_{X_r}}{\mu\left[1+\alpha S(\Omega(\Lambda_r(i)))\right]} \quad (47)$$

which can be substituted into (46) to obtain

$$p_{X_r}\left(\int\limits_{s\in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}}ds\right)^{\frac{1}{\sigma-1}} X_{ir}(s)^{\frac{\sigma-1}{\sigma}-1} = p_r(s) \quad (48)$$

Then we can rewrite (48) as

$$p_{X_r}\left(\int\limits_{s\in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}}ds\right)^{\frac{1}{\sigma-1}} X_{ir}(s)^{\frac{\sigma-1}{\sigma}} = p_r(s)X_{ir}(s)$$

Integrating this expression with respect to variety s and using the fact that the individual expenditure on manufactures in region r is $E_{ir} = \int\limits_{s\in N} p_r(s)X_{ir}(s)ds$, we obtain:

$$\begin{aligned} p_{X_r}\left(\int\limits_{v\in N} X_{ir}(v)^{\frac{\sigma-1}{\sigma}}dv\right)^{\frac{1}{\sigma-1}} \int\limits_{s\in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}}ds &= \int\limits_{s\in N} p_r(s)X_{ir}(s)ds; \\ p_{X_r}\left(\int\limits_{s\in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}}ds\right)^{\frac{\sigma}{\sigma-1}} &= E_{ir} \end{aligned}$$

From the previous expression we find that $\int\limits_{s\in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}}ds = \left(\frac{E_{ir}}{p_{X_r}}\right)^{\frac{\sigma-1}{\sigma}}$, which can be substituted into (48) to obtain the individual demand in region r for variety s

$$X_{ir}(s) = \frac{p_r(s)^{-\sigma}}{p_{X_r}^{1-\sigma}}E_{ir}$$

Let us notice that from (45) we obtain that

$$X_{ir}(s) = \left(\frac{p_r(v)}{p_r(s)}\right)^{\sigma} X_{ir}(v)$$

which can be substituted into the definition $X_{ir} \equiv \left(\int\limits_{s\in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}} ds\right)^{\frac{\sigma}{\sigma-1}}$ to obtain

$$X_{ir}(v) = \frac{p_r(v)^{-\sigma}}{\left(\int\limits_{s\in N} (p_r(s))^{1-\sigma} ds\right)^{\frac{\sigma}{\sigma-1}}} X_{ir}$$

Finally, we derive the expression for the (minimum) expenditure of attaining X_{ir} by substituting the expression fo $X_{ir}(v)$ found previuosly into $E_{ir} = \int\limits_{v\in N} p_r(v)X_{ir}(v)dv$. We obtain

$$E_{ir} = \left(\int\limits_{s\in N} (p_r(s))^{1-\sigma} ds\right)^{\frac{1}{1-\sigma}} X_{ir}$$

which, using the definition of price index $p_{X_r} \equiv \left(\int\limits_{s\in N} (p_r(s))^{1-\sigma} ds\right)^{\frac{1}{1-\sigma}}$, can be rewriten as $E_{ir} = p_{X_r} X_{ir}$.

I.C. Individual demand of the differentiated good by skilled and unskilled workers in region r.

In this section we show how we derive the individual demand by the skilled and by the unskilled consumers in region r. As $\int_{\Lambda_r(i)} dj = \Omega(\Lambda_r(i))$, the demands for each *skilled* individual i in region r as given by (7) and (8) can be rewritten as

$$\begin{aligned} A_{H_r} &= (1-\mu)\left[w_{H_r} - p_{X_r}\tfrac{\alpha S(\Omega(\Lambda_{H_r}))}{1+\alpha S(\Omega(\Lambda_{H_r}))}\tfrac{(h_{H_r}X_{H_r}+l_{H_r}X_{L_r})}{\Omega(\Lambda_{H_r})}\right], \qquad (49)\\ X_{H_r} &= \tfrac{\mu}{p_{X_r}}\left[w_{H_r} + p_{X_r}\tfrac{1-\mu}{\mu}\tfrac{\alpha S(\Omega(\Lambda_{H_r}))}{1+\alpha S(\Omega(\Lambda_{H_r}))}\tfrac{(h_{H_r}X_{H_r}+l_{H_r}X_{L_r})}{\Omega(\Lambda_{H_r})}\right]; \end{aligned}$$

while for each *unskilled* individual i in region r the demands are

$$\begin{aligned} A_{L_r} &= (1-\mu)\left[w_{L_r} - p_{X_r}\tfrac{\alpha S(\Omega(\Lambda_{L_r}))}{1+\alpha S(\Omega(\Lambda_{L_r}))}\tfrac{(h_{L_r}X_{H_r}+l_{L_r}X_{L_r})}{\Omega(\Lambda_{L_r})}\right], \qquad (50)\\ X_{L_r} &= \frac{\mu}{p_{X_r}}\left[w_{L_r} + p_{X_r}\tfrac{1-\mu}{\mu}\tfrac{\alpha S(\Omega(\Lambda_{L_r}))}{1+\alpha S(\Omega(\Lambda_{L_r}))}\tfrac{(h_{L_r}X_{H_r}+l_{L_r}X_{L_r})}{\Omega(\Lambda_{L_r})}\right]. \end{aligned}$$

Considering the second equations in (49) and in (50), we obtain a system of two equations in the two unknowns X_{H_r} and X_{L_r} given by

$$\begin{cases} X_{H_r} = w_{H_r}\frac{\mu}{p_{X_r}} + \frac{\alpha(1-\mu)S(\Omega(\Lambda_{H_r}))}{1+\alpha S(\Omega(\Lambda_{H_r}))}\frac{(h_{H_r}X_{H_r}+l_{H_r}X_{L_r})}{\Omega(\Lambda_{H_r})} \\ X_{L_r} = \frac{\mu}{p_{X_r}} + \frac{\alpha(1-\mu)S(\Omega(\Lambda_{L_r}))}{1+\alpha S(\Omega(\Lambda_{L_r}))}\frac{(h_{L_r}X_{H_r}+l_{L_r}X_{L_r})}{\Omega(\Lambda_{L_r})} \end{cases} \tag{51}$$

where we used the fact that $w_{L_r} = 1$ and where $\Omega(\Lambda_{H_r}) = h_{H_r}+l_{H_r}$, $\Omega(\Lambda_{L_r}) = h_{L_r} + l_{L_r}$.

I.D. More on the local demand in the complete additive specification

In the text we have introduced the new direct centrifugal force created by the introduction of conspicuous consumption and we have also discussed how it reinforces the original market size effect in the core-periphery model. For the sake of completeness, in this Appendix we notice that when some skilled workers move in a region, associated with the increase in the number of firms producing in the region, there are also two more additional effects produced by conspicuous consumption that strengthen the traditional forces at work in Forslid and Ottaviano (2003). The first one is produced by the subsequent reduction in the local price index that tends to further attract workers because of the increase in their local real wage, and this shows that conspicuous consumption strengthens the traditional cost-of-living effect. The other is produced because the traditional centrifugal force generated by the market-crowding effect is sharpened by conspicuous consumption effects. This is due to the fact that when a skilled worker moves in a region this increases also the number of firms producing in the same region, depressing the local price index and inducing a fall in local demand per firm (or per worker, given that the two are proportional), for given expenditures on manufactures (e.g. when $L + H_r w_{H_r}$ in (29) is treated as given). Hence, because of the conspicuous good effect, all other workers in the region decrease their demand given that they observe a reduction in the demand of their neighbours. Lower demand leads to lower operating profits for firms and, therefore, lower skilled wages, strenghtening the traditional centrifugal force. This latter result can be shown as follows. The local demand per firm, in region r, is $\frac{X_{rr}(s)}{\frac{H_r}{f}}$. Defining $L + H_r w_{H_r} = E_r$ (where E_r are the total expenditures in region r), we know that:

$$\frac{X_{rr}(s)}{\frac{H_r}{f}} = \mu \frac{(\sigma-1)E_r}{\sigma\beta\left(\frac{H_r}{f}+\frac{H-H_r}{f}\phi\right)} \frac{\frac{\alpha(H_r+L)+1}{\frac{H_r}{f}}}{\alpha\mu(H_r+L)+1}$$

Then, we can apply a logarithmic transformation and get that $log\frac{X_{rr}(s)}{\frac{H_r}{f}} =$ $= log\left(\mu\left(\sigma-1\right)E_r\right)$-$log\left(\sigma\beta\left(\frac{H_r}{f}+\frac{H-H_r}{f}\phi\right)\right)$+$log\frac{\frac{\alpha(H_r+L)+1}{\frac{H_r}{f}}}{\alpha\mu(H_r+L)+1}$. Hence, we can write that the rate of change of $\frac{X_{rr}(s)}{\frac{H_r}{f}}$ (with $\widehat{.}$ denoting the rate of change of a variable) is:

$$\widehat{\left(\frac{X_{rr}(s)}{\frac{H_r}{f}}\right)} = \widehat{E_r} - \sigma\beta\widehat{\left(\frac{H_r}{f}+\frac{H-H_r}{f}\phi\right)} + \widehat{\left(\frac{\frac{\alpha(H_r+L)+1}{\frac{H_r}{f}}}{\alpha\mu\left(H_r+L\right)+1}\right)} < 0$$

The rate of change is negative because for a given value of E_r we know that $\widehat{E_r} = 0$, while $\sigma\beta\widehat{\left(\frac{H_r}{f}+\frac{H-H_r}{f}\phi\right)} > 0$ and $\widehat{\left(\frac{\frac{\alpha(H_r+L)+1}{\frac{H_r}{f}}}{\alpha\mu(H_r+L)+1}\right)} < 0$.

APPENDIX II. NETWORKS AND ADDITIVE SPECIFICATION

In the case of the additive specification, we know that $S(\Omega(\Lambda_r(i))) = \Omega(\Lambda_r(i))$. Hence, with $\Omega(\Lambda_{H_r}) = h_{H_r} + l_{H_r}$ and $\Omega(\Lambda_{L_r}) = h_{L_r} + l_{L_r}$, the system in (51) becomes

$$\begin{cases} \frac{1+\alpha(h_{H_r}+l_{H_r})-\alpha(1-\mu)h_{H_r}}{1+\alpha(h_{H_r}+l_{H_r})} X_{H_r} = w_{H_r}\frac{\mu}{p_{X_r}} + \frac{\alpha(1-\mu)}{1+\alpha(h_{H_r}+l_{H_r})} l_{H_r} X_{L_r} \\ \frac{1+\alpha(h_{L_r}+l_{L_r})-\alpha(1-\mu)l_{L_r}}{1+\alpha(h_{L_r}+l_{L_r})} X_{L_r} = \frac{\mu}{p_{X_r}} + \frac{\alpha(1-\mu)}{1+\alpha(h_{L_r}+l_{L_r})} h_{L_r} X_{H_r} \end{cases} \tag{52}$$

This system is solved to obtain the individual demands of the differentiated good by the skilled workers and by the unskilled workers in region r, i.e., X_{H_r} and X_{L_r}.

II.A. Complete network

Let us now consider the complete network and derive the expression for skilled wages as a function of H_r. Substituting $X_{rr}(s)$ and $X_{rv}(s)$ from (29) and (30) into (22) and making use of (14), the wage paid to skilled workers in region r must satisfy the following equation

$$w_{H_r} = \frac{\mu}{\sigma}\left[\frac{(L+H_r w_{H_r})}{H_r+(H-H_r)\phi}\frac{1+\alpha\Lambda_r}{1+\alpha\mu\Lambda_r} + \phi\frac{(L+H_v w_{H_v})}{(H-H_r)+H_r\phi}\frac{1+\alpha\Lambda_v}{1+\alpha\mu\Lambda_v}\right] \tag{53}$$

and an analogous expression holds for w_{H_v}. Hence, we get a system of two linear equations in w_{H_r} and w_{H_v} that can be solved to obtain the two regional wages for skilled workers as an explicit function of a given distribution of workers, H_r and H_v, between the two regions, and we find that the wage paid in region r to skilled workers is

$$w_{H_r} = \mu L\frac{\sigma A(H_v+\phi H_r)+\phi\sigma B(H_r+\phi H_v)+H_v\mu BA(\phi-1)(\phi+1)}{\phi\sigma^2(H_r^2+H_v^2)-\phi\mu\sigma(AH_r^2+BH_v^2)-H_vH_r\left[\mu\sigma(A+B)-\sigma^2\left(\phi^2+1\right)+\mu^2BA(\phi-1)(\phi+1)\right]} \tag{54}$$

where $A = \frac{\alpha\Lambda_r+1}{\alpha\mu\Lambda_r+1}$ and $B = \frac{\alpha\Lambda_v+1}{\alpha\mu\Lambda_v+1}$.[29] This expression shows that the wage depends on the value of α, and A and B represent a measure of the effects produced by the proximity of neighbors, respectively, in r (where the number of neighbours is given by $\Lambda_r = H_r + L$) and in v (where the number of neighbours is given by $\Lambda_v = H_v + L$).

The wage of skilled workers in (54) evaluated in the case in which they are evenly distributed in the two regions is given by the following expression

$$w_{H/2} = \frac{2\mu L}{H}\frac{\alpha\Lambda_s+1}{\sigma-\mu+\mu\alpha\Lambda_s(\sigma-1)}$$

where $\Lambda_s \equiv (H/2 + L)$ is the number of neighbours in each region at the symmetric equilibrium.[30] It can be readily shown that the wage (w_{H_r}) increases in the symmetric equilibrium with α. Finally, evaluating the wage of skilled workers in (54) when they are all in region r we get that

$$w_{H_r} = \frac{\mu L}{H}\frac{2+\alpha(H+2L)(\mu+1)+2\alpha^2\mu L(H+L)}{(\mu\alpha L+1)[\sigma-\mu+\alpha\mu(H+L)(\sigma-1)]}$$

II.B. Segregated network

In the case of the segregated network the system in (52) becomes

$$\begin{cases} \frac{1+\alpha H_r-\alpha(1-\mu)H_r}{1+\alpha H_r}X_{H_r} = w_{H_r}\frac{\mu}{p_{X_r}} \\ \frac{1+\alpha L-\alpha(1-\mu)L}{1+\alpha L}X_{L_r} = \frac{\mu}{p_{X_r}} \end{cases} \tag{55}$$

[29] An analogous expression to (54) holds for w_{H_v}.

[30] Specifically, we observe that $A\,(H_r = H_v = H/2){=}B\,(H_r = H_v = H/2){=}\frac{\alpha\Lambda_s+1}{\alpha\mu\Lambda_s+1}$.

which can be solved for X_{H_r} and X_{L_r} to find respectively that

$$\begin{aligned} X_{H_r} &= \mu \frac{w_{H_r}}{p_{X_r}} \frac{1+\alpha H_r}{1+\alpha\mu H_r} \quad \text{and} \\ X_{L_r} &= \mu \frac{1}{p_{X_r}} \frac{1+\alpha L}{1+\alpha\mu L}. \end{aligned} \tag{56}$$

Making use of these solutions, we can rewrite the total demand in region r for variety s produced in region k in (13) as follows

$$X_{kr}(s) = \mu \frac{p_{kr}(s)^{-\sigma}}{p_{X_r}^{1-\sigma}} \left(w_{H_r} \frac{1+\alpha H_r}{1+\alpha\mu H_r} H_r + \frac{1+\alpha L}{1+\alpha\mu L} L \right) \tag{57}$$

where the price indices and the prices are respectively given by (19), (20), (16) and (17). Making use of (57), the wage in (22) can be rewritten as follows

$$w_{H_r} = \frac{\mu}{\sigma} \left[\frac{(A_r w_{H_r} + A_L)}{H_r + (H-H_r)\phi} + \frac{\phi(A_v w_{H_v} + A_L)}{(H-H_r) + H_r\phi} \right]$$

where $A_r \equiv \frac{1+\alpha H_r}{1+\alpha\mu H_r} H_r$, $A_v \equiv \frac{1+\alpha(H-H_r)}{1+\alpha\mu(H-H_r)} (H-H_r)$ and $A_L \equiv \frac{1+\alpha L}{1+\alpha\mu L} L$. Previous expression can be considered together with the analogous expression obtained for w_{H_v} to get a system of two equations in two unknowns w_{H_r} and w_{H_v}, that can be solved to find the two skilled regional wages given by

$$w_{H_r} = \frac{\mu A_L \left[2\sigma\phi H_r + \sigma H_v \left(1+\phi^2\right) - A_v\mu(1-\phi)(\phi+1) \right]}{D_s} \tag{58}$$

and

$$w_{H_v} = \frac{\mu A_L \left[2\sigma\phi H_v + \sigma H_r \left(\phi^2+1\right) - A_r\mu(1-\phi)(\phi+1) \right]}{D_s} \tag{59}$$

with the denominator of both wages given by $D_s \equiv \sigma^2 (H_v + \phi H_r)(H_r + \phi H_v)$ $- A_r\sigma\mu (H_v + \phi H_r) - \sigma\mu A_v (H_r + \phi H_v) + \mu^2 (1-\phi)(1+\phi) A_r A_v$.

Then, $W_{\alpha ir}$ in (9) can be written for skilled workers in region r as

$$W_{\alpha H_r} = \frac{w_{H_r}}{1+\alpha\mu H_r},$$

while for unskilled it is given by

$$W_{\alpha L_r} = \frac{1}{1+L\alpha\mu}$$

Hence, we can rewrite (26) as follows

$$V(H_r, \phi, \alpha) = \ln \left\{ \left[\frac{(H-H_r) + H_r\phi}{H_r + (H-H_r)\phi} \right]^{\frac{\mu}{1-\sigma}} \left[\frac{1+\alpha H_r}{1+\alpha(H-H_r)} \right]^{\mu} \frac{\frac{w_{H_r}}{1+\alpha\mu H_r}}{\frac{w_{H_v}}{1+\alpha\mu(H-H_r)}} \right\} \tag{60}$$

where the two regional wages for skilled workers w_{H_r} and w_{H_v} can be substituted from (58) and (59).

II.C. Star network

In the case of the star the system in (52) becomes

$$\begin{cases} X_{H_r} = w_{H_r}\frac{\mu}{p_{X_r}} + \frac{\alpha(1-\mu)}{1+\alpha}X_{L_r} \\ X_{L_r} = \frac{\mu}{p_{X_r}} + \frac{\alpha(1-\mu)}{1+\alpha H_r/L}X_{H_r}H_r/L \end{cases}$$

which can be solved to find X_{H_r} and X_{L_r}, whose values are respectively given by

$$X_{H_r} = \frac{\mu}{p_{X_r}}\left(L+\alpha H_r\right)\frac{\alpha(1-\mu)+w_{H_r}(1+\alpha)}{L+\alpha(L+H_r)+H_r\mu\alpha^2(2-\mu)}$$

and

$$X_{L_r} = \frac{\mu}{p_{X_r}}\left(\alpha+1\right)\frac{L+\alpha H_r+H_r\alpha w_{H_r}(1-\mu)}{L+\alpha(L+H_r)+H_r\mu\alpha^2(2-\mu)}$$

Making use of these solutions, we can rewrite expression (13) in the text as follows

$$X_{kr}(s) = \mu\frac{p_{kr}(s)^{-\sigma}\{[\alpha(1-\mu)+w_{H_r}(1+\alpha)](L+\alpha H_r)H_r+(\alpha+1)[L+\alpha H_r+H_r\alpha w_{H_r}(1-\mu)]L\}}{p_{X_r}^{1-\sigma}[L+\alpha(L+H_r)+H_r\mu\alpha^2(2-\mu)]} \tag{61}$$

with the price indices given by (19) and (20), and the prices given by (16) and (17). Expression (61) is then used to rewrite the wage in (22) as follows

$$w_{H_r} = \frac{\mu}{\sigma}\left(a_r + b_r w_{H_r} + \phi a_v + \phi b_v w_{H_v}\right)$$

where

$$\begin{aligned} a_r &= \frac{(L+\alpha H_r)(L+L\alpha+\alpha H_r-\alpha\mu H_r)}{[H_r+(H-H_r)\phi][L+\alpha(L+H_r)+H_r\mu\alpha^2(2-\mu)]}, \\ a_v &= \frac{[L+\alpha(H-H_r)][L+L\alpha+\alpha(H-H_r)-\alpha\mu(H-H_r)]}{(H-H_r+H_r\phi)\{L+\alpha[L+(H-H_r)]+(H-H_r)\mu\alpha^2(2-\mu)\}}, \\ b_r &= \frac{(L+L\alpha-L\alpha\mu+\alpha H_r)(\alpha+1)H_r}{[H_r+(H-H_r)\phi][L+\alpha(L+H_r)+H_r\mu\alpha^2(2-\mu)]} \text{ and} \\ b_v &= \frac{(L+L\alpha-L\alpha\mu+\alpha(H-H_r))(H-H_r)(\alpha+1)}{(H-H_r+H_r\phi)\{L+\alpha[L+(H-H_r)]+(H-H_r)\mu\alpha^2(2-\mu)\}}. \end{aligned}$$

Writing an analogous expression for w_{H_v}, we can solve the system of two equations in two unknowns w_{H_r} and w_{H_v}, to find that:

$$w_{H_r} = \frac{\mu^2 a_r b_v - \sigma\mu\phi a_v - \sigma\mu a_r - \mu^2\phi^2 a_r b_v}{\sigma\mu b_r + \sigma\mu b_v - \sigma^2 - \mu^2 b_r b_v + \mu^2\phi^2 b_r b_v} \tag{62}$$

and

$$w_{H_v} = \frac{\mu^2 b_r a_v - \sigma\mu\phi a_r - \sigma\mu a_v - \mu^2\phi^2 b_r a_v}{\sigma\mu b_r + \sigma\mu b_v - \sigma^2 - \mu^2 b_r b_v + \mu^2\phi^2 b_r b_v} \tag{63}$$

Hence, in the specific case of the star, $W_{\alpha ir}$ in (9) can be written for skilled and unskilled workers in region r, respectively, as

$$W_{\alpha H_r} = w_{H_r} - \alpha\mu \frac{L+\alpha H_r + H_r \alpha w_{H_r}(1-\mu)}{L+\alpha(L+H_r)+H_r\mu\alpha^2(2-\mu)}$$

and

$$W_{\alpha L_r} = 1 - \mu\alpha H_r \frac{\alpha(1-\mu)+w_{H_r}(1+\alpha)}{L+\alpha(L+H_r)+H_r\mu\alpha^2(2-\mu)}$$

and the expression for $W_{\alpha H_r}$ is used to evaluate $V(H_r, \phi, \alpha)$ in (26) in the specific case of the star to obtain expression (36).

II.D. The mixed case

Let us consider the case of the additive specification. Then when region r has a complete (integrated) network, we know that the aggregate demand in r of variety s produced in r, $X_{rr}(s)$, is given by (29), while the aggregate demand in r of variety s produced in v can be obtained from (30) and it is given by

$$X_{vr}(s) = \mu \frac{(\sigma-1)\,\tau^{-\sigma}}{\sigma\beta\,(n_r + n_v\phi)} \frac{(L + H_r w_{H_r})\,(L\alpha + \alpha H_r + 1)}{1 + \alpha\mu\,(L + H_r)}$$

On the other hand, given that region v has two segregated networks, we know that the aggregate demands in v of variety s produced respectively in v and in r can be obtained from (57). Hence, the wage in (22) in region r in the mixed case can be rewritten as follows

$$w_{H_r} = \frac{\mu}{\sigma}\left[\frac{(L+H_r w_{H_r})}{(H_r+H_v\phi)}A + \phi\frac{(w_{H_v}A_v + A_L)}{(H_v+H_r\phi)}\right]$$

while that obtained in region v is given by

$$w_{H_v} = \frac{\mu}{\sigma}\left[\frac{(w_{H_v}A_v + A_L)}{(H_v+H_r\phi)} + \frac{\phi(L+H_r w_{H_r})}{(H_r+H_v\phi)}A\right]$$

We use these last two equations to find the wages of skilled workers in the two regions, which are respectively

$$w_{H_r} = \mu\frac{\sigma\phi^2 A_L H_v - AL\mu A_v + AL\sigma H_v + \sigma\phi A_L H_r + AL\mu\phi^2 A_v + AL\sigma\phi H_r}{\sigma^2 H_r H_v + \sigma^2\phi H_r^2 + \sigma^2\phi H_v^2 + A\mu^2 A_v H_r + \sigma^2\phi^2 H_r H_v - \sigma\mu A_v H_r - \sigma\mu\phi A_v H_v - A\sigma\mu\phi H_r^2 - A\mu^2\phi^2 A_v H_r - A\sigma\mu H_r H_v}$$

and

$$w_{H_v} = \mu \frac{\sigma A_L H_r - A\mu A_L H_r + \sigma\phi A_L H_v + AL\sigma\phi^2 H_r + A\mu\phi^2 A_L H_r + AL\sigma\phi H_v}{\sigma^2 H_r H_v + \sigma^2\phi H_r^2 + \sigma^2\phi H_v^2 + A\mu^2 A_v H_r + \sigma^2\phi^2 H_r H_v - \sigma\mu A_v H_r - \sigma\mu\phi A_v H_v - A\sigma\mu\phi H_r^2 - A\mu^2\phi^2 A_v H_r - A\sigma\mu H_r H_v}$$

where the two denominators in the two equations are equal.

Then, we find that with $W_{\alpha Hr}$ for skilled workers in the integrated region r given by

$$W_{\alpha Hr} \equiv w_{H_r} - \alpha\mu \frac{L + H_r w_{H_r}}{L\alpha\mu + \alpha\mu H_r + 1}$$

and $W_{\alpha Hv}$ for skilled workers in the segregated region v

$$W_{\alpha Hv} = \frac{w_{H_v}}{\alpha\mu H_v + 1},$$

the log of the indirect utility levels $V(H_r, \phi, \alpha)$ in the mixed case is given by

$$V^{mix}(H_r, \phi, \alpha) = \ln\left[\left(\frac{p_{X_v}}{p_{X_r}}\right)^{\mu} \left(\frac{1+\alpha(L+H_r)}{1+\alpha H_v}\right)^{\mu} \frac{w_{H_r} - \alpha\mu\frac{L+H_r w_{H_r}}{L\alpha\mu+\alpha\mu H_r+1}}{\frac{w_{H_v}}{\alpha\mu H_v+1}}\right] \tag{64}$$

Evaluating $V^{mix}(H_r, \phi, \alpha)$ in (64) when *all* skilled workers are located in the integrated region r we obtain the following expression[31]

$$V^{mix}(H, \phi, \alpha) = \ln \frac{(1+\alpha\mu L)\{2+\alpha[2L-H(\sigma-1)]\}[\alpha(H+L)+1]^{\mu}\sigma\phi^{1-\frac{\mu}{\sigma-1}}}{\{[L\alpha\mu(\sigma+1)+\mu+\sigma][1+\alpha(H+L)]\}\phi^2+(L\alpha+1)[\alpha\mu(\sigma-1)(H+L)+\sigma-\mu]}$$

From the inspection of $V^{mix}(H, \phi, \alpha)$ we know that agglomeration in r can be stable only for high or intermediate ϕ. When $\phi = 1$, the argument of the logarithm in $V^{mix}(H, 1, \alpha)$ is equal to 1 when $\alpha = 0$. With a positive value of α, we can show that the argument of the logarithm in $V^{mix}(H, 1, \alpha)$ is smaller than 1 and, thus, agglomeration is an equilibrium when $\phi = 1$, only for relatively large value of μ provided that the number of unskilled workers is relatively large with respect to that of skilled workers, that is $H < 2L/(\sigma - 1)$.[32] More generally, we know that in this last case full agglomeration is stable in r also for all values of $\phi \in (\phi_s, 1)$.

[31] Where $2 + \alpha[2L - H(\sigma - 1)]$ in the numerator has to be positive to have a positive value of $W_{\alpha Hr}$ when $H_r = H$. All the other factors in the numerator and denominator are positive.

[32] Proof. The argument of the logarithm in $V^{mix}(H, 1, \alpha)$ is smaller than 1 if $\frac{\left[H\alpha+2L\alpha+2+\alpha\mu\left(H+2L+2L^2\alpha+2HL\alpha\right)\right]}{(1+\alpha\mu L)\{2+\alpha[2L-H(\sigma-1)]\}} < [\alpha(H + L) + 1]^{\mu}$. We know that these two expressions, defined as $LHS = \frac{\left[H\alpha+2L\alpha+2+\alpha\mu\left(H+2L+2L^2\alpha+2HL\alpha\right)\right]}{(1+\alpha\mu L)\{2+\alpha[2L-H(\sigma-1)]\}}$ and $RHS = [\alpha(H + L) + 1]^{\mu}$,

Instead, evaluating $V^{mix}(H_r, \phi, \alpha)$ in (64) when all skilled workers are located in the segregated region v, we find that

$$V^{mix}(0, \phi, \alpha) = \ln \frac{\left\{(L\alpha+1)[H\alpha\mu(\sigma+1)+\sigma+\mu]\phi^2 - H\sigma\alpha[(\sigma-1)\alpha\mu H+\sigma-\mu]\phi+(L\alpha+1)[H\alpha\mu(\sigma-1)+\sigma-\mu]\right\}}{2\sigma(L\alpha+1)^{1-\mu}(\alpha H+1)^{\mu}\phi^{1-\frac{\mu}{\sigma-1}}}$$

From the inspection of $V^{mix}(0, \phi, \alpha)$ we know that agglomeration in r can be stable only for high or intermediate ϕ. When $\phi = 1$, the argument of the logarithm in $V^{mix}(0, 1, \alpha)$ is equal to 1 when $\alpha = 0$.

APPENDIX III. THE AVERAGE SPECIFICATION

In the case of the average specification, we know that $S(\Omega(\Lambda_r(i))) = 1$. Hence, with $\Omega(\Lambda_{H_r}) = h_{H_r} + l_{H_r}$ and $\Omega(\Lambda_{L_r}) = h_{L_r} + l_{L_r}$, the system in (51) becomes

$$\begin{cases} X_{H_r} = \frac{\mu}{p_{X_r}}\left[w_{H_r} + \frac{(1-\mu)}{\mu}p_{X_r}\frac{\alpha'}{(1+\alpha')}\frac{(h_{H_r}X_{H_r}+l_{H_r}X_{L_r})}{h_{H_r}+l_{H_r}}\right] \\ X_{L_r} = \frac{\mu}{p_{X_r}}\left[1 + \frac{(1-\mu)}{\mu}p_{X_r}\frac{\alpha'}{(1+\alpha')}\frac{(h_{L_r}X_{H_r}+l_{L_r}X_{L_r})}{h_{L_r}+l_{L_r}}\right] \end{cases} \tag{65}$$

This system is solved to obtain the individual demands of the differentiated good by the skilled workers and by the unskilled workers in region r, i.e., X_{H_r} and X_{L_r}.

III.A. Complete network

In the case of the complete network $h_{H_r} = h_{L_r} = H_r$ and $l_{H_r} = l_{L_r} = L$, and the system in (65) obtained with the additive specification can be solved to find the individual demand of the differentiated good by a skilled consumer in region r

$$X_{H_r} = \frac{\mu}{p_{X_r}}\frac{\{(1-\mu)\alpha' L + w_{H_r}[L+H_r+\alpha'(H_r+L\mu)]\}}{(L+H_r)(1+\mu\alpha')} \tag{66}$$

and that by an unskilled consumer in the same region

$$X_{L_r} = \frac{\mu}{p_{X_r}}\frac{[(1-\mu)\alpha' H_r w_{H_r} + L + H_r + \alpha'(L+\mu H_r)]}{(L+H_r)(1+\mu\alpha')} \tag{67}$$

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are respectively equal to $\frac{H\alpha+2L\alpha+2}{H\alpha+2L\alpha-H\alpha\sigma+2} > 1$ and to 1 when $\mu = 0$, and that they both increase in the range $\mu \in [0,1]$ and assume respectively the value $2\frac{H\alpha+L\alpha+1}{H\alpha+2L\alpha-H\alpha\sigma+2}$ and $H\alpha + L\alpha + 1$ when $\mu = 1$. If $H > 2L/(\sigma-1)$ we find always that $LHS > RHS$ for $\mu \in [0,1]$ and therefore full agglomeration in r with $\phi = 1$ is never an equilibrium. However, if $H < 2L/(\sigma-1)$ full agglomeration in r with $\phi = 1$ is an equilibrium for relatively large value of μ that ensure that $LHS < RHS$. Q.E.D.

Substituting (66) and (67) into (13) we find that

$$X_{rr}(s) = \mu \frac{(\sigma-1)(L+H_r w_{H_r})}{\sigma\beta(n_r+n_v\phi)} \frac{\alpha'+1}{\mu\alpha'+1} \tag{68}$$

and

$$X_{rv}(s) = \mu \frac{(\sigma-1)\tau^{-\sigma}(L+H_v w_{H_v})}{\sigma\beta(n_v+n_r\phi)} \frac{\alpha'+1}{\mu\alpha'+1} \tag{69}$$

Hence, substituting $X_{rr}(s)$ and $X_{rv}(s)$ from (68) and (69) into (22) and making use of (14), we find that the wage paid to skilled workers in region r must satisfy the following equation

$$w_{H_r} = \frac{\frac{\left(L+H_r w_{H_r}\right)}{(H_r+H_v\phi)} + \frac{\phi\left(L+H_v w_{H_v}\right)}{(H_v+H_r\phi)}}{\sigma} \mu \frac{\alpha'+1}{\mu\alpha'+1}$$

An analogous expression holds for w_{H_v}. Therefore, also in this case we get a system of two linear equations in w_{H_r} and w_{H_v} that can be solved to obtain the two regional wages for skilled workers as an explicit function of a given distribution of workers, H_r and H_v, between the two regions. Specifically, the wage paid in region r to skilled workers is

$$w_{H_r} = L\mu \frac{\alpha'+1}{\sigma-\mu+\alpha'\mu(\sigma-1)} \frac{H_v\left[\sigma\left(\phi^2+1\right)(\mu\alpha'+1)+\mu(\phi-1)(\phi+1)(\alpha'+1)\right]+2H_r\phi\sigma(\mu\alpha'+1)}{H_v H_r\left[\sigma\left(\phi^2+1\right)(\mu\alpha'+1)+\mu(\phi-1)(\phi+1)(\alpha'+1)\right]+\phi\sigma(H_r^2+H_v^2)(\mu\alpha'+1)} \tag{70}$$

Making use of (66) and (67), we can substitute the total amount of consumptions by the neighbours of skilled workers $\int_{\Lambda_{H_r}} X_{jr} dj = X_{H_r} H_r + X_{L_r} L$ in the definition of $W_{\alpha' H_r}$ and, thus, rewrite the wage of skilled workers net of the conspicuous effect in this case as follows[33]

$$W_{\alpha' H_r} = w_{H_r} - \mu \frac{\alpha'}{\mu\alpha'+1} \frac{L+H_r w_{H_r}}{L+H_r} = \frac{w_{H_r}(L+H_r)+L\mu\alpha'(w_{H_r}-1)}{(\mu\alpha'+1)(L+H_r)}$$

Finally, we can get an analogous expression to the previous one for the wage of unskilled workers net of the conspicuous effect. Specifically, we can substitute the total amount of consumptions by the neighbours of unskilled workers $\int_{\Lambda_{L_r}} X_{jr} dj = X_{H_r} H_r + X_{L_r} L$ in the definition of $W_{\alpha' L_r}$ from (9) and obtain that[34]

$$W_{\alpha' L_r} = 1 - \mu \frac{\alpha'}{\mu\alpha'+1} \frac{L+H_r w_{H_r}}{L+H_r} = \frac{L+H_r+H_r\mu\alpha'(1-w_{H_r})}{(\mu\alpha'+1)(L+H_r)}$$

[33] This expression is relevant to check the positivity of agricultural demand by skilled workers.

[34] This expression is relevant to check the positivity of agricultural demand by unskilled workers.

The *symmetric equilibrium*, with all variables assuming the same value in both regions, is one of the interior equilibria also in the case under scrutiny without size effects. To be stable it is required that the derivative of $V(H_r, \phi, \alpha')$ with respect to H_r evaluated at the symmetric equilibrium, that is $V_{H_r}(H/2, \phi, \alpha')$, is negative. We show that

$$V_{H_r}(H/2, \phi, \alpha') = \frac{f_2'\phi^2 + f_1'\phi + f_0'}{H(\phi+1)(\sigma-1)(H+2L)\{H+2L-\alpha'[H(\sigma-1)-2L]\}} * \frac{1}{\{[\sigma-\mu+\phi(\sigma+\mu)]+\alpha'\mu[\sigma-1+\phi(\sigma+1)]\}} \tag{71}$$

where the parameters f_0', f_1' and f_2' are functions of μ, σ, L, H and α'. All the factors in the denominator are positive.[35] Thus, the sign of $V_{H_r}(H/2, \phi, \alpha')$ depends on that of the expression $F' \equiv f_2'\phi^2 + f_1'\phi + f_0'$, and the symmetric equilibrium is stable when $F' < 0$, or unstable when the opposite sign holds. The analysis is then similar to that conducted in Section 4.3 for the additive specification.

If $\alpha = 0$ and $\phi = 1$ then $F' = a_0 = 0$. If $\phi = 1$ but α is positive, $F' = 16H(\sigma-1)\alpha'\sigma[\sigma H - \mu(H + 2L + H\alpha' + 2L\alpha' - H\sigma\alpha')]$. If $\alpha' < 1/(\sigma-1)$ then F' is negative and the symmetric equilibrium is stable for any value of H[36] If $\alpha' > 1/(\sigma-1)$, F' is negative and the symmetric equilibrium is stable for $\phi = 1$ only if $H < \frac{2L(1+\alpha')}{\frac{\sigma}{\mu}+[\alpha'(\sigma-1)-1]}$.[37] This proves the first Proposition (that on the symmetric equilibrium) in Section 6.2.

Moreover, in the the case of *full agglomeration in region* r, the value of $V(H_r, \phi, \alpha')$ when all skilled workers are located in region r can be written as follows:

$$V(H, \phi, \alpha') = \ln\left\{\frac{\phi^{1-\frac{\mu}{\sigma-1}}\frac{\sigma L}{(L+H)}\left[l_2(\alpha')^2 + l_1\alpha' + l_0\right]}{t_2\phi^2 + t_1\phi + t_0}\right\} \tag{72}$$

[35] Specifically, we know that $H + 2L - \alpha'[H(\sigma-1) - 2L] =$ $[1-\alpha'(\sigma-1)]H + 2L(1+\alpha') > 0$ to have a positive value of $W_{\alpha' H_r}$ when $H_r = H/2$. If $\alpha' < 1/(\sigma-1)$ this requires that $H > \frac{2L(1+\alpha')}{[1-\alpha'(\sigma-1)]}$. Instead, if $\alpha' > 1/(\sigma-1)$ this requires that $H < \frac{2L(1+\alpha')}{[\alpha'(\sigma-1)-1]}$

[36] Indeed, if $\alpha' < 1/(\sigma-1)$, F' is negative only if $\left(\frac{\sigma}{\mu} + [\alpha'(\sigma-1)-1]\right)H < 2L(1+\alpha')$. Since we know that $\frac{\sigma}{\mu} + [\alpha'(\sigma-1)-1] < 0$ if $\mu < \frac{\sigma}{[1-\alpha'(\sigma-1)]}$ (which is always true since $\mu < 1$ and $\frac{\sigma}{[1-\alpha'(\sigma-1)]} > 1$) F' is negative only if $H > -\frac{2L(1+\alpha')}{\left(-\frac{\sigma}{\mu}+[1-\alpha'(\sigma-1)]\right)}$ (which is always true).

[37] This value is smaller than the maximum value admissible for H that ensures a positive value of $W_{\alpha' H_r}$ when $\alpha' > 1/(\sigma-1)$, that is $H = \frac{2L(1+\alpha')}{[\alpha'(\sigma-1)-1]}$.

where the coefficients $l_2 = \mu(H + 2L - H\sigma)$, $l_1 = 2H + 2L + \mu H + 2\mu L - H\sigma$ and $l_0 = 2\,(H + L)$ do not depend on ϕ and where

$$\begin{aligned} t_2 &= L(\alpha' + 1)[\alpha'\mu\,(\sigma + 1) + \sigma + \mu] > 0; \\ t_1 &= -H\alpha'\sigma[\alpha'\mu\,(\sigma - 1) + \sigma - \mu] < 0; \\ t_0 &= L(\alpha' + 1)[\alpha'\mu\,(\sigma - 1) + \sigma - \mu] > 0 \end{aligned}$$

Given the sign of the parameters t_2, t_1 and t_0 specified above, expression $t_2\phi^2 + t_1\phi + t_0$ in the denominator is a parabola with positive value t_0 when $\phi = 0$.[38] Moreover, we know that expression $l_2\,(\alpha')^2 + l_1\alpha' + l_0$ in the numerator must be positive to have $W_{\alpha' H_r} > 0$, and that expression $t_2\phi^2 + t_1\phi + t_0$ in the denominator must be positive to have $W_{\alpha' H_v} > 0$ when workers are all agglomerated in r. The analysis is similar as in the additive case (Section 4.2). The numerator of the argument of the logarithm can be represented by an increasing function in ϕ which take the value 0 when $\phi = 0$. This term is represented by the dotted curve in Figure 1 already used in Section 4.2. Comparing the doted curve with the curve representing the denominator in the logarithm of (72), we conclude that we can have either: 1) stable full agglomeration for intermediate values of ϕ when the second higher parabola lies below the dotted curve or 2) a stable full agglomeration for high level of economic integration comparing the lowest parabola with the dotted curve. Finally, the case represented by the higher parabola, which says that full agglomeration is never stable, independently of ϕ, is excluded. Hence, we can state what written in the second Proposition (on full agglomeration) in Section 6.2.

III.B. Segregated network

In the case of the segregated network $h_{H_r} = H_r$, $l_{L_r} = L$ and $h_{L_r} = l_{H_r} = 0$, and the system in (65) obtained with the additive specification can be solved to find that the individual demands by each skilled and unskilled worker in r are, respectively, given by

$$X_{H_r} = \mu \frac{1 + \alpha'}{1 + \mu\alpha'} \frac{w_{H_r}}{p_{X_r}} \quad \text{and} \quad X_{L_r} = \mu \frac{1 + \alpha'}{1 + \mu\alpha'} \frac{1}{p_{X_r}} \tag{73}$$

Then, the indirect utility function (23) in the case of skilled workers can be rewritten as follows

$$U(A_{H_r}, \Phi\,(X_{H_r}, X_{-ir})) = \eta \frac{w_{H_r}}{(p_{X_r})^{\mu}} \frac{(1 + \alpha')^{\mu}}{1 + \mu\alpha'}$$

[38] Its mimum value is attained at $\phi = -\frac{t_1}{2t_2} = \frac{H\sigma\alpha'}{2L(\alpha'+1)} > 0$.

with the logarithm of the ratio of the current indirect utility levels in region r with respect to region v given by

$$V^s\left(H_r,\phi,\alpha'\right)=\ln\left[\left(\frac{p_{X_v}}{p_{X_r}}\right)^{\mu}\frac{w_{H_r}}{w_{H_v}}\right]$$

Substituting (73) into (13) we find the same expressions for $X_{rr}(s)$ and $X_{rv}(s)$ obtained in the case of the average complete network in (68) and (69). Hence, also the wages of skilled in the two regions are equivalent to those obtained in the complete average specification (70). Nevertheless, the expression for the indirect utility function is different and the logarithm of the ratio of the current indirect utility levels in region r with respect to region v in (37) for skilled workers is given by the followign expression

$$V^s\left(H_r,\phi,\alpha'\right)=\ln\left\{\left(\frac{p_{X_v}}{p_{X_r}}\right)^{\mu}\frac{H_v\left[\sigma\left(\phi^2+1\right)(\mu\alpha'+1)+\mu(\phi-1)(\phi+1)(\alpha'+1)\right]+2(\mu\alpha'+1)\sigma H_r\phi}{H_r\left[\sigma\left(\phi^2+1\right)(\mu\alpha'+1)+\mu(\phi-1)(\phi+1)(\alpha'+1)\right]+2(\mu\alpha'+1)H_v\sigma\phi}\right\} \tag{74}$$

Then, $V^s\left(H_r,\phi,\alpha'\right)$ in (74) is used to evaluate the stability of the symmetric and the fully agglomerated equilibria. The derivative of $V^s\left(H_r,\phi,\alpha'\right)$ with respect to H_r evaluated at the symmetric equilibrium, is given by the following expression

$$V^s_{H_r}\left(H/2,\phi,\alpha'\right)=\frac{4(1-\phi)}{H(\sigma-1)(1+\phi)\{[\sigma+\mu+\mu\alpha'(\sigma+1)]\phi+\sigma-\mu+\mu\alpha'(\sigma-1)\}}Z$$

with $Z=\phi\left(\sigma-1+\mu\right)\left[\sigma+\mu+\mu\alpha'\left(\sigma+1\right)\right]-\left(\sigma-1-\mu\right)\left[\sigma-\mu+\mu\alpha'\left(\sigma-1\right)\right]$. Direct computation shows that the faction is always positive when $\phi\in[0,1)$ and is equal to zero when $\phi=1$. Thus, when $\phi\in[0,1)$, the sign of $V^s_{H_r}\left(H/2,\phi,\alpha'\right)$ depends on that of Z. Given that $\sigma-1>\mu$,[39] we know that Z is positive if $\phi\in(\phi^s_b,1)$ with ϕ^s_b denoting the break point in the case under scrutiny with $0<\phi^s_b\equiv\frac{(\sigma-1-\mu)[\sigma-\mu+\mu\alpha'(\sigma-1)]}{(\sigma-1+\mu)[\sigma+\mu+\mu\alpha'(\sigma+1)]}<1$. On the other hand, Z is negative if $\phi\in[0,\phi^s_b)$. Importantly, ϕ^s_b is smaller than the corresponding value found for $\alpha'=0$, and ϕ^s_b. is a decreasing function of α'. As increases in α' reduce the breakpoint ϕ^s_b, and thus the width of the range $\phi\in[0,\phi^s_b)$ for which the symmetric equilibrium is stable, we can state that relative concerns destabilize the symmetric equilibrium and favors agglomeration. In other words, the symmetric equilibrium is stable only for relatively low levels

[39] We recall that we assume that the no black hole condition corresponding to the case of $\alpha'=0$ holds.

of economic integration, the lower the level the larger is α'. Finally, notice that $V^s_{H_r}(H/2, \phi, \alpha')$ is independent of L, as it happens in the case of the additive specification, and also independent from of H.

We now focus on the other equilibrium. In the case of full agglomeration, expression (74) can be manipulated to give

$$V^s(H, \phi, \alpha') = \ln\left[\frac{2\sigma\phi^{1-\frac{\mu}{\sigma-1}}}{\frac{\sigma+\mu+\mu\alpha'(1+\sigma)}{(1+\mu\alpha')}\phi^2 + \frac{\sigma-\mu+\mu\alpha'(\sigma-1)}{(1+\mu\alpha')}}\right]$$

First, notice that $V^s(H, \phi, \alpha')$ is not only independent of L, as it happens in the case of the additive specification, but also independent of H. The numerator in $V^s(H, \phi, \alpha')$ is increasing in $\phi \in [0,1]$ from 0 when $\phi = 0$ to 2σ when $\phi = 1$.[40] Moreover, the denominator in $V^s(H, \phi, \alpha')$ is a positive upward opening parabola in ϕ, which takes its minimum value $[\sigma - \mu + \mu\alpha'(\sigma - 1)]/(1 + \mu\alpha') > 0$ when $\phi = 0$, and increases to 2σ when $\phi = 1$. Both numerator and denominator are represented in Figure 8, as a function of ϕ. In Figure 8 the continuous curve passing through the origin is the numerator obtained for $\alpha' = 0$ while the continuous convex curve is the denominator also for $\alpha' = 0$. In this case, full agglomeration is stable only for ϕ larger than the "sustain point" ϕ^s_s, where ϕ^s_s corresponds to the intersection between the two curves in the range $\phi \in (0,1)$. Increases in α' shift downward the parabola corresponding to the denominator, which represented by the dashed line. Note that for $\phi = 1$, the argument of the logarithm in $V^s(H, 1, \alpha')$ is still equal to 1. However, given that the minimum value attained by the parabola in the denominator when $\phi = 0$ decreases with α', we find that the sustain point ϕ^s_s defining the interval $\phi \in [\phi^s_s, 1)$ for which full agglomeration is stable, decreases with α'.

[40] Let us recall that we assume that the no black hole condition corresponding to the case of $\alpha' = 0$ holds, that is $\mu < \sigma - 1$.

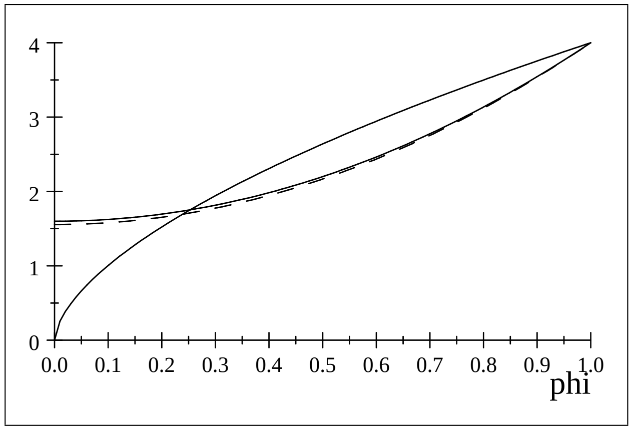


Figure 8

The results obtained in the case of the segregated network without group size effects are summarized in the Proposition in Section 6.3.

III.C. Star network

In the case of the star network $h_{L_r} = H_r/L,\ \ l_{H_r} = 1$ and $h_{H_r} = l_{L_r} = 0$, and the system in (65) can be solved to find the individual demand by each skilled worker in r

$$X_{H_r} = \frac{\mu}{p_{X_r}} \frac{(1+\alpha')[w_{H_r}(1+\alpha')+\alpha'(1-\mu)]}{(1+\mu\alpha')[1+(2-\mu)\alpha']} \tag{75}$$

and the individual demand by each unskilled worker in r

$$X_{L_r} = \frac{\mu}{p_{X_r}} \frac{(1+\alpha')[1+\alpha'+w_{H_r}\alpha'(1-\mu)]}{(1+\mu\alpha')[1+(2-\mu)\alpha']} \tag{76}$$

Then, the indirect utility function (23) in the case of skilled workers can be rewritten as follows

$$U(A_{H_r}, \Phi\left(X_{H_r}, X_{-ir}\right)) = \frac{\eta(1+\alpha')^{\mu}}{(p_{X_r})^{\mu}} \frac{\left\{w_{H_r}\left[2\alpha'+\mu(\alpha')^2+1\right]-\mu\alpha'(\alpha'+1)\right\}}{(2\alpha'-\mu\alpha'+1)(\mu\alpha'+1)}$$

while the logarithm of the ratio of the current indirect utility levels in region r with respect to region v in (37) for skilled workers is given by

$$V^{*}\left(H_r, \phi, \alpha'\right) = \ln\left\{\frac{(p_{X_v})^{\mu}}{(p_{X_r})^{\mu}} \frac{\left[w_{H_r} - \frac{\mu\alpha'(\alpha'+1)}{2\alpha'+\mu(\alpha')^2+1}\right]}{\left[w_{H_v} - \frac{\mu\alpha'(\alpha'+1)}{2\alpha'+\mu(\alpha')^2+1}\right]}\right\} \tag{77}$$

Making use of (75) and (76), we can rewrite expression (13) as follows

$$X_{kr}(s) = \frac{p_{kr}(s)^{-\sigma}}{p_{X_r}^{1-\sigma}} \mu \frac{(1+\alpha')}{(1+\mu\alpha')} \frac{L(\alpha'+1)+\alpha' H_r(1-\mu)+w_{H_r}[H_r(\alpha'+1)+L\alpha'(1-\mu)]}{[(2-\mu)\alpha'+1]} \tag{78}$$

Expression (78) is then used to rewrite the wage in (22) as follows

$$w_{H_r} = \frac{\mu(1+\alpha')}{\sigma(1+\mu\alpha')[(2-\mu)\alpha'+1]}\left(c_r w_{H_r} + d_r + \phi c_v w_{H_v} + \phi d_v\right)$$

with

$$\begin{aligned} c_r &= \frac{[L\alpha'(1-\mu)+(\alpha'+1)H_r]}{H_r+H_v\phi},\ c_v = \frac{[L\alpha'(1-\mu)+(\alpha'+1)H_v]}{(H_v+H_r\phi)}, \\ d_r &= \frac{(\alpha'+1)L+\alpha'(1-\mu)H_r}{H_r+H_v\phi} \text{ and } d_v = \frac{(\alpha'+1)L+\alpha'(1-\mu)H_v}{(H_v+H_r\phi)} \end{aligned}$$

Writing an analogous expression for w_{H_v}, we can solve the system of two equations in two unknowns w_{H_r} and w_{H_v} finding that

$$w_{H_r} = \frac{\mu(\alpha'+1)}{\sigma(\mu\alpha'+1)(1-\mu\alpha'+2\alpha')}\frac{d_r+d_v\phi+kc_v d_r\left(\phi^2-1\right)}{(1-kc_r)(1-kc_v)-\phi^2k^2c_rc_v} \tag{79}$$

and

$$w_{H_v} = \frac{\mu(\alpha'+1)}{\sigma(\mu\alpha'+1)(1-\mu\alpha'+2\alpha')}\frac{d_v+\phi d_r+kc_r d_v\left(\phi^2-1\right)}{(1-kc_r)(1-kc_v)-\phi^2k^2c_rc_v} \tag{80}$$

which can be substituted into (77) to find $V^*(H_r, \phi, \alpha')$.

The derivative of $V^*(H_r, \phi, \alpha')$ in (77) with respect to H_r evaluated at the symmetric equilibrium is given by the following expression

$$V^*_{H_r}(H/2, \phi, \alpha') = \frac{4(1-\phi)\left(s_1^*\phi+s_0^*\right)}{(1+\phi)H(\sigma-1)\left(h_1^*\phi+h_0^*\right)}$$

where the coefficients s_1^*, s_0^*, h_1^* and h_0^* are all functions of μ, σ, H, α' and L, and $V^*_{H_r}(H/2, 1, \alpha') = 0$.

As in Section 5.2, we focus the analysis on the calibrated model with parameters $H = 10$, $\sigma = 2$, $\mu = 0.11$ and $L = 20$. Figure 9.a compares the value of $V^*_{H_r}(H/2, \phi, \alpha')$ as a function of ϕ in the case of $\alpha = 0$ (black curve), $\alpha = 0.02$ (red curve), $\alpha = 0.04$ (blue curve), $\alpha = 0.06$ (green curve) and $\alpha = 0.08$ (yellow curve), the same parameters that are considered in Figure 2.b. The numerical analysis shows that the introduction of weak relative concerns (small α') in the star network without group size effects destabilizes the symmetric equilibrium for low values of integration ϕ while they have no effect for high levels of ϕ.

Insert Figure 9.a at the end of the manuscript about here

Finally we consider the full agglomeration equilibrium. The value of $V^*(H_r, \phi, \alpha')$ in (77) when all skilled workers are located in region r can be written as follows:

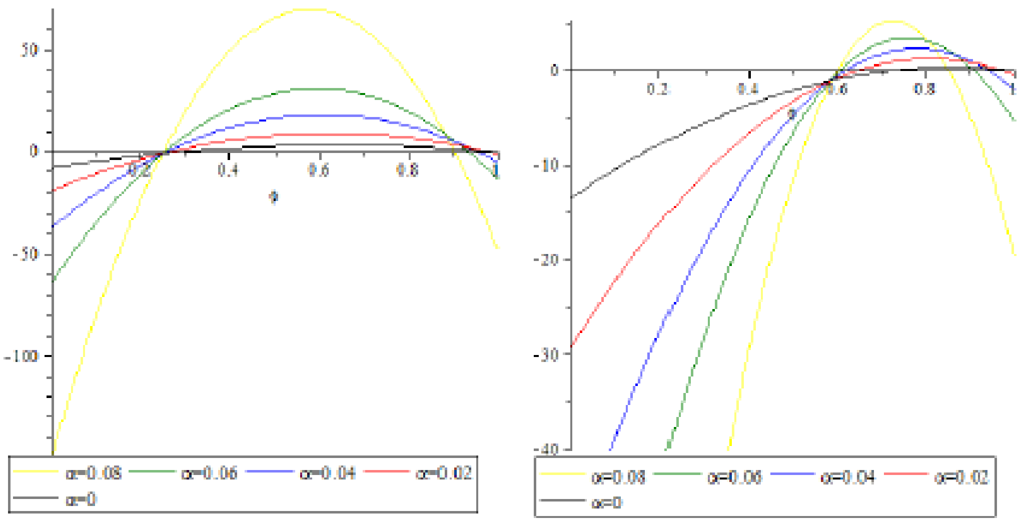
$$V^{*}(H,\phi,\alpha') = \ln \phi^{-\frac{\mu}{\sigma-1}} + \ln \frac{\left[w_{H_r} - \frac{\mu\alpha'(\alpha'+1)}{2\alpha'+\mu(\alpha')^2+1}\right]}{\left[w_{H_v} - \frac{\mu\alpha'(\alpha'+1)}{2\alpha'+\mu(\alpha')^2+1}\right]} \tag{81}$$

Indeed, $\ln \frac{p^{\mu}_{X_v}}{p^{\mu}_{X_r}} = \ln \phi^{-\frac{\mu}{\sigma-1}}$ and the skilled wages with full agglomeration in r can be obtained respectively from (79) and (80).

Direct computation shows that $V(H,1,\alpha') = 0$ when $\phi = 1$, while when $\phi \to 0$ (that is, in autarky), $V^{*}(H,0,\alpha') \to -\infty$. This behavior implies that full agglomeration in r is never sustainable with low values of ϕ. Then, considering the same calibrated model with the same parameters as in Figure 9.a, the curve representing $V^{*}(H,\phi,\alpha')$ in Figure 9.b show that full agglomeration is stable only for high level of integration ϕ. Figure 9.b also shows that increases in the value of α' allows full agglomeration to be stable for lower levels of integration ϕ (which is in line with the effects described in Figure 9.a).

Insert Figure 9.b at the end of the manuscript about here

The results obtained in the case of the multiple star network without group size effects are summarized in the Proposition in Section 6.4.



2.a 2.b

Figure 2.a-b: Stability symmetric equilibrium for the complete network: the plot of F.

Graphics drawn for *σ=2, H=10, L=20 and μ=0.4 (in case a) and μ=0.11 (in case b)*

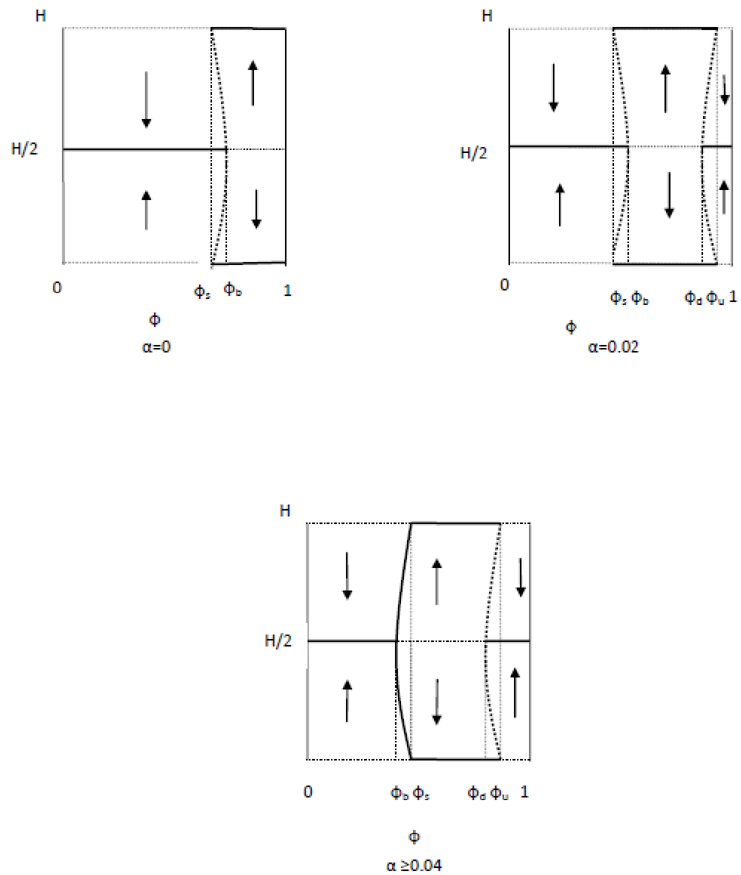


Figure 3

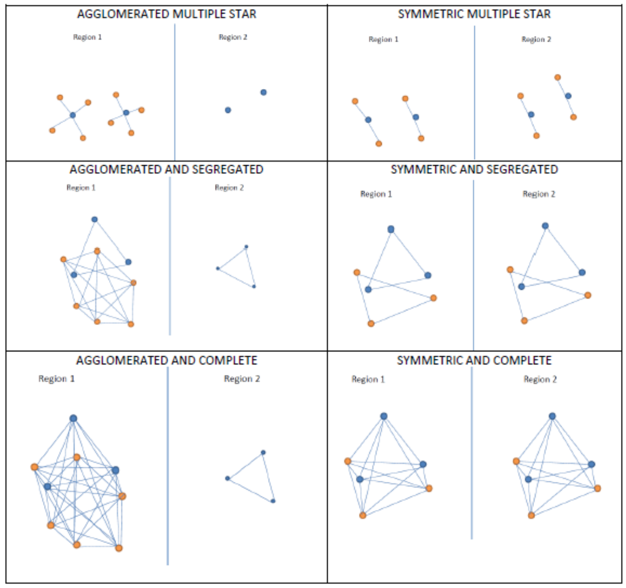
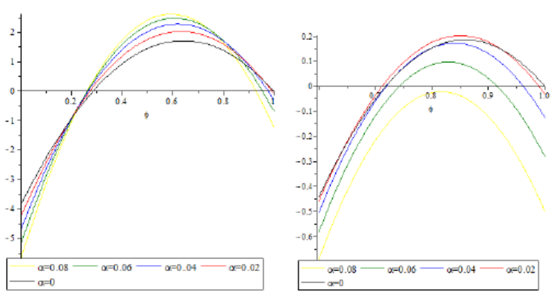


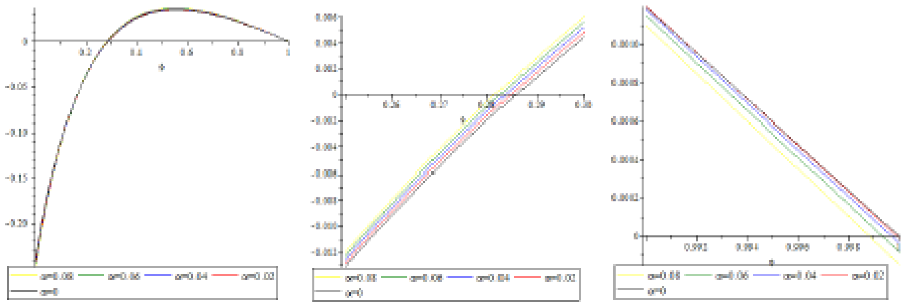
Figure 4: Networks, agglomeration and dispersion



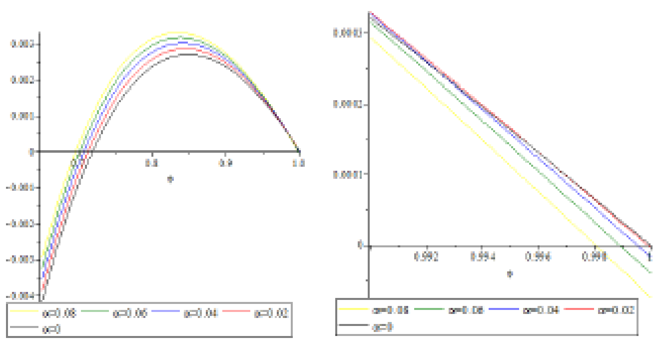
5.a 5.b

Figure 5.a-b: Stability symmetric equilibrium for the segregated network: the plot of G.

Graphics drawn for *σ=2, H=10, L=20 and μ=0.4 (in case a) and μ=0.11 (in case b)*



6.a



6.b

Figure 6.a-b: Stability symmetric equilibrium for the star network: the plot of V^*_{Hr}.

Graphics drawn for *σ=2, H=10, L=20 and μ=0.4 (in case a) and μ=0.11 (in case b*)

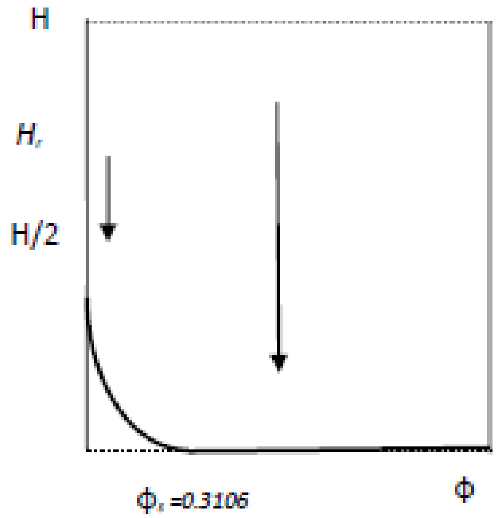


Figure 7.a

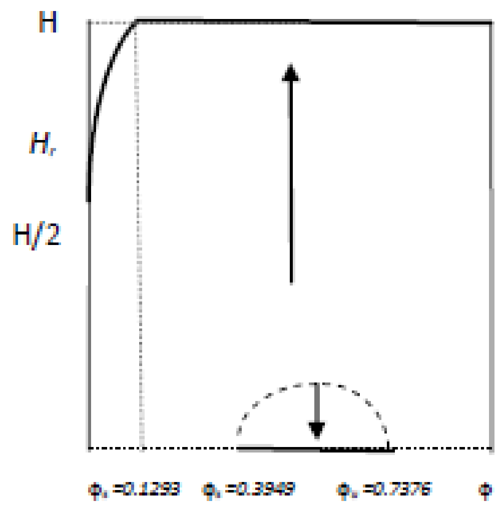


Figure 7.b

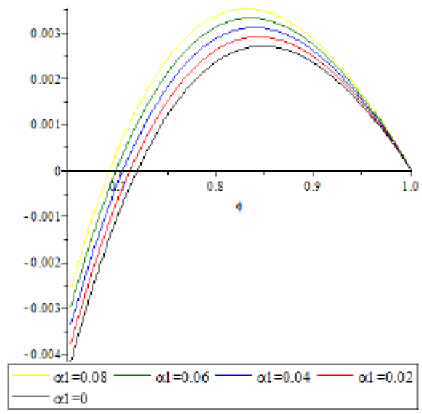


Figure 9.a: Stability symmetric equilibrium for the star network with average specification

(parameters equal to those in Figure 2.a: *μ=0.11, σ=2, H=10, L=20*)

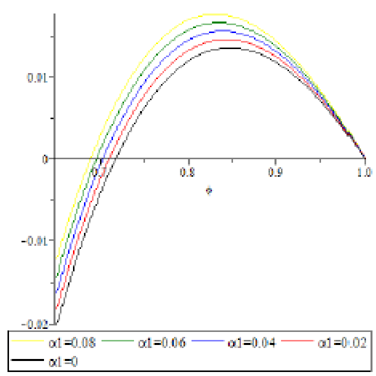


Figure 9.b: Stability full agglomeration for the star network with average specification

(parameters equal to those in Figure 2.b: *μ=0.11, σ=2, H=10, L=20*)

ORIGINAL CORE PERIPHERY MODEL (*α= α′=0*)					
Symmetric Equilibrium: $\Phi_L=(0, \phi_b)$			Full Agglomeration: $\Phi_H=(\phi_s,1)$		
CONSPICUOUS CONSUMPTION					
Specification / **Network**	**ADDITIVE *α>0***		**Specification** / **Network**	**AVERAGE *α′>0***	
Complete	Symm. Equilib. $\Phi_L=(0, \phi_b\downarrow)$ & $\Phi_{LA}=(\phi_d,1)$	Full Agglom. $\Phi_M=(\phi_s, \phi_u)$	**Complete**	Symm. Equilib. $\Phi_L=(0, \phi_b\,?)$ & $\Phi_{LA}=(\phi_d,1)$ Φ_{LA} *exists* $\forall H$ *if* α' small or *H small* & α' large	Full Agglom. $\Phi_M=(\phi_s, \phi_u)$ *or* $\Phi_H=(\phi_s,1)$
Segregated	Symm. Equilib. $\Phi_L=(0, \phi_b)$ & $\Phi_{LA}=(\phi_d,1)$ *E.g. if* μ *high* $\phi_b\downarrow$, *if* μ *low* $\phi_b\uparrow$	Full Agglom. $\Phi_M=(\phi_s, \phi_u)$	**Segregated**	Symm. Equilib. $\Phi_L=(0, \phi_b\downarrow)$ /	Full Agglom. $\Phi_H=(\phi_s\downarrow,1)$
Star	Symm. Equilib. *E.g.* $\Phi_L=(0, \phi_b\downarrow)$ & $\Phi_{LA}=(\phi_d,1)$	Full Agglom. $\Phi_M=(\phi_s, \phi_u)$ ($\Phi_H=(\phi_s,1)$)	**Star**	Symm. Equilib. *E.g.* $\Phi_L=(0, \phi_b\downarrow)$ /	Full Agglom. *E.g.* $\Phi_H=(\phi_s\downarrow,1)$

Table 2. Types of equilibria for given conspicuous effect specification, network structure and level of integration. The table represents the ranges of ϕ for which the various types of equilibrium are stable: Φ_L =low level of ϕ, Φ_M=medium level of ϕ, Φ_H =high level of ϕ and Φ_{LA}= large level of ϕ. The arrows show how the thresholds values of ϕ change with respect to the original core-periphery model (with *α= α′=0*), and the ? denotes an uncertain effect on the threshold ϕ_b (symm. equil. in the complete average specification). *E.g.* is used to present results obtained by means of simulations.