Trade Policy:
Home Market Effect versus Terms of Trade Externality

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Abstract

We study trade policy in a two-sector Krugman-type trade model with home market effects. We conduct a general analysis allowing for three different instruments: tariffs, export taxes and production subsidies. For each instrument, we consider unilateral trade policy without retaliation.

When carefully disentangling the different effects that determine policy makers' choices and modelling general equilibrium effects of taxes/tariffs, we find - contrary to the results of previous studies - that production subsidies are always inefficiently low and driven by terms of trade effects.

In the cases of tariffs and export taxes results depend crucially on whether the free trade allocation is efficient. When starting from an allocation that is distorted because of monopolistic competition, the home market effect (and in the case of export taxes also the desire to correct for the monopolistic inefficiency) induces policy makers to set a tariff (an export subsidy). However, when monopolistic distortions are corrected, terms of trade effects dominate the choice of trade policy and lead to an import subsidy (an export tax).

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1 Introduction

The aim of this paper is to study trade policy in a version of the Krugman (1980) model of intra-industry trade with monopolistic competition and increasing returns. We consider a two country version of the Krugman model with two sectors: one with monopolistic competition, increasing returns and iceberg trade costs and one that features perfect competition and constant returns. Within this framework, we study the unilateral incentives to set production subsidies/taxes, import subsidies/tariffs, as well as export taxes/subsidies.[1]

In our analysis, we disentangle three different effects that drive policy makers' incentives to set trade policy unilaterally and which they have to trade off against each other. First, a standard terms of trade externality - the desire to manipulate international prices in favor of the domestic economy by decreasing the relative price of imported bundles. Second, a home market (production relocation) externality. This externality leads policy makers to induce firms to locate in the domestic economy, so that domestic consumers can save on transport costs and benefit from a lower price level. Third, a misallocation between the differentiated and the homogeneous sector due to monopolistic pricing that implies a too low number of firms in the differentiated sector.

When a production subsidy/tax is the only instrument available, single country policy makers subsidize domestic output, but never up to the level that would implement the Pareto-optimal allocation. The reason is that the terms of trade externality always dominates the other effects. Lower subsidies imply lower relative prices of importables in world markets, whereas the home market effect would call for over-subsidizing production in order to attract firms to the domestic economy.

In the cases of import tariffs and export taxes, the novelty of our analysis is to show that optimal unilateral policy choices depend crucially on whether the initial allocation is efficient. There are two reasons for this. On the one hand, when the initial allocation is inefficient, the volume of trade is low because there are too few firms in the differentiated sector.[2] Consequently, the terms of trade externality is less relevant than in the case of efficiency. On the other hand, the price level is higher because less varieties are available, therefore the incentive to decrease

[1] We investigate cooperative and strategic determination of trade policy in Campolmi, Fadinger and Forlati (2010).

[2] In a symmetric equilibrium all trade is intra-industry.

it is stronger. Thus, the home market effect is more important than when the allocation is Pareto-optimal.

Specifically, when starting from an inefficient allocation – the case considered in the previous literature – single countries' policy makers subsidize domestic production using import tariffs or export subsidies. They do this in order to save on transport costs (home market effect) and, in the case of export subsidies, also to correct for the monopolistic distortion. In contrast, if the inefficiency of the initial allocation is eliminated with production subsidies, the terms of trade externality dominates. Thus, policymakers find it optimal to reduce the number of domestic varieties using an import subsidy or an export tax.

Finaly, if we allow for an elasticity of substitution between varieties that is larger than the trade elasticity, import subsidies (export taxes) are optimal for most of the relevant parameter space, even when not correcting the monopolistic distortions.

Our results differ from those of the previous literature that has analyzed trade policy in a two-sector Krugman model. There are several reasons for this. First, we consider income effects of trade and production taxes, while previous contributions have either assumed that tariffs are a pure waste (Venables (1987), Ossa (2008)) or that utility is quasi-linear (Helpman and Krugman (1989) Bagwell and Staiger (2009)). While these assumptions guarantee analytical tractability, both eliminate important general equilibrium effects. Second, we use a different – and as we will argue the relevant – definition of the terms of trade. This makes clear that also policy instruments which do not have a direct impact on world market prices of individual varieties (e.g. tariffs) have terms of trade effects. This statement holds true with constant returns in the homogeneous sector (which implies factor price equalization). The reason is that all policy instruments affect the number of domestic and foreign differentiated varieties and therefore the welfare relevant price indices. Third, we are the first to underline the role played by the inefficiency of the initial allocation on trade policy. This allows to interpret existing results in a new light.

For example, Venables (1987) interprets the result that a small production subsidy or an export subsidy increases welfare in the light of a home market effect, while we show that the subsidy is always smaller than the one that implements the Pareto-optimal allocation. This implies that domestic policy makers try to improve their terms of trade rather than to increase the number of domestic firms above the efficient level. Still, they choose a positive level of subsidy because the number of firms in the decentralized equilibrium without policy intervention is too low. Turning to his results on tariffs, which has recently been confirmed by Ossa (2008) in a strategic setting, he finds that a country's welfare is always raised by a unilateral increase in its import tariffs because of the home market effect. However, we show that this result does not go through and that the terms of trade effect dominates when the initial allocation is efficient.[3]

Finally, Bagwell and Staiger (2009) consider a variant of the two-sector Krugman model with quasi-linear utility and allow policy makers to simultaneously choose tariffs and export taxes in a strategic setting. They show that in this special case Nash-equilibrium policy choices are explained exclusively by the desire to manipulate international prices and not by the home market externality. We study strategic interaction in our more general framework in Campolmi et al. (2010).

Summarizing, the main contributions of our paper are the following ones. We isolate the different incentives that determine policy makers' objectives and show how they interact. Moreover, we show that home market effects never determine trade policy in the case of production subsidies/taxes and that when considering tariffs and export taxes as the trade policy instrument results depend crucially on the (in)efficiency of the initial allocation. We also clarify what the welfare relevant terms of trade are in this model and make clear that both home market and terms of trade effects coexist even when we consider a linear outside good and tariffs as the only policy instrument.

The paper proceeds as follows: The next section sets up the model and Section 3 presents the equilibrium conditions. In the following sections we discuss the definition of the terms of trade and the different incentives that determine policy makers' choices. Section 6 is dedicated to studying trade policy, while Section 7 concludes.

[3]As shown by Gros (1987), this would not be the case in a one-sector economy where it is always optimal to set an import tariff due to terms of trade effects. In the one-sector model, the number of varieties is fixed and the competitive equilibrium (with or without trade) is Pareto-optimal. Hence, neither monopolistic distortions nor the home market effect play any role for policy makers' incentives. At the same time, without the linear outside good, trade policy has an effect on factor prices and affects the terms of trade. Thus, our results are consistent with his.

2 The Model

The world economy consists of two countries: Home and Foreign. Each country produces a homogenous good and a continuum of differentiated goods. All goods are tradable but only the differentiated goods are subject to transport costs. The differentiated goods sector is characterized by monopolistic competition while there is perfect competition in the homogenous good sector. Both countries are identical in terms of preferences, production technology and market structure. In what follows Foreign variables will be denoted by a (*).

2.1 Households

Household's utility function in the Home country is given by:

$$U(C,Z) \equiv \; C^{\alpha} Z^{1-\alpha} \tag{1}$$

where C aggregates over the varieties of differentiated goods, Z represents the homogeneous good and α is the expenditure share of the differentiated bundle in the aggregate consumption basket. While the homogeneous good is identical across countries, each country produces a different subset of differentiated goods. In particular, N varieties are produced in the Home country while N^* are produced by Foreign. We allow for a general specification of the consumption aggregators with two different elasticity of substitutions, one between Home and Foreign goods (η) and one between goods produced in the same country (ε):[4]

$$C = \left[C_H^{\frac{\eta-1}{\eta}} + C_F^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \qquad \eta > 0 \tag{2}$$

$$C_H = \left[\int_0^N c(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad C_F = \left[\int_0^{N^*} c(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad \varepsilon > 1 \tag{3}$$

Foreign consumers have an analogous utility function. Let $p(h)$ ($p^*(h)$) be the price paid by Home (Foreign) consumers for domestically produced varieties while $p(f)$ ($p^*(f)$) is the price paid by home (foreign) consumers for imported varieties. In general, $p(h) \neq p^*(f)$ and

[4]Note that when $\eta = \varepsilon$, $C = \left[\int_0^N c(h)^{\frac{\varepsilon-1}{\varepsilon}} dh + \int_0^{N^*} c(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}}$ i.e. the model collapses to the standard one considered in this literature.

$p^*(h) \neq p(f)$ because of transport costs, taxes on production, on imports and on exports. Households inelastically supply L units of labor. The budget constraint of Home consumers reads as follows:

$$PC + p_Z Z = WL + T, \tag{4}$$

where W is the wage, p_Z is the price of the homogeneous good, P is the price of the differentiated bundle and T is a lump sum tax/transfer which depends on the tariff/subsidy scheme adopted by the domestic government and which will be defined later. The solution to the consumer problem gives the following demand functions and price indices:

- Home's and Foreign's demand for differentiated varieties produced by Home:

$$c(h) = \left[\frac{p(h)}{P_H}\right]^{-\varepsilon} C_H \qquad\qquad c^*(f) = \left[\frac{p^*(f)}{P_F^*}\right]^{-\varepsilon} C_F^* \tag{5}$$

$$C_H = \left[\frac{P_H}{P}\right]^{-\eta} C \qquad\qquad C_F^* = \left[\frac{P_F^*}{P^*}\right]^{-\eta} C^* \tag{6}$$

- Home's and Foreign's demand for differentiated varieties produced by Foreign:

$$c(f) = \left[\frac{p(f)}{P_F}\right]^{-\varepsilon} C_F \qquad\qquad c^*(h) = \left[\frac{p^*(h)}{P_H^*}\right]^{-\varepsilon} C_H^* \tag{7}$$

$$C_F = \left[\frac{P_F}{P}\right]^{-\eta} C \qquad\qquad C_H^* = \left[\frac{P_H^*}{P^*}\right]^{-\eta} C^* \tag{8}$$

- Demand for the homogeneous good in Home and Foreign:

$$Z = \frac{1-\alpha}{\alpha}\frac{P}{p_Z}C \qquad\qquad Z^* = \frac{1-\alpha}{\alpha}\frac{P^*}{p_Z^*}C^* \tag{9}$$

- Domestic price indices:

$$P = \left[P_H^{1-\eta} + P_F^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{10}$$

$$P_H = \left[\int_0^N p(h)^{1-\varepsilon} dh\right]^{\frac{1}{1-\varepsilon}} \qquad\qquad P_F = \left[\int_0^{N^*} p(f)^{1-\varepsilon} df\right]^{\frac{1}{1-\varepsilon}} \tag{11}$$

- Foreign price indices:

$$P^* = \left[P_H^{*\,1-\eta} + P_F^{*\,1-\eta}\right]^{\frac{1}{1-\eta}} \tag{12}$$

$$P_H^* = \left[\int_0^{N^*} p^*(h)^{1-\varepsilon} dh\right]^{\frac{1}{1-\varepsilon}} \qquad\qquad P_F^* = \left[\int_0^{N} p^*(f)^{1-\varepsilon} df\right]^{\frac{1}{1-\varepsilon}} \tag{13}$$

2.2 Firms in the Differentiated Sector

Firms in the differentiated sector operate in a regime of monopolistic competition. They pay a fixed cost in terms of labor f and then produce with a constant returns to scale technology:

$$y(h) = L_C(h) - f, \tag{14}$$

where $L_C(h)$ is the amount of labor allocated to the production of the differentiated good h. Goods sold in the foreign market are subject to an iceberg transport cost $\tau \geq 1$. Governments in both countries can use three policy instruments: a production tax/subsidy on fixed and marginal costs (τ_C), a tariff/subsidy on imports (τ_I) and a tax/subdsidy on exports (τ_X).[5] A

[5] In general τ_i indicates the gross subsidy/tax for $i \in \{C, I, X\}$ i.e., $\tau_i < 1$ indicates a subsidy, $\tau_i > 1$ indicates a tax while when $\tau_i = 1$ the policy instrument is not used.

(*) indicates the Foreign policy instruments. We assume that subsidies (taxes) are received (paid) directly by the firms. Equivalently, we could have consumers receiving (paying) them from (to) the government.

Given the constant price elasticity of demand, optimal prices charged by Home firms in the domestic market are a fixed markup over their perceived marginal cost $\tau_C W$ and optimal prices paid by Foreign consumers equal domestic prices augmented by transport costs and tariffs:[6]

$$p(h) = \tau_C \frac{\varepsilon}{\varepsilon - 1} W \qquad\qquad p^*(f) = \tau_I^* \tau_X \tau p(h) \tag{15}$$

In the same way, Foreign firms' optimal pricing decisions lead to:

$$p^*(h) = \tau_C^* \frac{\varepsilon}{\varepsilon - 1} W^* \qquad\qquad p(f) = \tau_I \tau_X^* \tau p^*(h) \tag{16}$$

2.3 Homogeneous good sector

Both countries can produce a homogenous good using the same production technology:

$$Q_Z = L_Z \tag{17}$$

where L_Z is the amount of labor allocated to producing the homogeneous good. The good is sold in a perfectly competitive market without trade costs. Consequently, the price equals marginal cost and is the same in both countries:

$$p_Z = W \qquad\qquad p_Z = p_Z^* \tag{18}$$

Due to the assumption of constant returns to scale, and as long as the homogeneous good is produced in both countries in equilibrium, which we will assume for the rest of the paper, there

[6]Following the previous literature (Venables (1987), Ossa (2008)), we assume that tariffs and export taxes are charged ad valorem on the factory gate price augmented by transport costs. This implies that transport services are taxed.

is factor price equalization:

$$p_Z = p_Z^* = W = W^* \tag{19}$$

2.4 Government

The government of each country disposes of three fiscal instruments. A production tax/subsidy (τ_C), a tariff/subsidy on imports (τ_I) and a tax/subsidy on exports (τ_X). All government revenues are redistributed to consumers through a lump sum transfer T. The government is assumed to run a balanced budget. Hence, the government's budget constraint is:

$$(\tau_I - 1)\tau_X^* \tau P_H^* C_F + (\tau_X - 1)\tau P_H C_F^* + (\tau_C - 1)W \int_0^N (y(h) + f)dh = T \tag{20}$$

Total government income consists of import revenues charged on imports of differentiated goods gross of transport costs and foreign export taxes; export taxes charged on exports gross of transport costs; and the production tax revenues.

3 Equilibrium

Given that firms share the same production technology, the equilibrium is symmetric − all firms in the differentiated sector of a given country charge the same price and produce the same quantity. This implies that in equilibrium price indices can be written as:

$$\frac{p(h)}{P_H} = N^{\frac{1}{\varepsilon-1}} \qquad \frac{p^*(h)}{P_H^*} = N^{*\frac{1}{\varepsilon-1}} \tag{21}$$

$$P_F = \tau_I \tau_X^* \tau P_H^* \qquad P_F^* = \tau_I^* \tau_X \tau P_H \tag{22}$$

3.1 Free Entry in the Differentiated Sector

The assumption of free entry in the differentiated sector implies that monopolistic producers make zero profits in equilibrium:[7]

$$\Pi(h) = c(h)\left[p(h) - \tau_C W\right] + c^*(f)\left[\tau p(h) - \tau\tau_C W\right] - f\tau_C W = 0 \tag{23}$$

Combining the optimal pricing rule with equation (23), we obtain:

$$c(h) + \tau c^*(f) = (\varepsilon - 1)f \tag{24}$$

Substituting the demand functions into (24) and using (21) and (22), the zero profit condition for firms in the domestic differentiated sector can be rewritten as:

$$(\varepsilon - 1)f = N^{\frac{\varepsilon}{1-\varepsilon}}\left(\frac{P_H}{p_z}\right)^{-\eta}\left[\left(\frac{P}{p_z}\right)^{\eta} C + \tau^{1-\eta}(\tau_I^*\tau_X)^{-\eta}\left(\frac{P^*}{p_z}\right)^{\eta} C^*\right] \tag{25}$$

An analogous condition can be derived for firms located in Foreign:

$$(\varepsilon - 1)f = N^{*\frac{\varepsilon}{1-\varepsilon}}\left(\frac{P_H^*}{p_z}\right)^{-\eta}\left[\left(\frac{P^*}{p_z}\right)^{\eta} C^* + \tau_I{}^{-\eta}\tau_X^*{}^{-\eta}\tau^{1-\eta}\left(\frac{P}{p_z}\right)^{\eta} C\right] \tag{26}$$

3.2 Goods and Labor Market Clearing Conditions

For each differentiated variety produced by Home the following market clearing condition must be satisfied:

$$y(h) = c(h) + \tau c^*(f) \tag{27}$$

Therefore, the zero profit condition (24) and market clearing (27) imply that the production of each variety is fixed and the same is true for the varieties produced by Foreign:

$$y(h) = (\varepsilon - 1)f \qquad\qquad y^*(h) = (\varepsilon - 1)f \tag{28}$$

[7]Remember that firms pay (receive) taxes (subsidies) to (from) the government. Taking this into account, firms' revenues from exporting are given by $c^*(f)\frac{p^*(f)}{\tau_I^*\tau_X} = c^*(f)\tau p(h)$.

The market clearing condition for the homogeneous good is given by:

$$Q_Z + Q_Z^* = Z + Z^*, \tag{29}$$

which, using the demand functions, can be written as:

$$Q_Z + Q_Z^* = \frac{(1-\alpha)}{\alpha}\left[\frac{P}{p_z}C + \frac{P^*}{p_z}C^*\right] \tag{30}$$

Finally, equilibrium in the labor market implies that $L = L_C + L_Z$ with $L_C = NL_C(h)$ in the symmetric equilibrium. Making use of (14) and (28), we have:

$$L_C = N\varepsilon f \qquad\qquad Q_Z = L - N\varepsilon f \tag{31}$$

and for Foreign:

$$L_C^* = N^*\varepsilon f \qquad\qquad Q_Z^* = L^* - N^*\varepsilon f \tag{32}$$

3.3 Balanced Trade Condition

The model is solved under the assumption of financial autarky, so trade is balanced. Net-exports of the homogenous good by Home are defined as:

$$Z^X - Z^M \equiv Q_Z - \frac{1-\alpha}{\alpha}\frac{P}{p_Z}C \tag{33}$$

Hence, the balanced trade condition reads as follows:[8]

$$\tau\tau_X P_H C_F^* + p_Z\left(Z^X - Z^M\right) = \tau\tau_X^* P_H^* C_F \tag{34}$$

The left hand side of the above expression is the sum of net export value of the homogeneous goods and the value of exports of differentiated varieties, while the right hand side is the value of imports of differentiated varieties.

[8] Import tariffs/subsidies are collected directly by the governments at the border so they do not enter into this condition.

Combining (33) with (22), (34) and the demand functions, we can rewrite the balanced trade condition as follows:

$$Q_Z = \frac{(1-\alpha)}{\alpha}\frac{P}{p_z}C + \tau_I^{-\eta}(\tau_X^*\tau)^{1-\eta}\left(\frac{P_H^*}{p_z}\right)^{1-\eta}\left(\frac{P}{p_z}\right)^{\eta}C - \tau_I^{*-\eta}(\tau_X\tau)^{1-\eta}\left(\frac{P_H}{p_z}\right)^{1-\eta}\left(\frac{P^*}{p_z}\right)^{\eta}C^* \tag{35}$$

3.4 Price Indices

Using the optimal pricing rules (15) and (18) together with equations (17) and (21) (and the corresponding ones for Foreign), relative prices can be written as follows:

$$\frac{P_H}{p_z} = \frac{\varepsilon}{\varepsilon-1}\tau_C N^{\frac{1}{1-\varepsilon}} \qquad \frac{P_H^*}{p_z} = \frac{\varepsilon}{\varepsilon-1}\tau_C^* N^{*\frac{1}{1-\varepsilon}} \tag{36}$$

$$\frac{P}{p_z} = \left[\left(\frac{P_H}{p_z}\right)^{1-\eta} + (\tau_I\tau_X^*\tau)^{1-\eta}\left(\frac{P_H^*}{p_z}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \quad \frac{P^*}{p_z} = \left[\left(\frac{P_H^*}{p_z}\right)^{1-\eta} + (\tau_I^*\tau_X\tau)^{1-\eta}\left(\frac{P_H}{p_z}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{37}$$

The free entry conditions for the two countries (25) and (26), the market clearing condition for the homogeneous good (30) and the balanced trade condition (35) together with the expressions for price indices just derived and (31) and (32) fully characterize the equilibrium of the economy. For the case $\eta = \varepsilon$ it is possible to solve the system explicitly for N and N^*. The expressions for these variables can be found in the Appendix.

Before going into the details of trade policy choice under different instruments, in the next two sections we clarify two main points. First, the relevant definition of the terms of trade. Second, the different economic incentives that determine unilateral trade policies.

4 Terms of Trade

One crucial aspect of our study of policy makers' incentives to set trade policy is to define the terms of trade in the way that is relevant for policy makers. All previous contributions in this literature (Venables (1987), Helpman and Krugman (1989), Ossa (2008), Bagwell and Staiger (2009)) have defined the terms of trade as the relative prices of individual varieties in international markets, $\frac{\tau\tau_X^* p^*(h)}{\tau\tau_X p(h)} = \frac{\tau_X^*\tau_C^*}{\tau_X\tau_C}$. Because of the assumption of a linear outside good, relative wages are one. Consequently, only export and production taxes can affect these relative prices.

However, these are not the relative prices in international markets that domestic policy makers care about. Domestic policy makers – like domestic consumers – are interested in how many units of foreign bundles they can buy for a given amount of domestic bundles.[9] This relation is reflected in the trade balance condition (34).

Dividing (34) by $(\tau\tau_X P_H)$ we obtain $C_F^* + \left(\frac{p_z}{\tau\tau_X P_H}\right)(Z^X - Z^M) = \left(\frac{\tau_X^* P_H^*}{\tau_X P_H}\right) C_F$, where the left hand side is the value of domestic exports and the right hand side is the value of domestic imports. In this relation all variables are written in terms of optimal consumption indices.[10] Consider the case in which Home imports the homogeneous good ($Z^X = 0$). In this case an increase in the relative world market price (before tariffs are applied) of the Foreign differentiated bundle in terms of the domestic bundle $\left(\frac{\tau_X^* P_H^*}{\tau_X P_H}\right)$ hurts domestic consumers because the amount of foreign differentiated goods they can buy for a given amount of domestic goods decreases. Similarly, an increase of the relative price of homogeneous goods in international markets $p_z/(\tau\tau_X P_H)$ in terms of domestic exports also lowers the purchasing power of domestic exports in terms of Foreign goods.

As a consequence, the two relative world market prices that are of interest for domestic policy makers – and thus the welfare relevant definition of the terms of trade – are $(\tau_X^* P_H^*)/(\tau_X P_H)$ and $p_z/(\tau\tau_X P_H)$ if Home is an importer of the homogeneous good.[11]

Using the definition of the price indices, we can write the welfare-relevant terms of trade as $\left(\frac{\tau_X^* P_H^*}{\tau_X P_H}\right) = \left(\frac{N}{N^*}\right)^{\frac{1}{\varepsilon-1}} \frac{\tau_X^*\tau_C^*}{\tau_X\tau_C}$. Hence, the relative international price of imports of differentiated

[9] This welfare based definition of the terms of trade is common in the international macroeconomics literature. See for example Corsetti and Pesenti (2001) and Epifani and Gancia (2009).

[10] The consumption indices are the Hicksian demand functions for the respective bundles, which minimize expenditure for a given utility level.

[11] If Home is an exporter of the homogeneous good, $(\tau\tau_X^* P_H^*)/p_z$ is the second relevant relative price.

goods in terms of exports depends positively on the relative number of varieties produced domestically. It also depends directly on domestic and Foreign production subsidies τ_C, τ_C^* and export taxes τ_X, τ_X^*. Note that the previous literature has only considered the second part of the above expression as the terms of trade, omitting the part that depends on the relative number of domestic varieties, $\left(\frac{N}{N^*}\right)^{\frac{1}{\varepsilon-1}}$. Intuitively, an increase in N^* for a given N increases the utility derived from a given expenditure on the Foreign bundle.

It becomes clear that with the definition of the terms of trade we propose here, all policy instruments (production subsidies, tariffs and export taxes) affect the terms of trade indirectly, by changing the distribution of firms located in the domestic and in the Foreign economy.

5 Single Country Policy Makers' Incentives

In this section we clarify the role played by different inefficiencies/externalities in affecting a country's decision over its trade policy.

For the case commonly studied in the literature $\eta = \varepsilon$, it is possible to derive closed form solutions for the equilibrium allocations and prices. In that case we can use the representative consumer's indirect utility function to disentangle the different effects that determine unilateral trade policy choices. In the rest of this section and whenever we want to provide analytical results, we will restrict preferences to this special case.

Up to a constant, indirect utility in logs can be written as[12]

$$\log\left(V(P/p_z, I)\right) = -\alpha \log\left(\frac{P}{p_z}\right) + \log\left(\frac{I}{p_z}\right) \tag{38}$$

The first term, which can be interpreted as an inverse measure of the real wage, is decreasing in the relative price of the differentiated bundle. The second term is income in terms of the homogeneous good (labor). Differentiating indirect utility with respect to the dummy trade policy instrument τ_i, we obtain

$$\frac{\partial \log(V(P/p_z, I))}{\partial \tau_i} = -\alpha \frac{\frac{\partial (P/p_z)}{\partial \tau_i}}{P/p_z} + \frac{\frac{\partial (I/p_z)}{\partial \tau_i}}{I/p_z}. \tag{39}$$

Thus, trade policy affects indirect utility through two channels. On the one hand, it impacts

[12]See appendix for derivations.

on indirect utility through its effect on the relative price of the differentiated good. Since $\frac{P}{p_z}$ can be written as $\frac{\varepsilon}{\varepsilon-1}[N\tau_c^{1-\varepsilon}+(\tau_C^*\tau_I\tau_X^*\tau)^{1-\varepsilon}N^*]^{\frac{1}{1-\varepsilon}}$, trade policy affects relative prices both directly and indirectly (through a change in N and N^*). On the other hand, indirect utility changes through trade policy's impact on domestic income.

Note that income can be expressed from the trade balance.[13] Using the definition of consumption indices, and substituting the labor market clearing condition, the trade balance condition can be rewritten as:

$$\left(\frac{P_F^*}{P^*}\right)^{-\varepsilon}\left(\frac{p_z}{P^*}\right)\alpha\frac{I^*}{p_z}+\left(\frac{p_z}{\tau\tau_X P_H}\right)\left(L-\varepsilon fN-(1-\alpha)\frac{I}{p_z}\right)=\left(\frac{\tau_X^*P_H^*}{\tau_X P_H}\right)\left(\frac{P_F}{P}\right)^{-\varepsilon}\left(\frac{p_z}{P}\right)\alpha\frac{I}{p_z} \tag{40}$$

Totally differentiating this equation with respect to the policy instrument and collecting terms, we obtain:

$$\frac{\partial\left(\frac{I}{p_z}\right)}{\partial\tau_i}=B^{-1}(B_1+B_2+B_3+B_4+B_5), \tag{41}$$

where $B_1\equiv\left[\left(L-\varepsilon fN-(1-\alpha)\frac{I}{p_z}\right)\frac{\partial\left(\frac{p_z}{\tau\tau_X P_H}\right)}{\partial\tau_i}-\alpha\left(\frac{P_F}{P}\right)^{-\varepsilon}\left(\frac{p_z}{P}\right)\frac{I}{p_z}\frac{\partial\left(\frac{\tau_X^*P_H^*}{\tau_X P_H}\right)}{\partial\tau_i}\right]$

$B_2\equiv\left[-\varepsilon f\left(\frac{p_z}{\tau\tau_X P_H}\right)\frac{\partial N}{\partial\tau_i}\right]$

$B_3\equiv\left[-\varepsilon\alpha\left(\frac{P_F^*}{P^*}\right)^{-\varepsilon-1}\left(\frac{p_z}{P^*}\right)\frac{I^*}{p_z}\frac{\partial\left(\frac{P_F^*}{P^*}\right)}{\partial\tau_i}+\alpha\left(\frac{P_F^*}{P^*}\right)^{-\varepsilon}\frac{I^*}{p_z}\frac{\partial\left(\frac{p_z}{P^*}\right)}{\partial\tau_i}\right]$

$B_4\equiv\left[\alpha\left(\frac{P_F^*}{P^*}\right)^{-\varepsilon}\left(\frac{p_z}{P^*}\right)\frac{\partial\left(\frac{I^*}{p_z}\right)}{\partial\tau_i}\right]$

$B_5\equiv\left[\varepsilon\alpha\left(\frac{\tau_X^*P_H^*}{\tau_X P_H}\right)\left(\frac{P_F}{P}\right)^{-\varepsilon-1}\left(\frac{p_z}{P}\right)\frac{I}{p_z}\frac{\partial\left(\frac{P_F}{P}\right)}{\partial\tau_i}-\alpha\left(\frac{\tau_X^*P_H^*}{\tau_X P_H}\right)\left(\frac{P_F}{P}\right)^{-\varepsilon}\frac{I}{p_z}\frac{\partial\left(\frac{p_z}{P}\right)}{\partial\tau_i}\right]$

$B\equiv\left[\alpha\left(\frac{\tau_X^*P_H^*}{\tau_X P_H}\right)\left(\frac{P_F}{P}\right)^{-\varepsilon}\left(\frac{p_z}{P}\right)+(1-\alpha)\left(\frac{p_z}{\tau\tau_X P_H}\right)\right]$

We are now ready to discuss the different channels that determine unilateral trade policy.

First, with positive transport costs, there is a *home market* (production relocation) *externality.* Domestic policy makers try to induce firms to relocate to the domestic economy, so that domestic consumers can benefit from lower prices, since they save on transport costs. In terms

[13] Alternatively, income in terms of the homogeneous good can also be expressed as $L+T/p_z$. However, starting from the trade balance makes it easier to identify the different economic channels.

of indirect utility, this effect works through a decrease in P/p_z by changing the relative weights of domestic and Foreign varieties.

Second, there is a *terms of trade externality.* Countries have market power both in import and in export markets – they face an upward sloping export supply and a downward sloping import demand curve. Hence, single country policy makers try to render the relative prices of imported bundles cheaper in order to maximize the total purchasing power of domestically produced goods in international markets. In this way they can increase domestic income by importing more for each unit of exports.

In fact, the terms of trade externality is an income effect. Condition (41) clarifies exactly this. In this expression B_1 is the impact of the policy instrument on the domestic terms of trade for given quantities of imports. If the domestic country is an importer of the homogeneous good the first term is negative. Hence, if the change in the policy instrument reduces the relative import price of homogeneous goods – for example by reducing the number of domestic varieties – this affects domestic income positively. The second term is the relative price of imports of the differentiated bundle in terms of exports of the differentiated bundle. A reduction in this price also increases domestic income. It is clear that there is an inherent trade-off between the terms of trade effect and the home market externality, since the first implies a reduction in the number of domestically produced varieties while the second one calls for an increase.[14]

Third, there is an *efficiency effect of trade policies.* Without trade policy intervention, the number of differentiated varieties at the world level is too low relative to the amount of production of the homogeneous good. This is due to monopolistic price setting in the differentiated sector. In particular, if not corrected by the production subsidy, the price markup charged by firms in the differentiated sector leads to an equilibrium with an inefficiently low number of varieties and an inefficiently high level of production of the homogeneous good because the marginal rate of substitution between the two sectors does not equal the marginal rate of transformation. In order to correct for such an inefficiency, policy makers have an incentive to subsidize production. By doing so, they can either completely (production subsidy) or partially

[14]As explained below, B_2 is the opportunity cost in terms of production of the homogeneous good of a change in domestic production of differentiated varieties that is caused by a change in trade policy . The terms B_3 and B_4 measure Foreign substitution and income effects induced by changes in domestic trade policy. Other things equal, an increase in Foreign demand for domestic goods due to a fall in their relative price or due to higher Foreign income, augments domestic income. This is because it allows Home to import more Foreign goods. Similarly, B_5 is the domestic substitution effect.

(import or export subsidy) eliminate the price markup in the differentiated sector, thus increasing the world number of varieties. This monopolistic distortion incentive enters P/p_Z where it induces policy makers to increase $N + N^*$. Hence, it is easy to confuse the desire to correct the inefficiency due to monopolistic competition with the home market effect. In addition, both the home market effect and the efficiency effect enter I/p_z through the term B_2 that reflects the opportunity cost in terms of production of the homogeneous good of increasing N.

6 Trade Policy

We now study optimal trade policy without retaliation for production taxes (τ_C, τ_C^*), import tariffs (τ_I, τ_I^*) and export taxes (τ_X, τ_X^*). In each case, we analyze only the choice of one instrument at a time, so we do not allow, for example, policy makers to set simultaneously import tariffs and export taxes.[15] A detailed analysis of coordinated trade policy and strategic trade policy interaction for each instrument is provided in Campolmi et al. (2010).[16]

In order to better understand the results for the different policy instruments, it is useful to consider the efficient allocation as a benchmark. While we refer to Campolmi et al. (2010) for the formal derivations, here it is enough to summarize the main result. The first best allocation can be reached by setting the production subsidy in each country at the level required to eliminate the price markup in the differentiated sector ($\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$, $\tau_I = \tau_I^* = \tau_X = \tau_X^* = 1$).

For each of the policy instruments we then investigate two cases. Under the first hypothesis, the monopolistic distortion in the differentiated sector is not corrected (i.e. $\tau_C = \tau_C^* = 1$), while under the second one it is offset by an appropriate production subsidy (i.e. $\tau_C = \tau_C^* = (\varepsilon - 1)/\varepsilon$). These two scenarios allow to disentangle the consequences of the inefficiency caused by monopolistic pricing in the differentiated sector from the other effects.

For comparison with the existing literature, our main set of results is derived under the assumptions $\eta = \varepsilon$. For this case we provide analytical proofs.

We also show how results change when $\eta \neq \varepsilon$. In this case the general equilibrium and the optimal policy problem do not have explicit analytical solutions. Thus, we have to rely on

[15] Hence, when policy makers choose import tariffs, τ_X and τ_X^* are always set equal to one. This is an important difference with respect to the analysis in Bagwell and Staiger (2009).

[16] In that paper we investigate optimal trade policies both from the perspective of single country policy makers, studying the Nash equilibrium of the game, and from the perspective of a cooperative authority that maximizes average welfare of the world economy.

numerical simulations.

6.1 Production Subsidies

In this section we consider the case of a production subsidy/tax. We assume that this is the only available policy instrument, i.e. we set $\tau_I = \tau_I^* = \tau_X = \tau_X^* = 1$.

6.1.1 Benchmark case: $\eta = \varepsilon$

We start from a situation where none of the two countries is using the production subsidy (i.e. $\tau_C = \tau_C^* = 1$) and ask what the optimal choice of τ_C for Home would be under the assumption that Foreign keeps $\tau_C^* = 1$. We first present a numerical example to illustrate the different economic mechanisms at work.[17]

Figure 1 shows the behavior of some key variables for both countries as functions of the domestic production subsidy. An increase in the domestic subsidy increases demand for domestic differentiated goods and thus, other things equal, generates positive profits for producers located in the domestic country. This causes firms in the differentiated sector to enter the domestic market and to leave the Foreign one until zero profits are reached. Hence, the subsidy to production causes agglomeration. Overall, the increase in N more than compensates the decrease in N^* and pushes the number of varieties available at the world level closer to efficiency. This comes at the cost of worsening domestic terms of trade. Domestic utility as a function of the subsidy is hump-shaped, reflecting this trade-off. Overall, the terms of trade externality predominates and utility is maximized at a level of domestic subsidy that is strictly smaller than the efficient one ($\tau_C = \frac{\varepsilon-1}{\varepsilon} = 0.75$). Interestingly, even though now production is more efficient at the world level, Foreign is worse off. This is due to the higher transportation costs Foreign consumers have to pay after the relocation of firms to Home.

To abstract from the role played by the desire to correct the inefficiency caused by monopolistic competition, we run a second experiment starting from the first best allocation, i.e. we set

[17] For our numerical example we consider $\varepsilon = 4$ and transport costs $\tau = 1.4$, which are standard values in the literature. Anderson and Wincoop (2004) estimate an international trade cost excluding policy barriers of around 60% for industrialized countries. This splits into a transport cost of 21% and a 32% international trade costs excluding policy barriers, such as language and information costs (0.6=1.21*1.32-1). We view this as rather high but our results are perfectly robust to choosing this number for trade costs. We set the expenditure share on the differentiated sector $\alpha = 0.4$, as in Fujita, Krugman and Venables (1999). We use this calibration throughout the paper.

$\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$. This is done in Figure 2. A unilateral decrease in the domestic subsidy from the efficient level increases domestic utility, even though it lowers domestic consumption of differentiated goods. Hence, the optimal strategy given that the other country chooses an efficient subsidy, is to deviate to a smaller subsidy. This causes exit of firms in Home and entry in Foreign and improves domestic terms of trade, while lowering the aggregate level of efficiency. Home is under-subsidizing and not over-subsidizing domestic production as the home market effect would require. We can then conclude that in the case of production subsidies the terms of trade externality dominates.

Having gained some intuition, we now move to proving these statements formally.[18] First, we show that an increase in the domestic subsidy indeed increases the number of domestic firms at the expense of Foreign.

Lemma 1 Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. If $\tau_C = \tau_C^* \leq 1$ then $\frac{\partial N}{\partial \tau_C} < 0$, $\frac{\partial N^*}{\partial \tau_C} > 0$ and $|\frac{\partial N^*}{\partial \tau_C}| - |\frac{\partial N}{\partial \tau_C}| < 0$.

Lemma 1 implies that, starting from a symmetric equilibrium where either no country is using the subsidy or both are subsidizing at the same rate, a unilateral increase in the domestic subsidy τ_C[19] increases N at the cost of a reduction in N^*. At the same time the world level of differentiated varieties, $N + N^*$, increases.

Next, we decompose the welfare effects of an increase in the domestic production subsidy using (39) and the following Lemma:

Lemma 2 Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. If $\tau_C = \tau_C^* \leq 1$ then $\frac{\partial P/p_z}{\partial \tau_C} > 0$ and $\frac{\partial I/p_z}{\partial \tau_C} > 0$.

Increasing the production subsidy has two opposing effects on indirect utility. On the one hand, it increases indirect utility by lowering the domestic price index. This welfare gain reflects both the home market effect and the reduction in the monopolistic distortion. Both increase indirect utility by improving the purchasing power for a given income level. On the other hand, it decreases indirect utility by reducing domestic income. Intuition for the sources of this welfare loss can be gained from (41). The term $B_1 > 0$[20] implies that an increase in

[18] All proofs can be found in the Appendix.
[19] Recall that higher subsidy means moving τ_C from 1 to 0.
[20] The proof of the signs of the B's can be found in the appendix.

the production subsidy causes the terms of trade to move against Home. Moreover, the cost of reducing monopolistic distortions augments as indicated by the term $B_2 > 0$ which captures the domestic opportunity cost in terms of reduced production of homogeneous goods.[21] Whether the price or the income effect predominates crucially depends on the (in)efficiency of the initial allocation. This is proved in the next Theorem:

Theorem 1 Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. Then:

(1) $\left.\frac{\partial W}{\partial \tau_C}\right|_{\tau_C=\tau_C^*=1} < 0$

(2) $\left.\frac{\partial W}{\partial \tau_C}\right|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}} > 0$

(3) $\left.\frac{\partial W}{\partial \tau_C}\right|_{\tau_C=\frac{\varepsilon-1}{\varepsilon},\tau_C^*=1} > 0$ if $\tau < 4$ or $\varepsilon - 1 > \alpha$.

The first part of Theorem 1 states that starting from a free trade equilibrium countries always have an incentive to set a small subsidy. Parts 2 and 3 state that this subsidy is always[22] inefficiently low, independently of whether the other country also subsidizes production. We therefore conclude that the terms of trade effect always outweighs the other effects in the choice of the production subsidy. This is an important result because it contradicts the standard wisdom that in the two-sector Krugman model countries have an incentive to oversubsidize production in order to attract more firms (Venables (1987)).

The intuition for Theorem 1 is the following. At the free trade allocation the gains from lowering the price index are high due to the inefficiently low number of varieties. In addition, the opportunity cost from reducing the homogeneous good is negligible due to its abundance. At the same time, the income effect is small because the volume of trade is low, thus the terms of trade externality is weak. However, as we move towards the efficient allocation, the gain from lowering the price index decreases while the opportunity cost increases, thus weakening both

[21]There are also some other effects but all go in the same direction as the home market effect. The Foreign substitution effect is positive ($B_3 < 0$), since Foreign demand shift towards Home differentiated varieties and this more than compensates for the shift of Foreign demand towards homogeneous goods. In the case in which Foreign sets a positive subsidy, there is a positive income effect in Foreign ($B_4 < 0$). The increase in the domestic subsidy increases Foreign income because it reduces the Foreign subsidy bill through its effect on firm delocation. Finally, Home demand shifts away from Foreign varieties and away from homogeneous goods. This also has a positive effect on domestic income ($B_5 < 0$).

[22]Note that the third part of the theorem is always satisfied for $\varepsilon \geq 2$, a standard value for the trade elasticity.

the home market and the inefficiency considerations. Moreover, the terms of trade externality is strengthen as the volume of trade increases.

6.1.2 Implications of $\eta \neq \varepsilon$

Next, we check if the previous results generalize to the case $\eta < \varepsilon$.[23] The main advantage of this more general specification is that we can shut off the home market effect[24] (by setting $\tau = 1$) or/and the monopolistic distortion (by letting $\varepsilon \to \infty$). The drawback of this more general model is that it cannot be solved analytically, so we have to rely on simulations.

In Figure 3 we report both the subsidy that implements the first best allocation (top) and the optimal domestic subsidy/tax (bottom) for the case when τ_C^* is set to one. We consider $\tau \in [1, 2]$ and $\varepsilon \in [2, 8]$.[25] Several things are worth noticing. First, the efficient production subsidy does not depend on the level of transport cost and approaches one (no subsidy) as ε increases. Second, the domestic country always chooses a production subsidy lower than the efficient one, independently of the level of the monopolistic distortion and the level of the transport costs. Third, without the home market effect ($\tau = 1$) and with sufficiently low monopolistic distortions (high ε) the terms of trade externality becomes strong enough to induce domestic policy makers to tax production at a positive rate. Results are qualitatively the same when considering the case $\tau_C = \frac{\varepsilon - 1}{\varepsilon}$.[26]

6.2 Tariffs

Here, we explore what drives single country policy makers' incentives when the only instrument available to them is an import tariff. Again, we analyze the impact of a unilateral change in the domestic tariff in the absence of Foreign policy intervention ($\tau_I^* = 1$).

[23] $\eta \leq \varepsilon$ guarantees that demand for varieties is increasing in the sectoral price index.

[24] With $\eta = \varepsilon$ and no transport costs domestic and foreign varieties are perfect substitutes, so that a positive production subsidy set by one country induces the whole differentiated sector to locate in that country i.e. it induces full specialization.

[25] Given that $\eta \leq \varepsilon$, for this exercise we set $\eta = 2$ in order to study also cases with very strong inefficiency due to monopolistic distortion.

[26] The figure is not reported to save space but is available on request.

6.2.1 Benchmark case: $\eta = \varepsilon$

First, we study the effects of a unilateral change in the domestic tariff when the number of varieties is inefficiently low (i.e. $\tau_C = \tau_C^* = 1$). In this scenario a positive tariff improves domestic welfare. This result is consistent with Venables (1987) and Ossa (2008) and can be explained as follows.

When unilaterally setting a tariff/subsidy on imports (see Figure 4), the domestic policy maker faces a trade off between different effects. On the one hand, a tariff induces firms to relocate to the domestic economy and allows domestic consumers to save on transport costs. On the other hand, a tariff worsens domestic terms of trade and exacerbates the inefficiency due to monopolistic competition by reducing $N + N^*$, the total number of the varieties produced in the differentiated sector. Differently from the case of the production subsidy, here the home market effect prevails: a tariff on imports increases domestic welfare by boosting consumption in the differentiated and in the homogenous sectors. The impact of a unilateral change in the domestic tariff on the terms of trade is weaker than in the case of the other policy instruments because tariffs affect terms of trade only through their indirect impact on the relative number of varieties ($\frac{N}{N^*}$). At the same time, the potential efficiency gain of subsidizing imports is smaller than the one of a production subsidy since tariffs do not allow to correct distortions in the price of domestically produced goods. However, under the assumption that the monopolistic distortion is removed by a production subsidy (Figure 5), the optimal unilateral policy is an *import subsidy*. An import subsidy renders local differentiated goods relatively more expensive and induces households to increase their demand for Foreign goods. As a consequence, firms agglomerate in the Foreign economy and the number of domestic varieties is reduced while the Foreign one is boosted. This increases the domestic price level but also improves domestic terms of trade, which allows Home to import more differentiated goods for each unit of exports. In other words, when the monopolistic distortion is removed by an appropriate production subsidy, the terms of trade externality more than compensates the rise in transport cost generated by an import subsidy.

We now turn to a more formal analysis. First, we show that an increase in the domestic tariff always increases the number of domestic varieties at the expense of Foreign. In addition, setting a higher tariff always reduces the total number of differentiated varieties. Lemma 3 summarizes these results.

Lemma 3 Let $\tau > 1$, $\varepsilon > 1$, $0 < \alpha < 1$ and $\tau_I = \tau_I^* = 1$. Then, when $\tau_C = \tau_C^* = 1$ or $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$, $\frac{\partial N}{\partial \tau_I} > 0$, $\frac{\partial N^*}{\partial \tau_I} < 0$ and $|\frac{\partial N}{\partial \tau_I}| - |\frac{\partial N^*}{\partial \tau_I}| < 0$.

Again, we can decompose the welfare effects of an increase in the domestic tariff using (39). We first consider the case in which the initial allocation is inefficient.

Lemma 4 Let $\tau > 1$, $\varepsilon > 1$, $0 < \alpha < 1$, $\tau_I = 1$ and $\tau_C = 1$. Then $\frac{\partial P/p_z}{\partial \tau_I}|_{\tau_C=\tau_C^*,\tau_I=\tau_I^*} < 0$ and $\frac{\partial I/p_z}{\partial \tau_I}|_{\tau_C=\tau_C^*,\tau_I=\tau_I^*} > 0$.

When there are no policy interventions a positive tariff always increases welfare. The mechanism behind this is the home market effect, which increases the relative weight of cheap domestic varieties in the domestic consumption basket. This outcome dominates the negative effect on the world number of varieties that exacerbates the inefficiencies due to the monopolistic distortions. Lemma 4 formally proves the impact of tariffs on the Home price index and on domestic income. It confirms Venables (1987)' and Ossa (2008)'s results and their claim that in their analysis (where there is no correction of the monopolistic distortion) ignoring income effects by considering tariffs to be a pure waste is not a restrictive assumption. In fact, in this special case the income effect due to a tariff is always positive and reinforces the home market effect.[27] However, results turn around when we consider an initial allocation that is efficient:

Lemma 5 Let $\tau > 1$, $\varepsilon > 1$, $0 < \alpha < 1$, $\tau_I = 1$ and $\tau_C = \frac{(\varepsilon-1)}{\varepsilon}$. Then $\frac{\partial P/p_z}{\partial \tau_I}|_{\tau_C=\tau_C^*,\tau_I=\tau_I^*} < 0$ if $\varepsilon - 1 > \alpha$ and $\frac{\partial I/p_z}{\partial \tau_I}|_{\tau_C=\tau_C^*,\tau_I=\tau_I^*} < 0$.

According to Lemma 5, as long as $\varepsilon - 1 > \alpha$, a tariff renders domestic differentiated goods cheaper even when the monopolistic inefficiency is completely offset by an appropriate production subsidy. Conversely the income effect is negative in this case. This switch in the sign of the derivative of income can be explained as follows. When starting from an efficient equilibrium,

[27] If we look at the decomposition of (41), it becomes clear that the income effect is positive because the terms of trade effect is dominated by other effects. A positive tariff implies a shift in Foreign demand towards domestic varieties ($B_3 > 0$). Similarly, domestic demand shifts away from Foreign varieties ($B_5 > 0$). Both effects increase domestic income. In contrast, the terms of trade externality and the opportunity cost decrease income ($B_1 < 0$, $B_2 < 0$). Finally, the Foreign income effect, B_4, is zero, since there is no Foreign policy intervention.

the volume of trade is larger than when starting from an inefficient equilibrium because the total number of varieties in the differentiated sector is higher. Therefore, the income loss due to a terms of trade worsening is larger as well. Similarly, the opportunity cost in terms of homogenous good of increasing the domestic production in the differentiated sector is higher because homogeneous goods are no longer inefficiently abundant.

Theorem 2 formally proves that the optimal unilateral tariff is positive when starting from a free trade allocation and negative (an import subsidy) when starting from the Pareto-optimal allocation implemented by a production subsidy.

Theorem 2 Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. If $\tau_C = \tau_C^* = 1$, then $\frac{\partial W}{\partial \tau_I}|_{\tau_I = \tau_I^* = 1} > 0$. However, if $\tau_C = \tau_C^* = \frac{\varepsilon - 1}{\varepsilon}$ then $\frac{\partial W}{\partial \tau_I}|_{\tau_I = \tau_I^* = 1} < 0$.

On the one hand, the first part of Theorem 2 confirms Venables (1987)'s and Ossa (2008)'s result that the optimal unilateral trade policy is a positive tariff, even when we allow for redistribution of tariff revenues. On the other hand, the second part of Theorem 2 makes clear that this result depends crucially on whether the initial allocation is efficient. When starting from the Pareto optimal allocation, the optimal unilateral policy turns out to be an import subsidy. Like in the case of the production subsidy, at the efficient allocation the high volume of trade strengthen the terms of trade externality. Simultaneously, the relative low price level implies a weak home market effect and a high opportunity cost of reducing the production of the homogeneous good.

6.2.2 Implications of $\eta \neq \varepsilon$

Here we check if these results are robust to allowing ε to be different from η.

Again, let us first treat a situation where the production subsidies τ_C and τ_C^* are set equal to one. Figure 6 plots the optimal tariff/subsidy on imports as functions of the elasticity of substitution among varieties $\varepsilon \in (2, 8)$ and the iceberg cost $\tau \in [1, 2]$ with η being equal to 2.

Consider first the case $\tau = 1$. This implies that the home market effect is absent and the only incentives for policy makers are terms of trade effects and the elimination of monopolistic distortions. Consequently, the optimal policy is an import subsidy (i.e. $\tau_I < 1$). As ε becomes larger and larger, policy makers are less willing to subsidize imports (i.e. the optimal import subsidy tends to one). When ε increases, the differentiated sector converges to a competitive sector which produces a single variety and the elasticity of N with respect to tariffs becomes zero. As a consequence, both motives (the correction of the monopolistic distortion and the incentive to improve the terms of trade) for subsidizing imports vanish.

Moving next to positive but sufficiently low levels of transport costs, single country policy makers still find it optimal to subsidize imports while for high transport costs the optimal policy is a tariff. The intuition is straightforward: while terms of trade effects and monopolistic distortions determine policy choices for low transport costs, the home market effect prevails for high transport costs. Note, however, that if ε is sufficiently bigger than η, the optimal policy is an import subsidy even for transport costs of around 40%.

We can then conclude that the results of Venables (1987) and Ossa (2008) are not robust to the plausible case in which the elasticity of substitution between varieties is greater than the trade elasticity, even when we start from an inefficient allocation.[28]

6.3 Export Taxes

Finally, we briefly discuss the case of export taxes under the assumption that $\tau_I = \tau_I^* = \tau_X^* = 1$.

6.3.1 Benchmark case: $\eta = \varepsilon$

Once more, in the first scenario we study unilateral deviations from free trade under the assumption that the monopolistic distortion has not been eliminated. Figure 7 shows how a deviation to a positive subsidy on exports that attracts more firms to the domestic economy and increases the overall number of varieties available at the word level improves domestic welfare. The intuition is the same as in the production subsidy case.[29]

The second scenario (Figure 8) considers the case where the monopolistic distortion has been eliminated by production subsidies ($\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$). Now a unilateral deviation to a positive tax on exports, that relocates firms to the Foreign economy and induces an overall reduction

[28]Our finding that a positive import subsidy is optimal if the monopolistic inefficiency is corrected remains valid independently of the values of τ and ε. The figure is omitted to save space but is available from the authors on request.

[29]Gains from reducing the price level are high and opportunity costs of reducing Z are low because the number of differentiated varieties is too low. At the same time, the terms of trade externality is weak due to the low trade volume.

in the number of differentiated varieties, is welfare improving since it improves domestic terms of trade. Again, at the efficient allocation, the high trade volume leads to a strong terms of trade externality. At the same time, the price level is low and the opportunity cost of reducing production of the homogeneous god is sizable, thus implying a weak home market externality.

6.3.2 Implications of $\eta \neq \varepsilon$

In Figure 9 we report the optimal domestic export tax/subsidy for the case $\tau_C = \tau_C^* = 1$ and $\tau_X^* = 1$ for different values of ε and τ. While the home market effect and the monopolistic distortion call for a subsidy, in the benchmark case the terms of trade externality would require a tax and the first two effects predominate. Figure 9 shows that this result does not go through in a more general setup where we allow for the intra-industry elasticity ε to exceed the trade elasticity η. On the one hand, one might consider this result as not so surprising given that, by increasing ε, we are reducing the monopolistic distortion and also, as explained earlier, the home market externality. On the other hand, ε needs to be only marginally bigger than η for the optimal strategy to be an export tax, thus underlining the weakness of the previous result. When considering the case $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$, it is always optimal to set an export tax, independently on the level of τ and ε, thus the terms of trade and revenue externalities always predominate in this context.[30]

7 Conclusions

In this paper we have studied unilateral trade policy in a two-sector variant of the Krugman (1980) model of intra-industry trade. We have isolated the different incentives that drive policy makers' choices. These are determined by three main effects: a terms of trade effect, a home market externality and a distortion in the aggregate allocation due to monopolistic pricing. In addition, our analysis has revealed what the welfare relevant terms of trade in this model are. Contrary to the point of view of the previous literature, which has considered the prices of individual varieties in international markets as the terms of trade, we have shown that policy makers care about aggregate relative price indices of importables and not only about the prices of varieties. This implies that terms of trade effects and the home market externality coexist

[30] To save space we do not include this figure. The relevant figure is available on request.

even when considering tariffs and a homogeneous good produced with constant returns that fixes relative factor prices.

Our main contribution has been to show that all previous results on trade policy in the two-sector Krugman model depend crucially on the inefficiency of the free trade allocation. Indeed, monopolistic distortion implies an inefficiently high price level and a low volume of trade, thus strengthening the incentive to agglomerate firms and weakening therms of trade externalities. This leads to the optimality of import tariffs/export subsidies and production subsidies. Differently, at the efficient allocation, gains from further reduction in the price level are small while opportunity costs of reducing the production of the homogeneous good are high (weak home market effect). At the same time, terms of trade externality becomes strong due to the high volume of trade. Thus, domestic policy makers optimally choose to set import subsidies/export taxes and inefficiently low production subsidies.

The analysis in this paper sets the foundations for studying strategic trade policy in this set-up. Now that policy makers' incentives have been clarified, Nash equilibrium policy outcomes, where many of the incentives that determine policies are obscured by strategic interaction, can be investigated. We relegate this analysis to our companion paper, Campolmi et al. (2010).

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APPENDIX

A Equilibrium allocation and prices for the case $\eta = \varepsilon$

Under the parametric restrictions $\eta = \varepsilon$, it is possible to recover the equilibrium allocations and prices (and then implicitly single country welfare) as a function of the parameters of the model and of the policy instruments. Since we are studying production subsidies and tariffs, we set $\tau_X = \tau_X^* = 1$.

Under those assumptions relative prices in (37) simplify to:

$$\frac{P}{p_z} = \frac{\varepsilon}{\varepsilon-1}\left[N\tau_C^{1-\varepsilon} + N^*\left(\tau_I\tau\tau_C^*\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \qquad \frac{P^*}{p_z} = \frac{\varepsilon}{\varepsilon-1}\left[N^*(\tau_C^*)^{1-\varepsilon} + N\left(\tau_I^*\tau\tau_C\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{42}$$

Combining the zero profit conditions (25) and (26) and substituting out the expressions for the relative prices (36), it is possible to derive the following expression for C and C^*:

$$C = \frac{f(\varepsilon-1)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}(\frac{P}{p_z})^{-\varepsilon}(\tau\tau_I)^{\varepsilon}\left((\tau\tau_I^*\tau_C)^{\varepsilon} - \tau(\tau_C^*)^{\varepsilon}\right)}{(\tau_I\tau_I^*)^{\varepsilon}\tau^{2\varepsilon} - \tau^2} \tag{43}$$

$$C^* = \frac{f(\varepsilon-1)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}(\frac{P^*}{p_z})^{-\varepsilon}(\tau_I^*\tau)^{\varepsilon}\left((\tau\tau_I\tau_C^*)^{\varepsilon} - \tau\tau_C^{\varepsilon}\right)}{(\tau_I\tau_I^*)^{\varepsilon}\tau^{2\varepsilon} - \tau^2} \tag{44}$$

Using the trade balance condition (35), the labor market clearing condition (31), the equivalent equations for the foreign country, and the expressions for C, C^*, $\frac{P}{p_z}$ and $\frac{P^*}{p_z}$ just derived, we have the following system of equations in N and N^*:

$$-L + A_1 N + A_2 N^* = 0 \tag{45}$$

$$-L + A_2^* N + A_1^* N^* = 0 \tag{46}$$

the solution of which is:

$$N = \frac{L(A_2 - A_1^*)}{A_2^* A_2 - A_1 A_1^*} \qquad N^* = \frac{L(A_2^* - A_1)}{A_2^* A_2 - A_1 A_1^*} \tag{47}$$

where:

$$A_1 = \frac{f(1-\alpha)\varepsilon(\tau\tau_I)^\varepsilon \tau_C^{1-\varepsilon}\left((\tau\tau_I^*\tau_C)^\varepsilon - \tau(\tau_C^*)^\varepsilon\right)}{\alpha\left(\tau^{2\varepsilon}(\tau_I\tau_I^*)^\varepsilon - \tau^2\right)} - \frac{f\varepsilon\tau\tau_C^{1-\varepsilon}\left((\tau\tau_I\tau_C^*)^\varepsilon - \tau\tau_C^\varepsilon\right)}{\tau^{2\varepsilon}(\tau_I\tau_I^*)^\varepsilon - \tau^2} + f\varepsilon \tag{48}$$

$$A_2^* = \frac{f(1-\alpha)\varepsilon\tau\tau_C^{1-\varepsilon}\left((\tau\tau_I\tau_C^*)^\varepsilon - \tau\tau_C^\varepsilon\right)}{\alpha\left(\tau^{2\varepsilon}(\tau_I\tau_I^*)^\varepsilon - \tau^2\right)} + \frac{f\varepsilon\tau\tau_C^{1-\varepsilon}\left((\tau\tau_I\tau_C^*)^\varepsilon - \tau\tau_c^\varepsilon\right)}{\tau^{2\varepsilon}(\tau_I\tau_I^*)^\varepsilon - \tau^2} \tag{49}$$

$$A_2 = \frac{f(1-\alpha)\varepsilon\tau\tau_I(\tau_C^*)^{1-\varepsilon}\left((\tau\tau_I^*\tau_C)^\varepsilon - \tau(\tau_C^*)^\varepsilon\right)}{\alpha\left(\tau^{2\varepsilon}(\tau_I\tau_I^*)^\varepsilon - \tau^2\right)} + \frac{f\varepsilon\tau(\tau_C^*)^{1-\varepsilon}\left((\tau\tau_C\tau_I^*)^\varepsilon - \tau(\tau_C^*)^\varepsilon\right)}{\tau^{2\varepsilon}(\tau_I\tau_I^*)^\varepsilon - \tau^2} \tag{50}$$

$$A_1^* = \frac{f(1-\alpha)\varepsilon(\tau\tau_I^*)^\varepsilon(\tau_C^*)^{1-\varepsilon}\left((\tau\tau_I\tau_C^*)^\varepsilon - \tau\tau_C^\varepsilon\right)}{\alpha\left(\tau^{2\varepsilon}(\tau_I\tau_I^*)^\varepsilon - \tau^2\right)} - \frac{f\varepsilon\tau(\tau_C^*)^{1-\varepsilon}\left((\tau\tau_I^*\tau_C)^\varepsilon - \tau(\tau_C^*)^\varepsilon\right)}{\tau^{2\varepsilon}(\tau_I\tau_I^*)^\varepsilon - \tau^2} + f\varepsilon \tag{51}$$

Finally substituting equation (9) into the budget constraint, it is possible to write consumption as a function of income:

$$C = \alpha\frac{I}{p_z}\left(\frac{P}{p_z}\right)^{-1} \tag{52}$$

where $I = WL + T$. Substituting (9) and (52) into the utility function, taking logs and disregarding the constant, gives the (log) indirect utility:

$$W = -\alpha\log\left(\frac{P}{p_z}\right) + \log\left(\frac{I}{p_z}\right) \tag{53}$$

An increase in the production subsidy on a one hand increases W due to the reduction in the price level, but on the other hand it also reduces it due to the negative effect on income.

B Production Subsidy τ_c

We can now proceed to prove the lemmas and theorem relative to the production subsidy. First, we compute the derivatives of N and N^* w.r.t. τ_C and evaluate their sign starting from a symmetric situation $\tau_C = \tau_C^*$.

Lemma 1 Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. If $\tau_C = \tau_C^* \leq 1$ then $\frac{\partial N}{\partial \tau_C} < 0$, $\frac{\partial N^*}{\partial \tau_C} > 0$ and $|\frac{\partial N^*}{\partial \tau_C}|_{\tau_C=\tau_C^*} - |\frac{\partial N}{\partial \tau_C}|$.

Proof.1 $\frac{\partial N}{\partial \tau_C}\Big|_{\tau_C=\tau_C^*} =$

$$\frac{L\alpha\left[\tau^2\left(\alpha^2+(1-\alpha^2)\,\tau_C\right)+\tau^{\varepsilon+1}\left(2(1-\alpha)(\varepsilon-1)\tau_C+\alpha(2\varepsilon-1)\right)+(1-\alpha)\tau^{2\varepsilon}\left((1-\alpha)\tau_C+\alpha\right)\right]}{f\varepsilon\left(\tau^\varepsilon-\tau\right)\left[\alpha-(\alpha-1)\tau_C\right]^2\left[\alpha(\tau+\tau^\varepsilon)(\tau_C-1)-\tau_C(\tau^\varepsilon-\tau)\right]} \tag{54}$$

$$\frac{\partial N^*}{\partial \tau_C}\Big|_{\tau_C=\tau_C^*} = \frac{L\alpha\tau\left[\alpha(\tau^\varepsilon-\tau)+\tau^\varepsilon\left(2(\alpha-1)\varepsilon\tau_C-2\alpha\varepsilon\right)\right]}{f\varepsilon\left(\tau^\varepsilon-\tau\right)\left[\alpha-(\alpha-1)\tau_C\right]^2\left[\alpha(\tau+\tau^\varepsilon)(\tau_C-1)-\tau_C(\tau^\varepsilon-\tau)\right]} \tag{55}$$

The denominator of both expressions is negative whenever $\tau_C \leq 1$. The numerator of the first expression is always positive being the sum of only positive terms. For the numerator of the second expression to be positive we would need $\tau_C < \alpha\frac{1-\tau^{1-\varepsilon}-2\varepsilon}{2(1-\alpha)\varepsilon}$, not possible given that $\tau_C >= 0$ by definition. Finally, $|\frac{\partial N^*}{\partial \tau_C}| - |\frac{\partial N}{\partial \tau_C}| = \frac{\partial N^*}{\partial \tau_C} + \frac{\partial N}{\partial \tau_C} = -\frac{L(1-\alpha)\alpha}{f\varepsilon[\alpha-(\alpha-1)\tau_C]^2} < 0$ ■

Next, we prove the effects of a change in the production subsidy on relative price of the differentiated goods and on domestic income.

Lemma 2 Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. If $\tau_C = \tau_C^* \leq 1$ then $\frac{\partial \frac{P}{p_z}}{\partial \tau_C} > 0$ and $\frac{\partial \frac{I}{p_z}}{\partial \tau_C} > 0$.

Proof.1

$$\partial\left(\frac{P}{p_z}\right)/\partial\tau_C\Big|_{\tau_C=\tau_C^*} = \frac{1}{\varepsilon-1}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{(1-\varepsilon)}\left(\frac{P}{p_z}\right)^{\varepsilon}\left[\tau_C^{-\varepsilon}(\varepsilon-1)N-\tau_C^{1-\varepsilon}\left(\frac{\partial N}{\partial \tau_C}+\frac{1}{\tau^{\varepsilon-1}}\frac{\partial N^*}{\partial \tau_C}\right)\right] \tag{56}$$

For the derivative of the price index it suffices to notice that $\frac{\partial N}{\partial \tau_C} + \frac{1}{\tau^{\varepsilon-1}}\frac{\partial N^*}{\partial \tau_C} = L\alpha\tau^{-\varepsilon}\left[\frac{\alpha\tau^2+(1-\alpha)\tau^{2\varepsilon}(\alpha+(1-\alpha)\tau_C)+\tau^{1+\varepsilon}(\alpha(2\varepsilon-\alpha)+(1-\alpha)(2\varepsilon-1-\alpha)\tau_C}{f\varepsilon(\tau^\varepsilon-\tau)[\alpha-(\alpha-1)\tau_C]^2[\alpha(\tau+\tau^\varepsilon)(\tau_C-1)-\tau_C(\tau^\varepsilon-\tau)]}\right] < 0$ being the numerator a sum of positive terms while for the denominator we already proved it to be negative. For the derivative of income it is enough to remember that $\frac{I}{p_z} = L + N\varepsilon f(\tau_C - 1)$ and that $\frac{\partial N}{\partial \tau_C} < 0$. ■

Before proving the theorem on the welfare consequences of a change in the production subsidy, it is useful to decompose $\frac{\partial \frac{I}{p_z}}{\partial \tau_C}$ into the different effects as in (41).

Lemma A.1 Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. If $\tau_C = \tau_C^* \leq 1$ then $B_1 > 0$, $B_2 > 0$, $B_4 < 0$ and $B_5 < 0$. If $\tau_C = \tau_C^* = 1$ or $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ then also $B_3 < 0$.

Proof.1

- $B_1 > 0$: When $\tau_C = \tau_C^*$ the equilibrium is symmetric thus there is no trade in the homogeneous good and $B_1 = -\alpha \left(\frac{P_F}{P}\right)^{-\varepsilon} \left(\frac{p_z}{P}\right) \frac{I}{p_z} \frac{\partial\left(\frac{P_H^*}{P_H}\right)}{\partial \tau_C}$. Note that $\frac{\partial\left(\frac{P_H^*}{P_H}\right)}{\partial \tau_C} = \frac{\frac{\partial P_H^*/p_z}{\partial \tau_C} P_H - \frac{\partial P_H/p_z}{\partial \tau_C} P_H^*}{P_H^2} < 0$ given that $\partial \left(\frac{P_H^*}{p_z}\right) / \partial \tau_C = \frac{\varepsilon}{\varepsilon-1} N^{*\left(\frac{1}{1-\varepsilon}\right)} \left(-\frac{\tau_C^*}{(\varepsilon-1)N^*} \frac{\partial N^*}{\partial \tau_C}\right) < 0$ and $\partial \left(\frac{P_H}{p_z}\right) / \partial \tau_C = \frac{\varepsilon}{\varepsilon-1} N^{\left(\frac{1}{1-\varepsilon}\right)} \left(1 - \frac{\tau_C}{(\varepsilon-1)N} \frac{\partial N}{\partial \tau_C}\right) > 0$.

- $B_2 > 0$: This is so given that $B_2 = \left[-\varepsilon f \left(\frac{p_z}{\tau P_H}\right) \frac{\partial N}{\partial \tau_C}\right]$ and $\frac{\partial N}{\partial \tau_C} < 0$.

- $B_3 < 0$: This is so given that $B_3 = \alpha \left(\frac{P_F^*}{P^*}\right)^{-\varepsilon} \frac{I^*}{p_z} \left(\frac{p_z}{P^*}\right)^2 \left[-\varepsilon \frac{P^*/p_z}{P_F^*/p_z} \frac{\partial P_F^*/p_z}{\partial \tau_C} + (\varepsilon - 1) \frac{\partial P^*/p_z}{\partial \tau_C}\right]$, $\frac{\partial P_F^*/p_z}{\partial \tau_C} = \tau \frac{\varepsilon}{\varepsilon-1} N^{\left(\frac{1}{1-\varepsilon}\right)} \left(1 - \frac{\tau_C}{(\varepsilon-1)N} \frac{\partial N}{\partial \tau_C}\right) > 0$, $\frac{\partial P^*/p_z}{\partial \tau_C}|_{\tau_C=\tau_C^*=1} = -\frac{P^*}{p_z} \frac{\varepsilon \tau}{(\varepsilon-1)(\tau^\varepsilon - \tau)} < 0$ and $\frac{\partial P^*/p_z}{\partial \tau_C}|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}} = -\frac{P^*}{p_z} \frac{\varepsilon^2 \tau}{(\alpha+\varepsilon-1)((\alpha+\varepsilon-1)\tau^\varepsilon + (\alpha-\varepsilon+1)\tau)} < 0$.

- $B_4 < 0$: This is so given that $B_4 = \left[\alpha \left(\frac{P_F^*}{P^*}\right)^{-\varepsilon} \left(\frac{p_z}{P^*}\right) \frac{\partial\left(\frac{I^*}{p_z}\right)}{\partial \tau_i}\right]$, $\frac{I^*}{p_z} = L + (\tau_C^* - 1)\varepsilon f N^*$ and $\frac{\partial I/p_z}{\partial \tau_C} = \varepsilon f \frac{\partial N^*}{\partial \tau_C} < 0$

- $B_5 < 0$: Note that $B_5 = \alpha \frac{P_H^*}{P_H} \left(\frac{P_F}{P}\right)^{-\varepsilon} \frac{I}{p_z} \frac{p_z}{P} \left[\varepsilon \frac{P/p_z}{P_F/p_z} \left(\frac{p_z}{P}\right)^2 \left(\frac{\partial P_F/p_z}{\partial \tau_C} \frac{P}{p_z} - \frac{\partial P/p_z}{\partial \tau_C} \frac{P_F}{p_z}\right) + \frac{\partial P/p_z}{\partial \tau_C} \frac{p_z}{P}\right] =$

  $\alpha \frac{P_H^*}{P_H} \left(\frac{P_F}{P}\right)^{-\varepsilon} \frac{I}{p_z} \frac{p_z}{P} \left[\varepsilon \left(\frac{P_F}{p_z}\right)^{-1} \frac{\partial P_F/p_z}{\partial \tau_C} - (\varepsilon - 1) \frac{\partial P/p_z}{\partial \tau_C} \frac{p_z}{P}\right] < 0$
  given that $\partial \left(\frac{P_F}{p_z}\right) / \partial \tau_C = \tau \frac{\varepsilon}{\varepsilon-1} N^{*\left(\frac{1}{1-\varepsilon}\right)} \left(-\frac{\tau_C^*}{(\varepsilon-1)N^*} \frac{\partial N^*}{\partial \tau_C}\right) < 0$ and $\frac{\partial P/p_z}{\partial \tau_C} > 0$.

■

Before proving the effect on domestic welfare of a change in the domestic production subsidy (Theorem 1), we derive the necessary and sufficient condition for $N > 0$ and $N^* > 0$ i.e. no specialization in the case in which $\tau_C = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_C^* = 1$. This will then be used in the proof of Theorem 1.

Lemma A.2 Let $\tau > 1$, $\varepsilon > 1$, $0 < \alpha < 1$, $\tau_C = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_C^* = 1$. Then, $N > 0$ and $N^* > 0$ if an only if $\left(1 + \frac{\alpha}{\varepsilon-1}\right) \tau^{\varepsilon-1} + \left(1 - \frac{\alpha}{\varepsilon-1}\right) \tau^{1-\varepsilon} > 2 \left(\frac{\varepsilon}{\varepsilon-1}\right)^\varepsilon$.

Proof.1 When $\tau_C = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_C^* = 1$, the equilibrium number of varieties is given by:

$$N = \frac{L\alpha \frac{\varepsilon^\varepsilon}{\varepsilon-1}\tau^\varepsilon\left(2\left(\frac{\varepsilon-1}{\varepsilon}\right)^\varepsilon - \left(\tau^{\varepsilon-1}+\tau^{1-\varepsilon}\right)\right)}{f((\varepsilon-1)^\varepsilon\tau - \varepsilon^\varepsilon\tau^\varepsilon)\left((1+\frac{\alpha}{\varepsilon-1})\tau^{\varepsilon-1} - \left(\frac{\varepsilon}{\varepsilon-1}\right)^\varepsilon\right)}$$

$$N^* = \frac{L\alpha\varepsilon^{\varepsilon-1}\tau^\varepsilon\left(2\left(\frac{\varepsilon}{\varepsilon-1}\right)^\varepsilon - \left((1+\frac{\alpha}{\varepsilon-1})\tau^{\varepsilon-1} + (1-\frac{\alpha}{\varepsilon-1})\tau^{1-\varepsilon}\right)\right)}{f(\varepsilon^\varepsilon\tau^\varepsilon - (\varepsilon-1)^\varepsilon\tau)\left(\left(\frac{\varepsilon}{\varepsilon-1}\right)^\varepsilon - (1+\frac{\alpha}{\varepsilon-1})\tau^{\varepsilon-1}\right)}$$

Note that $2\left(\frac{\varepsilon-1}{\varepsilon}\right)^\varepsilon - \tau^{\varepsilon-1} + \tau^{1-\varepsilon} < 0$ given that $\tau^{\varepsilon-1} + \tau^{1-\varepsilon} > 1$ and $\left(\frac{\varepsilon-1}{\varepsilon}\right)^\varepsilon \leq \frac{1}{e}$. The last inequalities follows from $\lim_{\varepsilon\to\infty}\left(\frac{\varepsilon-1}{\varepsilon}\right)^\varepsilon = \frac{1}{e}$ and $\frac{\partial\left(\frac{\varepsilon-1}{\varepsilon}\right)^\varepsilon}{\partial\varepsilon} = \left(\frac{\varepsilon-1}{\varepsilon}\right)^\varepsilon\left(\log\left(\frac{\varepsilon-1}{\varepsilon}\right) + \frac{1}{\varepsilon-1}\right) > 0$ for all ε. Also, note that $(\varepsilon-1)^\varepsilon\tau - \varepsilon^\varepsilon\tau^\varepsilon < 0$. Therefore, $N > 0$ if and only if $(1+\frac{\alpha}{\varepsilon-1})\tau^{\varepsilon-1} - \left(\frac{\varepsilon}{\varepsilon-1}\right)^\varepsilon > 0$.

Moving to N^*, we have that the denominator is always negative when $N > 0$. Therefore, N^* is positive when $N > 0$ if and only if $2\left(\frac{\varepsilon}{\varepsilon-1}\right)^\varepsilon - \left((1+\frac{\alpha}{\varepsilon-1})\tau^{\varepsilon-1} + (1-\frac{\alpha}{\varepsilon-1})\tau^{1-\varepsilon}\right) < 0$.

Finally, note that the condition on N^* implies the one on N. Indeed,
$(1+\frac{\alpha}{\varepsilon-1})\tau^{\varepsilon-1} + (1-\frac{\alpha}{\varepsilon-1})\tau^{1-\varepsilon} > 2\left(\frac{\varepsilon}{\varepsilon-1}\right)^\varepsilon \implies (1+\frac{\alpha}{\varepsilon-1})\tau^{\varepsilon-1} > \left(\frac{\varepsilon}{\varepsilon-1}\right)^\varepsilon$
Why this is true it can be proven by contradiction. If $(1+\frac{\alpha}{\varepsilon-1})\tau^{\varepsilon-1} < \left(\frac{\varepsilon}{\varepsilon-1}\right)^\varepsilon$ then $(1-\frac{\alpha}{\varepsilon-1})\tau^{1-\varepsilon} < \left(\frac{\varepsilon}{\varepsilon-1}\right)^\varepsilon$. But hence $(1+\frac{\alpha}{\varepsilon-1})\tau^{\varepsilon-1} + (1-\frac{\alpha}{\varepsilon-1})\tau^{1-\varepsilon} < 2\left(\frac{\varepsilon}{\varepsilon-1}\right)^\varepsilon$ which contradicts our initial assumption.

■

Theorem 1 Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. Then:

(1) $\frac{\partial W}{\partial\tau_C}|_{\tau_C=\tau_C^*=1} < 0$

(2) $\frac{\partial W}{\partial\tau_C}|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}} > 0$

(3) $\frac{\partial W}{\partial\tau_C}|_{\tau_C=\frac{\varepsilon-1}{\varepsilon},\tau_C^*=1} > 0$ if $\tau < 4$ or $\varepsilon - 1 > \alpha$.

Proof.1

(1) $\frac{\partial W}{\partial\tau_C}|_{\tau_C=\tau_C^*=1} = \frac{\alpha((\alpha-1)\tau^\varepsilon - \tau(\alpha+\varepsilon-1))}{(\varepsilon-1)(\tau^\varepsilon-\tau)} < 0$

(2) $\frac{\partial W}{\partial\tau_C}|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}} = \frac{\alpha\varepsilon^2\tau(\tau^\varepsilon+\tau)}{(\varepsilon-1)(\tau^\varepsilon-\tau)(\alpha(\tau+\tau^\varepsilon)+(\varepsilon-1)(\tau^\varepsilon-\tau))} > 0$

(3) We prove it in two steps. First, we show that a sufficient condition for $\frac{\partial W}{\partial\tau_C}|_{\tau_C=\frac{\varepsilon-1}{\varepsilon},\tau_C^*=1} > 0$ is $\tau > \left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}$. Second, we show that under the assumption of no specialization ($N > 0$ and $N^* > 0$), $\tau > \left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}$ if $\tau < 4$ or $\varepsilon - 1 > \alpha$.

3a $\frac{\partial W}{\partial \tau_C}|_{\tau_C=\frac{\varepsilon-1}{\varepsilon},\tau_C^*=1} =$

$-\frac{\alpha(\varepsilon-1)^{-2}\varepsilon^{\varepsilon}\tau^{\varepsilon-2}}{\left(\tau^{\varepsilon-1}-\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}\right)\left(\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}-\left(\frac{\alpha}{\varepsilon-1}+1\right)\tau^{\varepsilon-1}\right)} + \frac{\alpha\varepsilon^{-\varepsilon}(\varepsilon-1)^{\varepsilon-2}(\alpha+\varepsilon-1)}{(\varepsilon^{\varepsilon}\tau^{\varepsilon}-(\varepsilon-1)^{\varepsilon}\tau)\left(\tau^{\varepsilon-1}-\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\varepsilon}\left(1-\frac{\alpha}{(\varepsilon-1)}\right)\right)(\varepsilon-1)\varepsilon^{\varepsilon}}$

Note that a sufficient condition for the first term to be positive is $\tau > \left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}$. Indeed, this implies that $\tau^{\varepsilon-1} - \left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon} > 0$ and $\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon} - (\frac{\alpha}{\varepsilon-1}+1)\tau^{\varepsilon-1} < 0$.

Next, since $\varepsilon^{\varepsilon}\tau^{\varepsilon} - (\varepsilon-1)^{\varepsilon}\tau > 0$ then the second term is positive if and only if $\tau^{\varepsilon-1} - \left(\frac{\varepsilon-1}{\varepsilon}\right)^{\varepsilon}\left(1-\frac{\alpha}{(\varepsilon-1)}\right) > 0$. However this last condition is always verified when $\tau > \left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}$ given that $\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon} > \left(\frac{\varepsilon-1}{\varepsilon}\right)^{\varepsilon} > \left(\frac{\varepsilon-1}{\varepsilon}\right)^{\varepsilon}\left(1-\frac{\alpha}{(\varepsilon-1)}\right)$.

3b Finally, we prove by contradiction that when $N > 0$ and $N^* > 0$, if $\tau < 4$ or $\varepsilon-1 > \alpha$ then $\tau > \left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}$. Suppose that $\tau^{\varepsilon-1} \leq \left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}$, then for $N > 0$ and $N^* > 0$ by Lemma 4 the following must hold:

$$\left(1+\frac{\alpha}{\varepsilon-1}\right)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}+\left(1-\frac{\alpha}{\varepsilon-1}\right)\tau^{1-\varepsilon} > \left(1+\frac{\alpha}{\varepsilon-1}\right)\tau^{\varepsilon-1}+\left(1-\frac{\alpha}{\varepsilon-1}\right)\tau^{1-\varepsilon} > 2\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}$$

$\Leftrightarrow \left(1-\frac{\alpha}{\varepsilon-1}\right)\tau^{1-\varepsilon} > \left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}\left(1-\frac{\alpha}{\varepsilon-1}\right)$.

It follows that if $1 > \frac{\alpha}{\varepsilon-1}$ then $\tau^{1-\varepsilon} > \left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}$. This however contradicts $\tau^{\varepsilon-1} \leq \left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}$. Thus, if $N > 0$ and $N^* > 0$ then $\tau^{\varepsilon-1} \leq \left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}$ only if $\varepsilon - 1 \leq \alpha$. This proves that $\varepsilon - 1 > \alpha$ is a sufficient condition for the welfare to be decreasing in the production subsidy. It only remains to show that another sufficient condition is $\tau < 4$. If $1 \leq \frac{\alpha}{\varepsilon-1}$ by Lemma A.2:

$$\left(1+\frac{\alpha}{\varepsilon-1}\right)\tau^{\varepsilon-1} > \left(1+\frac{\alpha}{\varepsilon-1}\right)\tau^{\varepsilon-1} + \left(1-\frac{\alpha}{\varepsilon-1}\right)\tau^{1-\varepsilon} > 2\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}$$

which implies:

$$\tau > 2^{\frac{1}{\varepsilon-1}}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}\left(1+\frac{\alpha}{\varepsilon-1}\right)^{\frac{1}{1-\varepsilon}} >= 4$$

To see why this last condition is true let's recall that $\frac{\partial 2^{\frac{1}{\varepsilon-1}}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}\left(1+\frac{\alpha}{\varepsilon-1}\right)^{\frac{1}{1-\varepsilon}}}{\partial\alpha} = -2^{\frac{1}{\varepsilon-1}}\left(\frac{\alpha}{\varepsilon-1}+1\right)^{\frac{\varepsilon}{1-\varepsilon}}(\varepsilon-1)^{\frac{1}{1-\varepsilon}-3}\varepsilon^{\frac{\varepsilon}{\varepsilon-1}} < 0$ and that $\frac{\partial 2^{\frac{1}{\varepsilon-1}}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}\left(1+\frac{\alpha}{\varepsilon-1}\right)^{\frac{1}{1-\varepsilon}}}{\partial\varepsilon}|_{\alpha=1} = -2^{\frac{1}{\varepsilon-1}}\left(\frac{\alpha}{\varepsilon-1}+1\right)^{\frac{\varepsilon}{1-\varepsilon}}(\varepsilon-1)^{\frac{1}{1-\varepsilon}-3}\varepsilon^{\frac{\varepsilon}{\varepsilon-1}} < 0$. Moreover if $\varepsilon - 1 \leq \alpha$, $\varepsilon < 2$. It follows if $\varepsilon - 1 \leq \alpha$, $\tau > 2^{\frac{1}{\varepsilon-1}}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}\left(1+\frac{\alpha}{\varepsilon-1}\right)^{\frac{1}{1-\varepsilon}} > 2^{\frac{1}{\varepsilon-1}}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}\left(1+\frac{\alpha}{\varepsilon-1}\right)^{\frac{1}{1-\varepsilon}}|_{\alpha=1,\varepsilon=2} = 4$. Therefore $\tau^{\varepsilon-1} \leq \left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}$ only if $\varepsilon - 1 \leq \alpha$ and $\tau > 4$.

■

C Import Tariff τ_I

In this section while retaining the assumptions $\eta = \varepsilon$ and $\tau_X = \tau_X^*$, we allow for the use of an import tariff as main policy instrument.

Lemma 3 Let $\tau > 1$, $\varepsilon > 1$, $0 < \alpha < 1$ and $\tau_I = \tau_I^* = 1$. Then, when $\tau_C = \tau_C^* = 1$ or $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$, $\frac{\partial N}{\partial \tau_I} > 0$, $\frac{\partial N^*}{\partial \tau_I} < 0$ and $|\frac{\partial N}{\partial \tau_I}| - |\frac{\partial N^*}{\partial \tau_I}| < 0$.

Proof.1 First we compute the derivatives for the case $\tau_C = \tau_C^* = 1$:

$$\left.\frac{\partial N}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=1} = \frac{L\alpha\tau^{\varepsilon+1}\left[(1+\varepsilon-\alpha)\tau + (\alpha+\varepsilon-1)\tau^{\varepsilon}\right]}{f\varepsilon(\tau-\tau^{\varepsilon})^2(\tau+\tau^{\varepsilon})} > 0 \tag{57}$$

$$\left.\frac{\partial N^*}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=1} = -\frac{L\alpha\tau\left[(1-\alpha)\tau^2 + \varepsilon\tau^{2\varepsilon} + (\alpha+\varepsilon-1)\tau^{\varepsilon+1}\right]}{f\varepsilon(\tau-\tau^{\varepsilon})^2} < 0 \tag{58}$$

$$\left|\left.\frac{\partial N}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=1}\right| - \left|\left.\frac{\partial N^*}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=1}\right| = -\frac{L(1-\alpha)\alpha\tau}{f\varepsilon(\tau+\tau^{\varepsilon})} < 0 \tag{59}$$

Next, we compute the derivatives for the case $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$:

$$\left.\frac{\partial N}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}} = \frac{L\alpha\tau(\varepsilon-1)\left[(1-\alpha)\alpha\tau^2 + (\alpha+\varepsilon-1)^2\tau^{2\varepsilon} + (\varepsilon^2+\alpha-1)\tau^{1+\varepsilon}\right]}{f(\alpha+\varepsilon-1)^2\left[\alpha(\tau+\tau^{\varepsilon}) + (\varepsilon-1)(\tau^{\varepsilon}-\tau)\right](\tau^{2\varepsilon}-\tau^2)} > 0 \tag{60}$$

$$\left.\frac{\partial N^*}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}} = \frac{L\alpha\tau(\varepsilon-1)\left[(\varepsilon-1)(1-\alpha)\tau^2 + \varepsilon(\alpha+\varepsilon-1)\tau^{2\varepsilon} + ((\varepsilon-1)^2+\alpha(2\varepsilon-1))\tau^{\varepsilon+1}\right]}{f(\alpha+\varepsilon-1)^2\left[\alpha(\tau+\tau^{\varepsilon}) + (\varepsilon-1)(\tau^{\varepsilon}-\tau)\right](\tau^2-\tau^{2\varepsilon})} < 0 \tag{61}$$

$$\left|\left.\frac{\partial N}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}}\right| - \left|\left.\frac{\partial N^*}{\partial \tau_I}\right|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon}}\right| = -\frac{L(1-\alpha)\alpha\tau(\varepsilon-1)}{f(\alpha+\varepsilon-1)^2(\tau+\tau^{\varepsilon})} < 0 \tag{62}$$

■

Lemma 4 Let $\tau > 1$, $\varepsilon > 1$, $0 < \alpha < 1$, $\tau_I = 1$ and $\tau_C = 1$. Then $\frac{\partial P/p_z}{\partial \tau_I}|_{\tau_C=\tau_C^*,\tau_I=\tau_I^*} < 0$ and $\frac{\partial I/p_z}{\partial \tau_I}|_{\tau_C=\tau_C^*,\tau_I=\tau_I^*} > 0$.

Proof.1

$\partial\left(\frac{P}{p_z}\right)/\partial\tau_I|_{\tau_C=\tau_C^*=\tau_I=\tau_I^*=1} = \frac{P}{p_z}\ \frac{\tau(\varepsilon\tau+\alpha(\tau^\varepsilon-\tau))}{(\varepsilon-1)(\tau^2-\tau^{2\varepsilon})} > 0$

$\partial\left(\frac{I}{p_z}\right)/\partial\tau_I|_{\tau_I=\tau_I^*=1,\tau_C=\tau_C^*=1} = \frac{L\alpha\tau}{\tau^\varepsilon+\tau} > 0$ ■

Lemma 5 Let $\tau > 1$, $\varepsilon > 1$, $0 < \alpha < 1$,$\tau_I = 1$ and $\tau_C = \frac{(\varepsilon-1)}{\varepsilon}$. Then $\frac{\partial P/p_z}{\partial \tau_I}|_{\tau_C=\tau_C^*,\tau_I=\tau_I^*} < 0$ if $\varepsilon - 1 > \alpha$ and $\frac{\partial I/p_z}{\partial \tau_I}|_{\tau_C=\tau_C^*,\tau_I=\tau_I^*} < 0$.

Proof.1

$\partial\left(\frac{P}{p_z}\right)/\partial\tau_I|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon},\tau_I=\tau_I^*=1} = -\frac{P}{p_z}\frac{(\alpha(1-\alpha)+(\varepsilon-1-\alpha)\varepsilon)\tau^2}{((\alpha+\varepsilon-1)(\tau^\varepsilon+\tau)((\alpha+\varepsilon-1)\tau^\varepsilon+(\alpha-\varepsilon+1)\tau)} < 0$ if $\varepsilon - 1 > \alpha$

$\partial\left(\frac{I}{p_z}\right)/\partial\tau_I|_{\tau_C=\tau_C^*=\frac{\varepsilon-1}{\varepsilon},\tau_I=\tau_I^*=1} = -\frac{L\alpha(\varepsilon-1)\tau^2\left(\left((\varepsilon-1)^2+2\varepsilon(\varepsilon-1)+\alpha(2\varepsilon-1)\right)\tau^\varepsilon-\left((\varepsilon-1)^2-\alpha\right)\tau\right)}{(\alpha+\varepsilon-1)^2(\tau^\varepsilon-\tau)(\tau^\varepsilon+\tau)((\alpha+\varepsilon-1)\tau^\varepsilon+(\alpha-\varepsilon+1)\tau)} < 0$ ■

Lemma A.3 Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$. If $\tau_C = \tau_C^* = 1$ or $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ then $B_1 < 0$, $B_2 < 0$, $B_3 > 0$ and $B_5 > 0$. If $\tau_C = \tau_C^* = 1$ then $B_4 = 0$ while if $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ then $B_4 > 0$.

Proof.1

- $B_1 < 0$: Since we start from a symmetric equilibrium, $B_1 = -\alpha\left(\frac{P_F}{P}\right)^{-\varepsilon}\left(\frac{p_z}{P}\right)\frac{I}{p_z}\frac{\partial\left(\frac{P_H^*}{P_H}\right)}{\partial\tau_I}$. Note that $\frac{\partial\left(\frac{P_H^*}{P_H}\right)}{\partial\tau_I} = \frac{\frac{\partial P_H^*/p_z}{\partial\tau_I}P_H-\frac{\partial P_H/p_z}{\partial\tau_I}P_H^*}{P_H^2} > 0$ given that $\partial\left(\frac{P_H^*}{p_z}\right)/\partial\tau_I = \frac{\varepsilon}{\varepsilon-1}N^{*\left(\frac{1}{1-\varepsilon}\right)}\left(-\frac{\tau_C^*}{(\varepsilon-1)N^*}\frac{\partial N^*}{\partial\tau_I}\right) > 0$ and $\partial\left(\frac{P_H}{p_z}\right)/\partial\tau_I = \frac{\varepsilon}{\varepsilon-1}N^{\left(\frac{1}{1-\varepsilon}\right)}\left(-\frac{\tau_C}{(\varepsilon-1)N}\frac{\partial N}{\partial\tau_I}\right) < 0$.

- $B_2 < 0$: This is so given that $B_2 = \left[-\varepsilon f\left(\frac{p_z}{\tau P_H}\right)\frac{\partial N}{\partial\tau_I}\right]$ and $\frac{\partial N}{\partial\tau_I} > 0$.

- $B_3 > 0$: This is so given that $B_3 = \alpha\left(\frac{P_F^*}{P^*}\right)^{-\varepsilon}\frac{I^*}{p_z}\left(\frac{p_z}{P^*}\right)^2\left[-\varepsilon\frac{P^*/p_z}{P_F^*/p_z}\frac{\partial P_F^*/p_z}{\partial\tau_I}+(\varepsilon-1)\frac{\partial P^*/p_z}{\partial\tau_I}\right]$, $\partial\left(\frac{P_F^*}{p_z}\right)/\partial\tau_I = \tau\tau_C\frac{\varepsilon}{\varepsilon-1}N^{\left(\frac{1}{1-\varepsilon}\right)}\left(-\frac{\tau_I^*}{(\varepsilon-1)N}\frac{\partial N}{\partial\tau_I}\right) < 0$ and $\partial\left(\frac{P^*}{p_z}\right)/\partial\tau_I = \frac{1}{\varepsilon-1}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{(1-\varepsilon)}\left(\frac{P}{p_z}\right)^\varepsilon\tau_C^{1-\varepsilon}\left[-\left(\frac{\partial N^*}{\partial\tau_I}+(\tau_I\tau)^{1-\varepsilon}\frac{\partial N}{\partial\tau_I}\right)\right] > 0$.

- $B_4 \geq 0$: This is so given that $B_4 = \left[\alpha\left(\frac{P_F^*}{P^*}\right)^{-\varepsilon}\left(\frac{p_z}{P^*}\right)\frac{\partial\left(\frac{I^*}{p_z}\right)}{\partial\tau_I}\right]$, $\frac{I^*}{p_z} = L + (\tau_C^*-1)\varepsilon f N^*$ and $\frac{\partial I/p_z}{\partial\tau_I} = (\tau_C^*-1)\varepsilon f\frac{\partial N^*}{\partial\tau_I} \geq 0$.

- $B_5 > 0$: Note that $B_5 = \alpha \frac{P_H^*}{P_H} \left(\frac{P_F}{P}\right)^{-\varepsilon} \frac{I}{p_z} \frac{p_z}{P} \left[\varepsilon \frac{P/p_z}{P_F/p_z} \left(\frac{p_z}{P}\right)^2 \left(\frac{\partial P_F/p_z}{\partial \tau_I} \frac{P}{p_z} - \frac{\partial P/p_z}{\partial \tau_I} \frac{P_F}{p_z} \right) + \frac{\partial P/p_z}{\partial \tau_I} \frac{p_z}{P} \right] =$

  $\alpha \frac{P_H^*}{P_H} \left(\frac{P_F}{P}\right)^{-\varepsilon} \frac{I}{p_z} \frac{p_z}{P} \left[\varepsilon \left(\frac{P_F}{p_z}\right)^{-1} \frac{\partial P_F/p_z}{\partial \tau_I} - (\varepsilon - 1) \frac{\partial P/p_z}{\partial \tau_I} \frac{p_z}{P} \right] > 0$
  given that $\partial \left(\frac{P_F}{p_z}\right) / \partial \tau_I = \tau \tau_C^* \frac{\varepsilon}{\varepsilon-1} N^{*\left(\frac{1}{1-\varepsilon}\right)} \left(1 - \frac{\tau_I}{(\varepsilon-1)N^*} \frac{\partial N^*}{\partial \tau_I}\right)$ and
  $\partial \left(\frac{P}{p_z}\right) / \partial \tau_I = \frac{1}{\varepsilon-1} \left(\frac{\varepsilon}{\varepsilon-1}\right)^{(1-\varepsilon)} \left(\frac{P}{p_z}\right)^{\varepsilon} \tau_C^{1-\varepsilon} \left[(\varepsilon-1)\tau^{1-\varepsilon} \tau_I^{-\varepsilon} N - \left(\frac{\partial N}{\partial \tau_I} + (\tau_I \tau)^{1-\varepsilon} \frac{\partial N^*}{\partial \tau_I} \right) \right] < 0.$

■

Theorem 2 Let $\tau > 1$, $\varepsilon > 1$ and $0 < \alpha < 1$ and $\tau_I = \tau_I^* = 1$. If $\tau_C = \tau_C^* = 1$, then $\frac{\partial W}{\partial \tau_I} > 0$.

However, if $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$, then $\frac{\partial W}{\partial \tau_I} < 0$.

Proof.1 In the first case $\frac{\partial W}{\partial \tau_I}|_{\tau_I = \tau_I^* = 1, \tau_C = \tau_C^* = 1} = \frac{\alpha \tau((\alpha+\varepsilon-1)\tau^{\varepsilon} + (1-\alpha)\tau)}{(\varepsilon-1)(\tau^{2\varepsilon} - \tau^2)} > 0$. In the second case $\frac{\partial W}{\partial \tau_I}|_{\tau_I = \tau_I^* = 1, \tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}} = -\frac{\alpha \tau^2((\alpha+2\varepsilon-1)\tau^{\varepsilon} + (1-\alpha)\tau)}{((\alpha(\tau^{\varepsilon}+\tau) + (\varepsilon-1)(\tau^{\varepsilon}-\tau))(\tau^{2\varepsilon}-\tau^2)} < 0$. ■

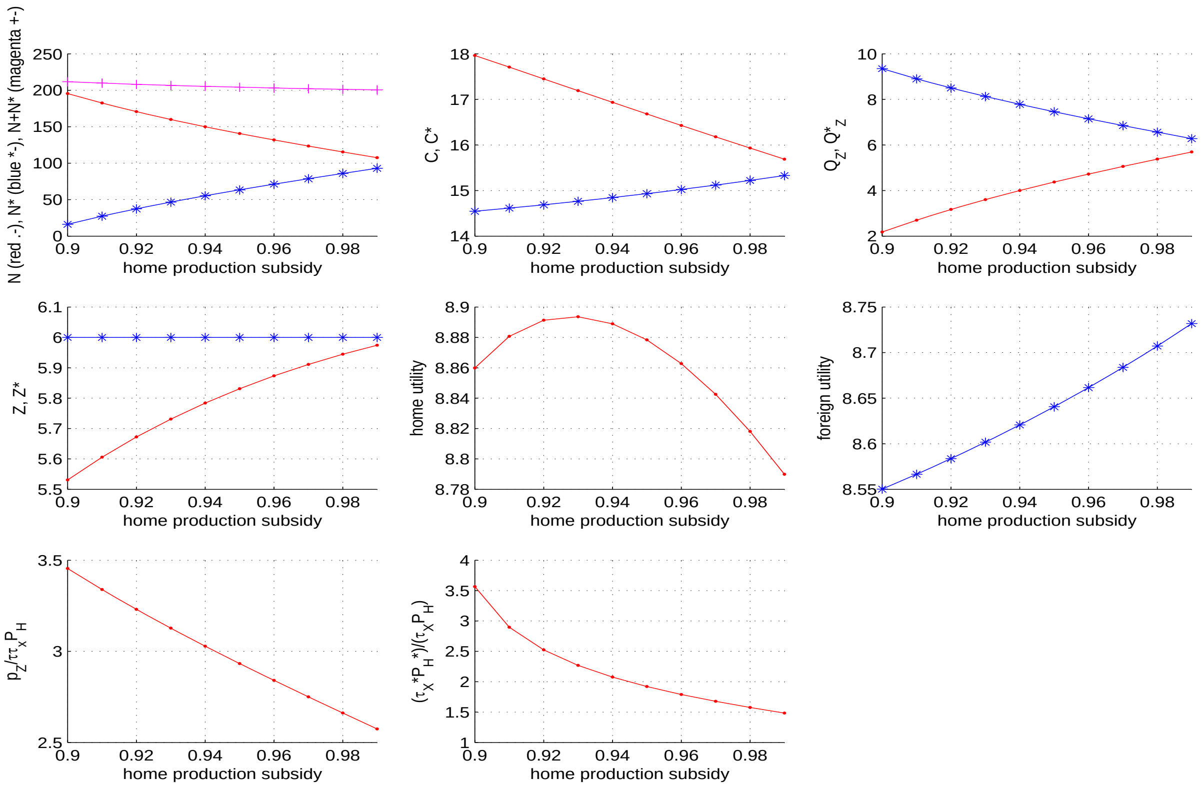

Figure 1: Effects of an unilateral shift of the domestic production subsidy when $\tau_C^* = 1$.

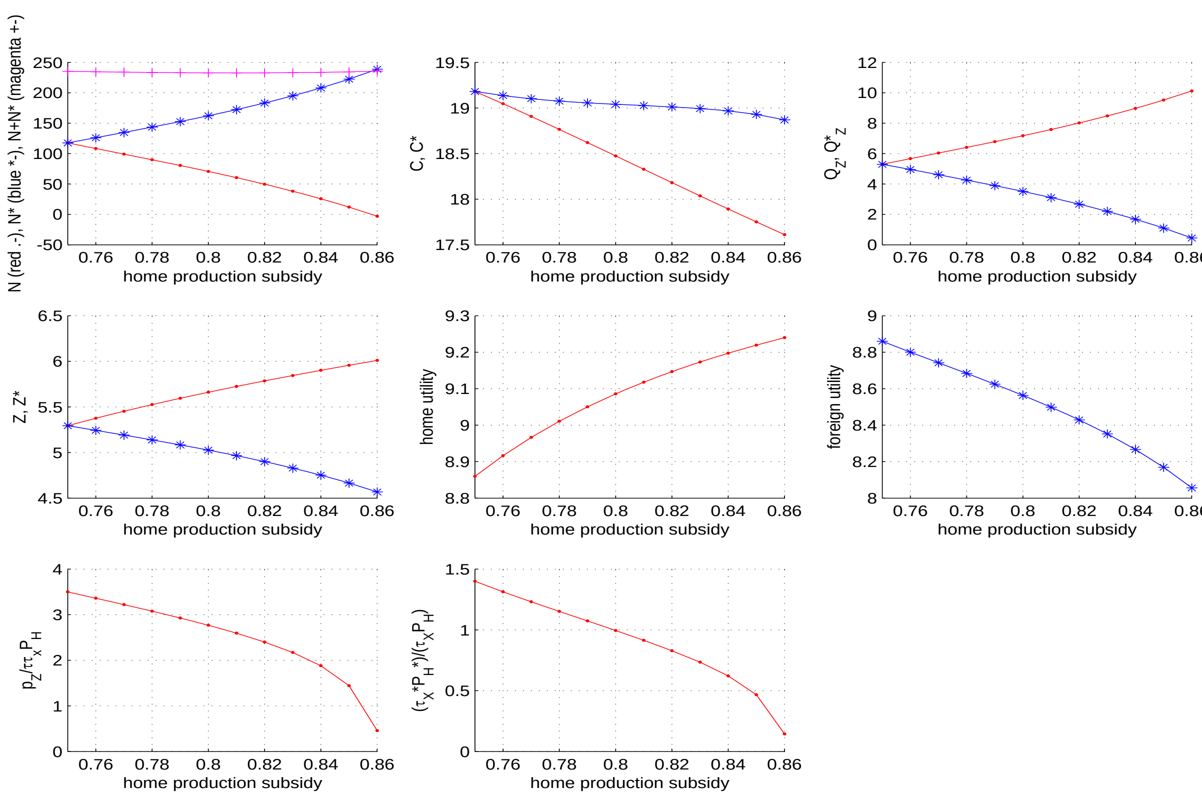

Figure 2: Effects of an unilateral shift of the domestic production subsidy when $\tau_C^* = \frac{\varepsilon-1}{\varepsilon}$.

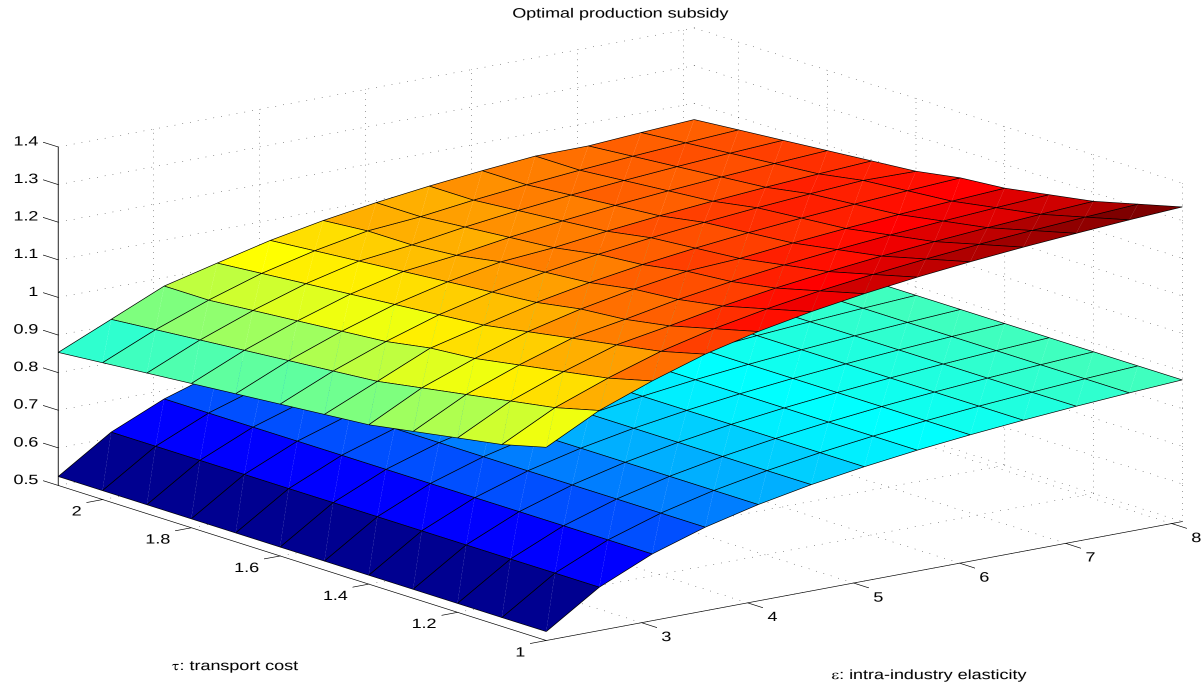

Figure 3: First Best allocation (bottom) and domestic (top) production subsidy when $\tau_C^* = 1$.

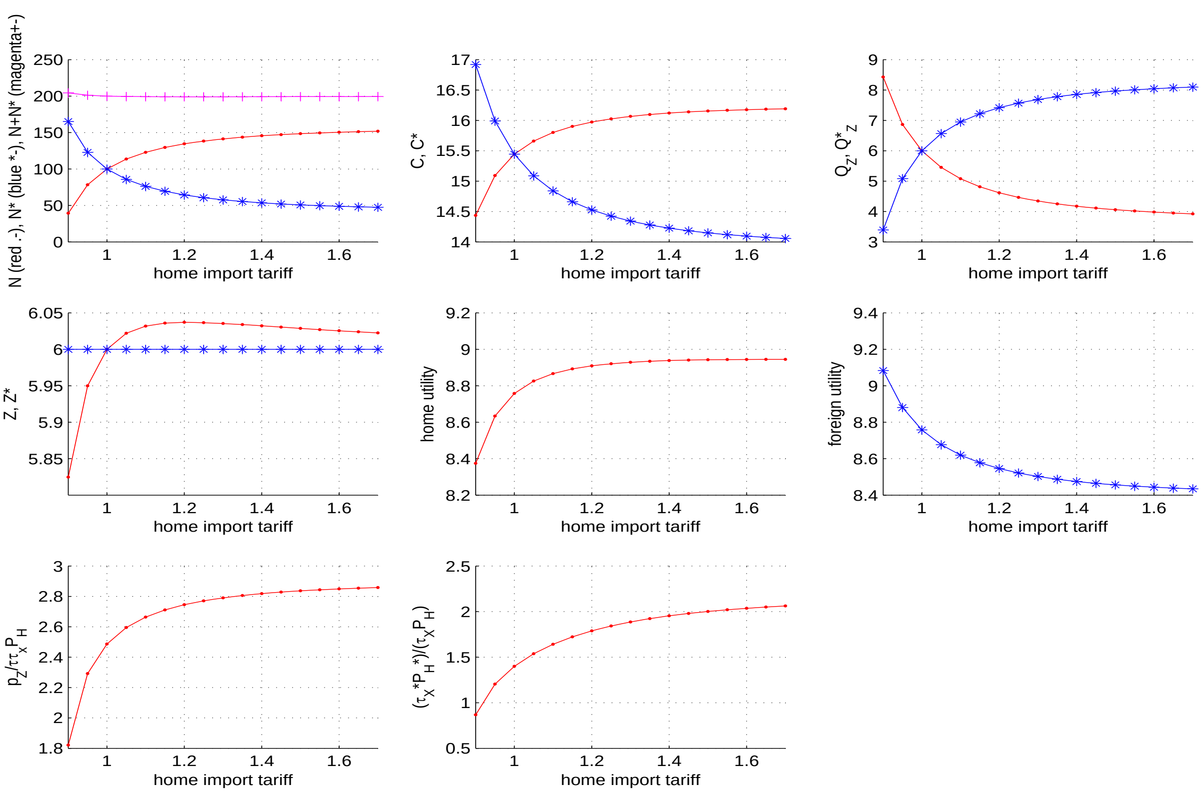

Figure 4: Effects of an unilateral shift of the domestic tariff when $\tau_C = \tau_C^* = 1$.

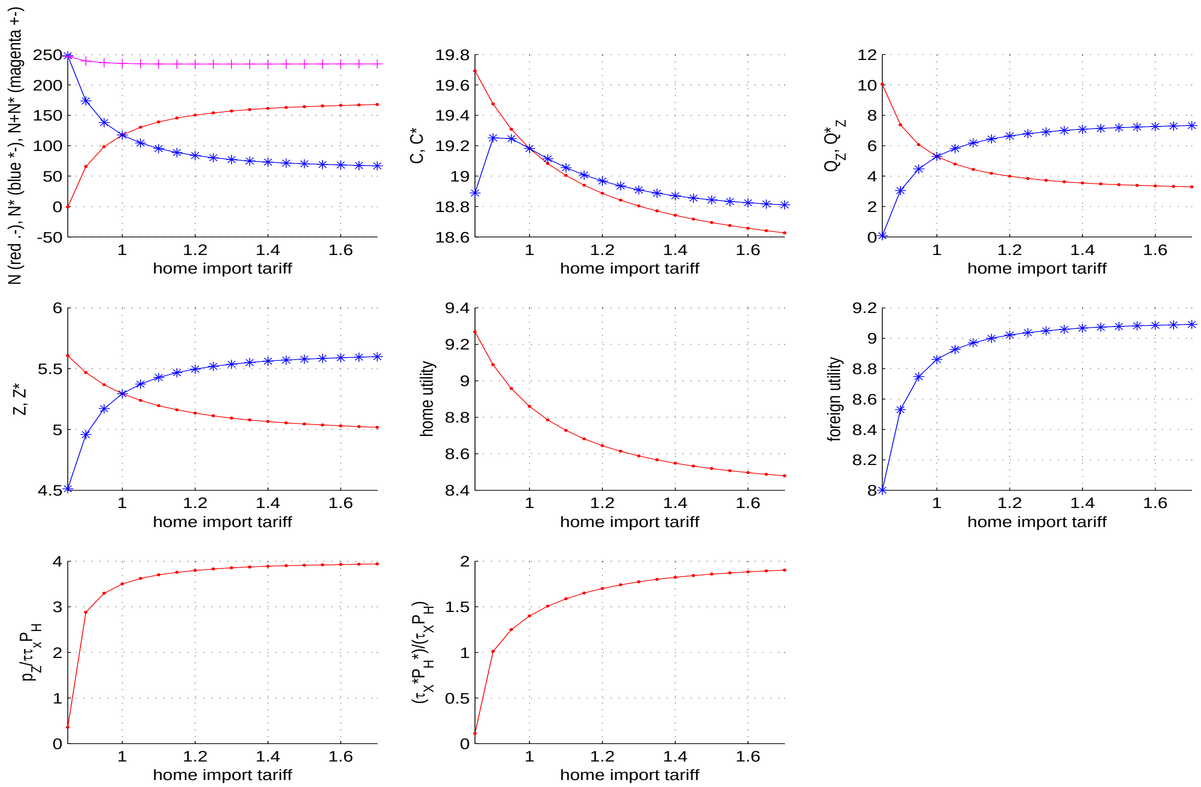

Figure 5: Effects of an unilateral shift of the domestic tariff when $\tau_C = \tau_C^* = (\varepsilon - 1)/\varepsilon$.

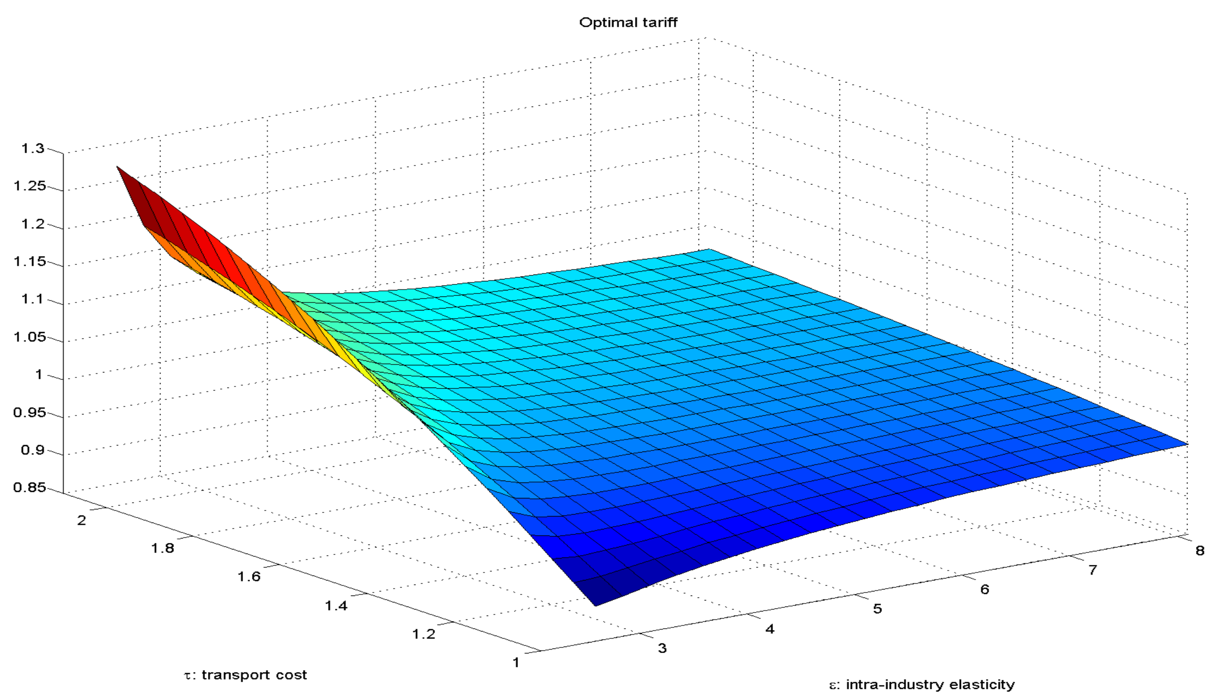

Figure 6: Domestic tariff/ subsidy on imports when $\tau_C = \tau_C^* = \tau_I^* = 1$.

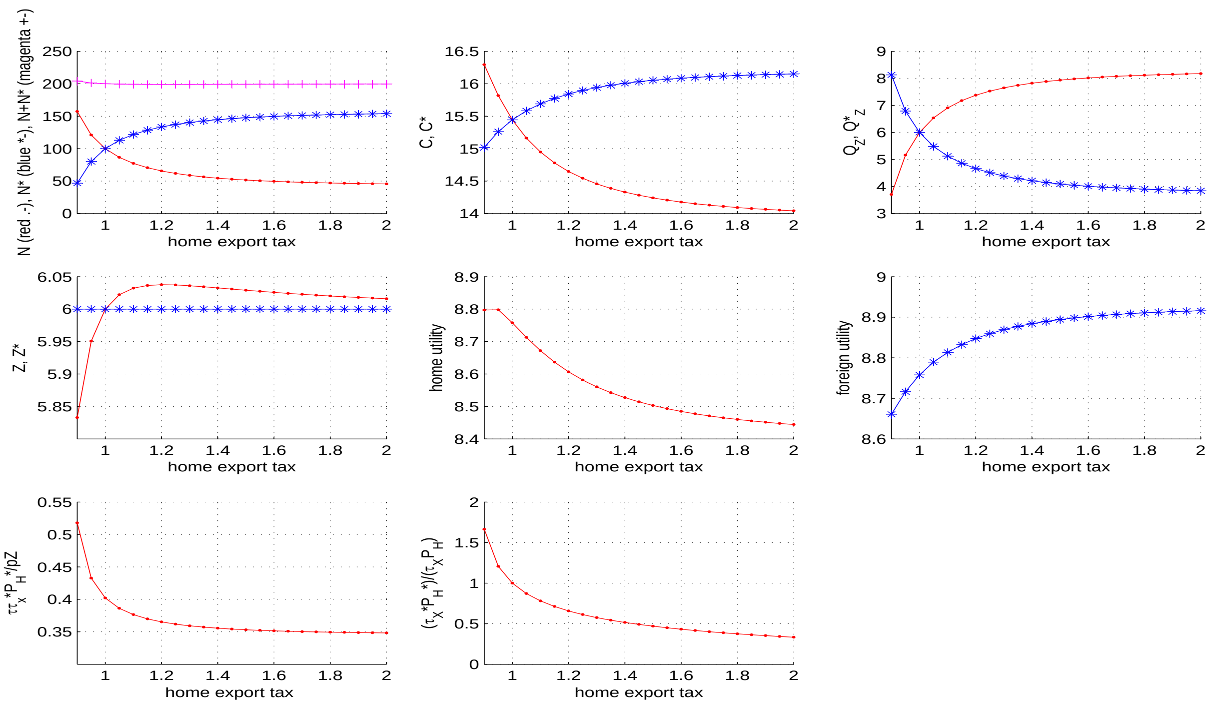

Figure 7: Effects of an unilateral shift of the domestic export tax when $\tau_C = \tau_C^* = \tau_X^* = 1$.

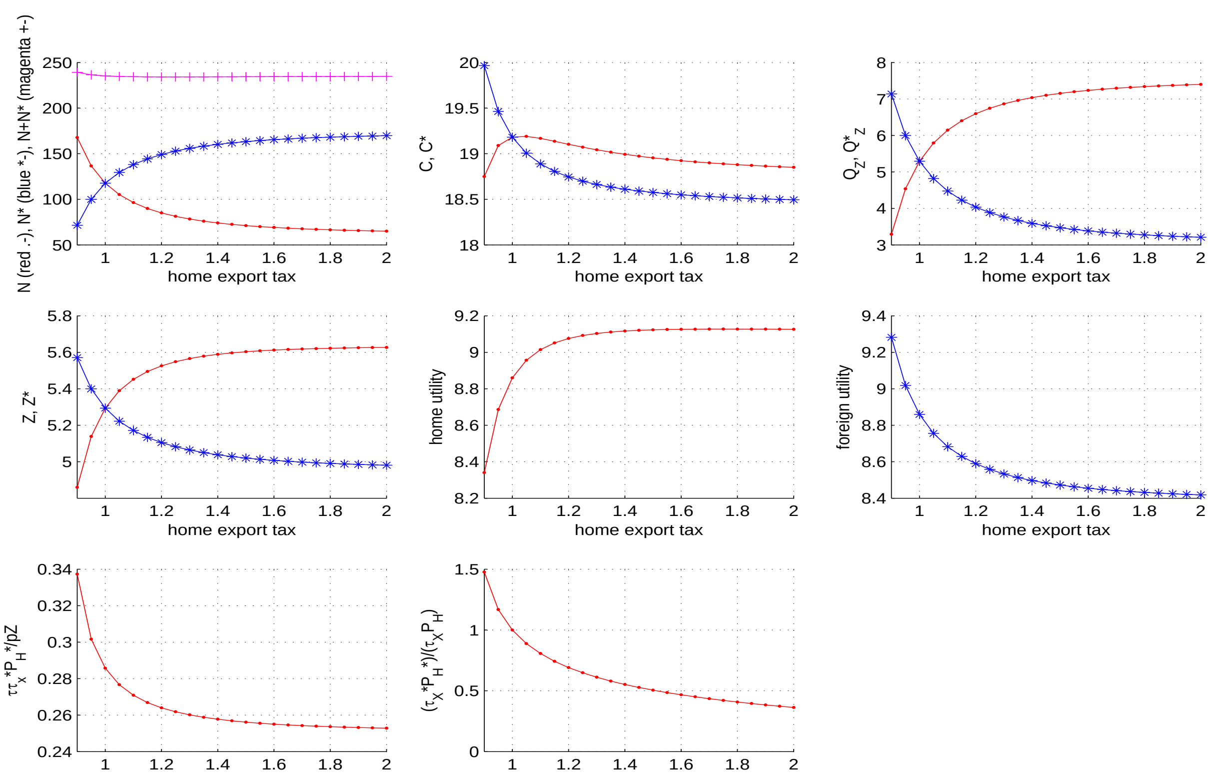

Figure 8: Effects of an unilateral shift of domestic export tax when $\tau_C = \tau_C^* = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_X^* = 1$.

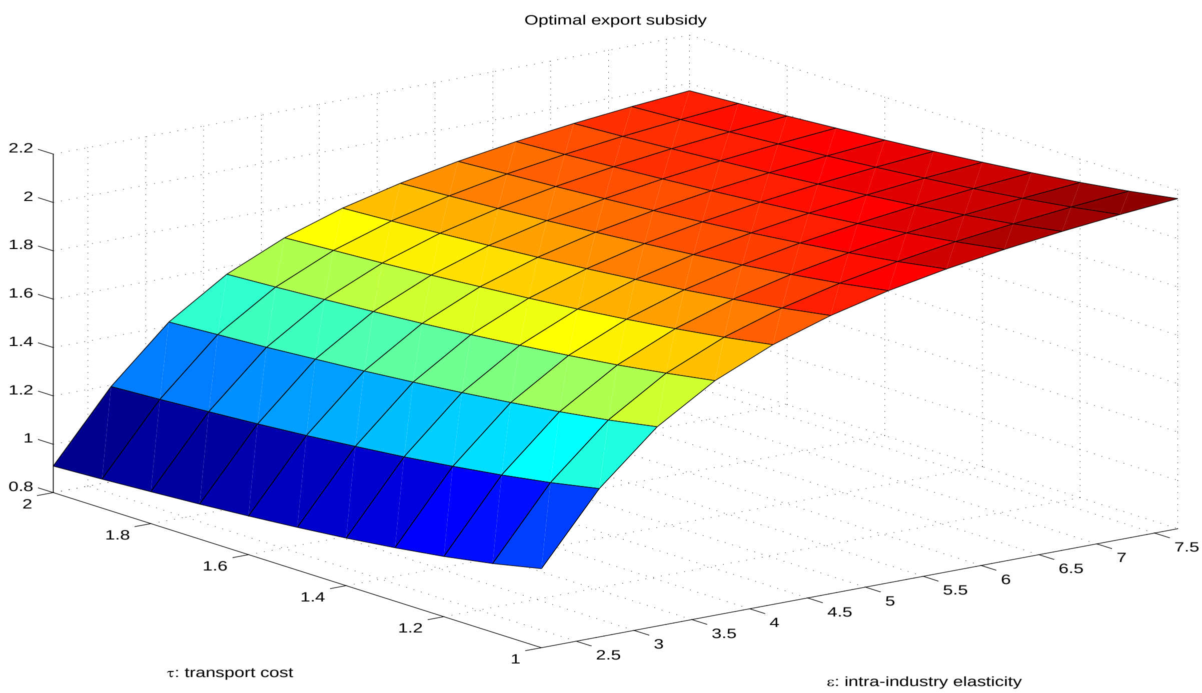

Figure 9: Optimal Export tax/subsidy when $\tau_C = \tau_C^* = 1$ and $\tau_X^* = 1$.