Deconstructing the Gains from Trade:
Selection of Industries vs. Reallocation of Workers*

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Abstract

In a Ricardian model with CES preferences and general distributions of industry efficiencies, the sources of the welfare gains from trade can be precisely decomposed into a selection and a reallocation effect. The former is the change in average efficiency due to the selection of industries that survive international competition. The latter is the rise in the weight of exporting industries in domestic production, due the reallocation of workers from the non-exporting industries. The analytical expression of this decomposition simplifies dramatically if industry efficiencies are Fréchet distributed, providing simple model-based measures of these two effects. Under this assumption, we also show that when the gains from trade are small, it is the selection effect that matters mostly; as the gains from trade rise and the export sector grows, so does the importance of the reallocation effect.

JEL classification: F10, D24, O40

Keywords: Eaton-Kortum model; selection effect; reallocation effect

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1 Introduction

In a very influential paper, Arkolakis, Costinot, and Rodríguez-Clare (2012) have shown that the welfare gains from trade implied by a very large class of models depend on only two sufficient statistics: (i) the share of expenditure on domestic goods; and (ii) the elasticity of imports with respect to variable trade costs (“trade elasticity”). This result is remarkable because it applies to frameworks as different as the simple Armington model, in which goods are differentiated by country of origin; the perfect competition model with heterogeneous industries and Fréchet-distributed efficiencies of Eaton and Kortum (2002); the monopolistic competition model of Krugman (1980); as well as variants of the monopolistic competition model with heterogeneous firms of Melitz (2003) featuring Pareto-distributed efficiencies (such as those developed by Chaney, 2008, and Eaton, Kortum, and Kramarz, 2011). This class of models is now commonly referred to as “quantitative trade models,” given their importance for quantitative analysis.

This key result, however, does not yield any implication about the sources of the welfare gains. In this paper, we explore this issue for one specific class of models, which is the Ricardian model with perfect competition, many countries and goods, CES preferences, and general distributions of industry efficiencies. With respect to Arkolakis, Costinot, and Rodríguez-Clare (2012), then, we restrict our attention to only one family of models. At the same time, however, we extend the scope of our analysis by providing general results for Ricardian models in which efficiencies follow a generic distribution, and not necessarily a Fréchet.

For this general family of models, we show that the welfare gains of the open economy with respect to the autarky economy can be decomposed into two distinct sources: a selection and a reallocation effect. The former is the effect on average efficiency of the selection of industries that, thanks to their sufficiently low marginal costs of production relative to foreign industries, survive international competition. This average efficiency is computed by considering, for the sole industries that survive international competition, the same relative weights in domestic production as the autarky economy. The latter effect, instead, is related to the rise in the weight in domestic production of the exporting industries, which is due to the reallocation of workers from non-exporting industries to the industries that start servicing the foreign market.

While the model provides very precise theoretical definitions for both effects, their analytical expression is, in general, too cumbersome to be used for empirical purposes. In most applications, in fact, it would require computing several billions of distributions of efficiencies. On the other hand, this decomposition simplifies dramatically if we impose that industry efficiencies are Fréchet distributed, thereby returning to one of the quantitative trade models of Arkolakis, Costinot, and Rodríguez-Clare (2012). Under this assumption, we can derive exact model-based measures of these two effects. Therefore, an important insight of our analysis is that quantitative trade models appear to be useful not only to assess the
overall welfare gains, but also to properly measure their sources.

For this model, we also demonstrate that, when the gains from trade are small and there are still few exporters in the domestic economy, the largest share of the gains pertains to the selection effect. As the export sector grows and the gains from trade increase, the importance of the reallocation effect also rises. Because the importance of the reallocation effect rises with the size of the overall gains from trade, it follows that the determinants of the former are exactly the same as the determinants of the latter. In particular, both the welfare gains and their share due to the reallocation effect are higher for small, open and productive economies that are nearer to large and rich markets, and that are less efficient and, then, easier to penetrate.

Under the assumption of Fréchet distributed efficiencies, we quantify the selection and the reallocation effect for a sample of 46 advanced and emerging economies in the years 2000 and 2005. In our sample, the selection effect is, on average, somewhat more important than the reallocation effect (accounting for about 60% of the gains from trade). However, the latter is responsible for over 70 percent of the gains from trade for small open economies such as Denmark, Ireland, the Netherlands, Singapore, Thailand, and Vietnam. On the other hand, the selection effect is dominant for large countries: the United States and Japan, among the advanced countries, and Brazil, Russia, India, and China, among the emerging countries, are the sole economies in which the share of gains pertaining to the selection effect is above 80 percent.

Our paper is related to several strands of the literature. Many recent empirical and theoretical studies have focused on one particular source of the welfare gains, that is the gain in aggregate productivity. An early example is Pavcnik (2002), who estimates productivity improvements in Chile using firm-level data. Other papers, instead, have focused on model-based measures of the “productivity gains from trade,” computed as increases in average efficiency, such as Bernard, Eaton, Jensen, and Kortum (2003), Costinot, Donaldson, and Komunjer (2012), Finicelli, Pagano and Sbracia (2012a and 2012b), and Bolatto (2012). To better grasp the link between these papers and our own, it is worth recalling that, in the Ricardian model, the growth in world-wide aggregate productivity induced by international trade is the basic source of the welfare gains for all countries. In other words, countries benefit from the fact that the world, in the aggregate, produces more of each good. Our paper sheds light, then, on how each individual country, by adjusting its domestic production after a trade liberalization, both contributes to the improvement in world-wide aggregate productivity and reaps the benefits of international trade.

Another related strand of the literature is the wave of papers focusing on empirical estimates of the gains from trade, such as Feenstra (1994 and 2009), Broda and Weinstein (2006), Goldberg, Khandelwal, Pavcnik, and Topalova (2009), and many others. These papers use different estimation methods to quantify either the contribution of specific sources of gains (usually those from consuming new varieties) or the size of the overall welfare gains. Our approach, instead, follows more closely the one of, among the others, Eaton and Kortum
(2002), Alvarez and Lucas (2007), Arkolakis, Demidova, Klenow, and Rodríguez-Clare (2008), Ravikumar and Waugh (2009), and Arkolakis, Costinot and Rodríguez-Clare (2012), in that we derive model-based measures of the welfare gains. Unlike those papers, however, we are also able to quantify the contribution of the different sources of gains.

A close relative of our study is also the paper by Demidova and Rodríguez-Clare (2009), who decompose the welfare gains from trade of a small open economy under monopolistic competition into four terms: productivity, terms of trade, number of varieties, and curvature (i.e. the degree of heterogeneity across varieties). Here, instead, we consider a general equilibrium model with perfect competition and, most importantly, we derive a quantifiable expression of the two sources that, in our framework, provide the welfare gains.

The rest of the paper is organized as follows. Section 2 describes the model, that extends Eaton and Kortum (2002) to general distribution of industry efficiencies. Section 3 proves that welfare can be decomposed into two distinct effects, related to the selection of industries and the reallocation of workers, induced by international trade. Section 4 focuses on the assumption of Fréchet-distributed industry efficiencies, shows that the analytical expressions of the two effects simplifies, and quantifies them for a sample of countries and years. Section 5 draws the main conclusions.

2 The model

We consider a continuum of tradable goods, indexed by $j \in [0, +\infty)$, that can potentially be produced in any of the $N$ countries of the world economy. Each good $j$ can be produced in country $i$ with an efficiency $z_i(j)$ that, in turn, is defined as the amount of output that can be produced with one unit of input — where both output and input are measured in units of constant quality. Any country has a fixed labor endowment $L_i$. Inputs include labor as well as a bundle of intermediates goods, which comprises the full set of tradable goods $j$. Technology is described by a Cobb-Douglas production function with constant returns to scale, in which labor has a constant share $\beta \leq 1$ for all industries and countries; namely:

$$q_i(j) = z_i(j) L_i^\beta (j) I_i^{1-\beta} (j),$$

(1)

where $q_i(j)$ is the quantity of output $j$ in country $i$, $L_i(j)$ is the number of workers, and $I_i(j)$ is the quantity of the bundle of intermediate goods.

Consumer preferences are the same across countries. The representative consumer in country $i$ purchases individual goods in amounts $c_i(j)$ in order to maximize a CES utility function:

$$U_i = \left[ \int \left[ c_i(j) \right]^{\frac{\sigma - 1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma - 1}},$$

1We can ignore physical capital in the production function because the model is static and, then, intermediate inputs play a very similar role.
where $\sigma > 0$ is the elasticity of substitution. While the model allows us to deal with both inelastic ($\sigma \leq 1$) and elastic demand ($\sigma > 1$), we will discuss mostly the latter case, because the goods that we consider are all tradable and, in this setting, the typical calibration is $\sigma > 1$.

Consumers maximize their utility function subject to a standard budget constraint. Because we assume that trade is balanced in the open economy, income available for consumption is $Y_i = w_i L_i$, where $w_i$ is the (nominal) wage.

International trade is constrained by barriers, which are modeled using the standard assumption of iceberg costs à la Samuelson (1952); i.e., delivering one unit of a good from country $i$ to country $n$ requires shipping $d_{ni}$ units, with $d_{ni} > 1$ for $i \neq n$ and $d_{ii} = 1$ for any $i$. By arbitrage, trade barriers obey the triangle inequality, so that $d_{ni} \leq d_{nk} d_{ki}$ for any $n$, $i$ and $k$.

Perfect competition implies that the price of one unit of good $j$ produced by country $n$ and delivered to country $i$ is:

$$p_{in}(j) = \frac{c_n d_{in}}{z_n(j)},$$

where $c_n = w_n^{1-\beta} p_n^{1-\beta}$ is the cost of one unit of input, $L_n^\beta(j) I_n^{1-\beta}(j)$, in the source country $n$, with $p_n$ being the unit price of the optimal bundle of intermediate goods, which is the same as the unit price of the optimal bundle of final goods (see equation (3) below). In other words, we assume (as Eaton and Kortum, 2002) that industries combine intermediate goods using the same CES aggregator that consumers use to combine final goods (with final and intermediate goods being the same goods).

Consumers purchase each good from the country that can supply it at the lowest price; therefore, the price of good $j$ in country $i$ is:

$$p_i(j) = \min_n \left( \frac{c_n d_{in}}{z_n(j)} \right).$$

We assume that, in each country $i$, industry efficiencies $z_i(j)$ are the realizations of a random variable $Z_i$, with a country-specific cumulative distribution function (c.d.f.) $F_i$. Because the $z_i(j)$ represent industry efficiencies and there is a continuum of goods, it is natural to assume that $Z_i$ is non-negative and absolutely continuous (in Section 4, in order to quantify the selection and reallocation effects, we will impose that the $Z_i$ are Fréchet distributed for any $i$). The continuum-of-goods assumption and the conventional application of the law of large numbers imply that the share of goods for which country $i$’s efficiency is below any real number $z$ is the probability $\Pr (Z_i < z) = F_i(z)$. It is worth noting that, in the autarky economy, all goods are made at home and, then, $Z_i$ is the efficiency distribution of the closed economy.

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2 For an extension of the model that encompasses both tradable and non-tradable goods and different elasticities of substitution between tradable goods, between non-tradable goods and between tradable and non-tradable goods, see Di Nino, Eichengreen, and Sbracia (2012).
Given the cost of inputs, the distribution of industry efficiencies translates into a distribution of good prices. More formally, let us denote with $P_i$ the random variable that describes the distribution of good prices in country $i$; this distribution is:

$$P_i = \min_n \left( \frac{c_n d_{in}}{Z_n} \right) = \left[ \max_n \left( \frac{Z_n}{c_n d_{in}} \right) \right]^{-1}. \quad (2)$$

The price index in country $i$, $p_i$, computed using the correct CES aggregator, is simply the moment of order $1 - \sigma$ of the random variable $P_i$, at the $1/(1 - \sigma)$ power; that is:

$$p_i = \left[ E \left( P_i^{1-\sigma} \right) \right]^{1/(1-\sigma)}. \quad (3)$$

After a simple manipulation of equations (2) and (3), we obtain:

$$p_i = c_i \cdot \left[ E \left( M_i^{\sigma-1} \right) \right]^{1/(\sigma-1)},$$

where $M_i = \max_n \left( \frac{c_i Z_n}{c_n d_{in}} \right)$, (4)

from which we derive the real wage, which measures welfare:$^3$

$$\frac{w_i}{p_i} = \left[ E \left( M_i^{\sigma-1} \right) \right]^{1/(\beta(\sigma-1))}. \quad (5)$$

The welfare gain from trade can be obtained by comparing the real wage of the open and the closed economy, where the latter can be obtained from the former, letting $d_{in} \to +\infty$ for $i \neq n$ (using equations (4) and (5)). In this case, we have $M_i \to Z_i$, and the real wage is $\left[ E \left( Z_i^{\sigma-1} \right) \right]^{1/\beta(\sigma-1)}$. Hence, the gain from trade for country $i$ is:

$$g_i = \left[ \frac{E \left( M_i^{\sigma-1} \right)}{E \left( Z_i^{\sigma-1} \right)} \right]^{1/\beta(\sigma-1)}. \quad (6)$$

The welfare gain, then, arises from the transformation, that occurs in the open economy, of the “source of the production efficiencies” (which determine good prices) from $Z_i$ to $M_i$. Note, in particular, that the latter random variable is a maximum between a set of random variables and that this set includes also $Z_i$. Because the maximum of a set of random variables first-order stochastically dominates any variable included in the set, then $M_i \succeq Z_i$, so that $g_i \geq 1$; i.e., the real wage is higher in the open economy.$^4$ Thus, the result that trade is welfare improving is here proven using the language of probability rather than the tools of general equilibrium.$^5$

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$^3$Recall that, in the competitive equilibrium of both the open and the closed economy, welfare is $w_i L_i / p_i$, where $L_i$ is exogenous.

$^4$We remind the reader that the random variable $X$ first-order stochastically dominates the random variable $Y$, and we write $X \succeq Y$, if and only if $F_X(z) \leq F_Y(z)$ for any $z \in \mathbb{R}$, where $F_X$ and $F_Y$ are the c.d.f. of, respectively, $X$ and $Y$.

$^5$The finding that $g_i \geq 1$ for any $i$ generalizes a result of Finicelli, Pagano, and Sbracia (2012a), extending it to a framework in which there are also intermediate goods.
3 Welfare decomposition

Let us now focus on how labor units are reallocated after opening to trade. To foster the intuition, we start by considering the case of two countries, say $i$ and $n$, before generalizing the result to $N$ countries.

The first-order conditions (FOCs) of the consumer’s problem imply that the overall consumption of good $j$ in country $i$ is:

$$c_i(j) = \left[ \frac{p_i(j)}{p_i} \right]^{-\sigma} U_i,$$  

where $U_i = w_i L_i / p_i$ is the level of utility achieved by country $i$.

The FOCs of the producer’s problem, on the other hand, imply that the quantities of intermediate goods and labor used to produce good $j$ in country $i$ are chosen according to the following proportions:

$$I_i(j) = \frac{1 - \beta}{\beta} \frac{w_i}{p_i} L_i(j).$$ (8)

By aggregating across industries both sides of equation (8), we find that the overall amount of intermediate goods used in country $i$ is $I_i = (1 - \beta) / \beta \cdot (w_i / p_i) \cdot L_i$.\(^6\)

The assumption that intermediate goods are combined using the same CES aggregator used to combine final goods implies that, for any country $i$, the demand for $j$ as intermediate good, $m_i(j)$, is proportional to the demand as consumption good, $c_i(j)$; that is: $c_i(j) / U_i = m_i(j) / I_i$. Because $I_i / U_i = (1 - \beta) / \beta$, it follows that, in country $i$, the demand for good $j$ as an intermediate input is $m_i(j) = (1 - \beta) \cdot c_i(j) / \beta$. Hence, in any country $i$, the overall demand for good $j$ is $c_i(j) / \beta$.

In the two-country model that we are examining, each good can either be produced abroad and imported at home; or be produced at home and sold only in the domestic market; or be produced at home and sold both in the domestic and the foreign market. Therefore, the resource constraint for country $i$ requires that:

$$q_i(j) = \begin{cases} 
0 & \text{if } j \in O_{i,z} \\
\frac{1}{\beta} c_i(j) & \text{if } j \in O_{i,d} \\
\frac{1}{\beta} \left[ c_i(j) + c_n(j) d_n \right] & \text{if } j \in O_{i,e} 
\end{cases}$$ (9)

for any $j$, where $O_{i,z}$ denotes the set of “zombie” industries in country $i$, i.e. those industries that shut down right after opening to trade;\(^7\) $O_{i,d}$ is the set of industries that sell their goods

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\(^6\)Notice that the proportionality between intermediate goods and labor endowment depends on the real wage that, in this setting, is the price of the primary input (labor) relative to the one of the secondary input (intermediate goods).

\(^7\)We borrow the terminology "zombie industries" from Caballero, Hoshi, and Kashyap (2008) who use it to refer to industries that are kept alive only by misdirected or subsidized bank lending. In the context of our model, instead, these industries would be kept alive by trade protectionist policies.
only on the domestic market; and $O_{i,e}^n$ is the set of industries that sell both at home and in country $n$. By construction, the sets $O_{i,z}$, $O_{i,d}$, and $O_{i,e}^n$ form a partition of the set of tradable goods; hence, the intersection between any subset of them is empty and their union spans the whole set of tradable goods. The set $O_{i,o} = O_{i,d} \cup O_{i,e}^n$, on the other hand, comprises the sole industries that survive international competition.\(^8\)

By plugging equations (1) and (7) into equation (9) (using also equation (8)), and solving the resource constraint for the number of workers in industry $j$, we obtain:

$$L_i(j) = \begin{cases} 
0 & \text{if } j \in O_{i,z} \\
\gamma_i^{-1}(j) \cdot \left( \frac{w_i}{p_i} \right)^{\beta(1-\sigma)} L_i & \text{if } j \in O_{i,d} \\
\gamma_i^{-1}(j) \cdot \left( \frac{w_i}{p_i} \right)^{\beta(1-\sigma)} L_i \cdot (1 + k_{ni}) & \text{if } j \in O_{i,e}^n 
\end{cases} \quad (10)$$

where:

$$k_{ni} = \frac{w_i L_i}{w_i L_i / p_i} d_{ni}^{-\sigma} \left( \frac{p_i}{p_n} \right)^{-\sigma} \quad (11)$$

The term $k_{ni}$ is related to the demand that comes from country $n$, since it depends on the relative size of this country in terms of real GDP, the iceberg cost between countries $i$ and $n$, and their relative price levels.

In the autarky economy, $O_{i,z} = O_{i,e}^n = \emptyset$ and the resource constraint returns, for any good $j$, $L_i(j) = \gamma_i^{-1}(j) \cdot (w_i/p_i)^{\beta(1-\sigma)} L_i$. Let us consider, then, how labor is reallocated after trade liberalization. With respect to the autarky economy, in the open economy the number of workers in the zombie industries goes to zero. The number of workers in the industries that produce goods that are sold only domestically declines (provided that $\sigma > 1$), because these industries face a tougher competition. In particular, imported goods are cheaper than those that were made at home in the autarky economy.\(^9\) The number of workers in the exporting industries rises, absorbing all the workers “in excess” from the other domestic industries. More specifically, these industries sell less in the domestic market (as international competition brings in cheaper imported goods), so they would need less workers to serve this market. Foreign demand, however, allows them not only to avoid firing workers, but also to hire new workers in order to produce more goods to be sold abroad.\(^11\)

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8 In the two-country model, these sets are defined as follows: $O_{i,z} = \{ j : \frac{z_i(j)}{c_i} > \frac{z_n(j)}{c_n d_{ni}} \}$, $O_{i,d} = \{ j : \frac{z_n(j)}{c_n d_{ni}} < \frac{z_i(j)}{c_i} > \frac{z_n(j)d_{ni}}{c_n} \}$, and $O_{i,e}^n = \{ j : \frac{z_n(j)}{c_n} \leq \frac{z_i(j)}{c_i} \leq \frac{z_n(j)d_{ni}}{c_n} \}$.

9 The term $c_n(j) d_{ni} / \beta$ in equation (9) represents the foreign demand that benefits only the exporting industries. In particular, the representative consumer of country $n$ demands the quantity $c_n(j) / \beta$, but iceberg costs imply that $d_{ni}$ units must be shipped from country $i$ to deliver one unit of good to country $n$. Thus, the overall quantity produced to serve the latter market is $c_n(j) d_{ni} / \beta$.

10 In the case $\sigma < 1$ ($\sigma = 1$), industries that produce goods sold only at home would employ more (the same number of) workers.

11 The two terms of equation (10) in the case $j \in O_{i,e}^n$ reflect exactly this dichotomy: the number of workers in the exporting industry that serve the domestic market (which declines) and the number of workers hired to start servicing the foreign market.
Notice that, in any industry, the number of workers is proportional to the efficiency of this industry, at the $\sigma - 1$ power (i.e. to $z_i^{\sigma - 1}(j)$). By aggregating across industries both sides of equation (10), we can derive the following decomposition of the real wage (which is proven in Appendix A for the general $N$-country case):

$$\frac{w_i}{p_i} = \frac{\lambda_{i,o} \cdot E \left( Z_{i,o}^{\sigma - 1} \right) + \lambda_{i,e} \cdot k_{ni} \cdot E \left( Z_{i,e;n}^{\sigma - 1} \right)}{\beta (\sigma - 1)}$$

(12)

where $\lambda_{i,o}$ is the probability that an industry of country $i$ survives international competition; $\lambda_{i,e}$ is the probability that an industry is also an exporter (with $\lambda_{i,e} \leq \lambda_{i,o}$); $Z_{i,o}$ is the random variable that describes the efficiencies of the surviving industries; and $Z_{i,e;n}$ describes the efficiencies of the industries that export in country $n$.\(^{13}\)

Equation (12) shows — together with equation (10), from which it is derived — the two sources of welfare gains in this model. The first one comes from impact of the selection of industries due to international competition, that brings the average efficiency of the economy from $E(Z_i^{\sigma - 1})$ to $E(Z_{i,o}^{\sigma - 1})$.\(^{14}\) The second one comes from the reallocation of workers to the exporting industries that, with respect to the autarky economy, in the open economy provides a contribution to welfare that is separate and additional to the previous one (measured by the second term inside the square brackets on the right-hand side of (12)).\(^{15}\) This contribution, which depends on the strength of the foreign demand (as measured by $k_{ni}$), is key to the result that trade is welfare improving. In fact, although we know that the real wage always rises after trade openness, the average efficiency does not necessarily rise.\(^{16}\) Hence, economies in which average efficiency is lower under trade openness, still benefit from trade thanks to this additional reallocation effect. It is known, however, that under broad conditions about the distribution of industry efficiencies, also the selection effect provides a positive contribution to the welfare gain and, in the next section, we will discuss and quantify both effects for one specific model that fulfills those conditions.\(^{17}\)

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\(^{12}\)The triangle inequality implies that if an industry is an exporter, than it must necessarily sell its goods also in its domestic market.

\(^{13}\)In the two-country model, these four variables are defined as follows: $\lambda_{i,o} = \Pr \left( \frac{Z_i}{e_i} \geq \frac{Z_n}{e_n} \right)$, $\lambda_{i,e} = \Pr \left( \frac{Z_i}{e_i} \geq \frac{Z_{i,e;n}}{e_n} \right)$, $Z_{i,o} = Z_i | Z_i \geq Z_n \frac{e_i}{e_n} d_{in}$, $Z_{i,e;n} = Z_i | Z_i \geq Z_n \frac{d_{i,n}}{e_n}$.

\(^{14}\)In the autarky economy, $\lambda_{i,o} = 1$, $Z_{i,o} = Z_i$, and $\lambda_{i,e} = 0$.

\(^{15}\)The efficiencies of the exporting industries are included also in $Z_{i,o}$ (that describes the efficiency of all the surviving industries, including the exporters). Therefore, the contribution of the reallocation effect is distinct from the one that comes from the selection effect.

\(^{16}\)In other words, the result that $M_i \geq Z_i$ implies that $E \left( M_i^{\sigma - 1} \right) \geq E \left( Z_i^{\sigma - 1} \right)$ (i.e. welfare rises after trade openness), even though $E \left( Z_i^{\sigma - 1} \right)$ can be either larger of smaller that $E \left( Z_{i,o}^{\sigma - 1} \right)$ (average efficiency does not necessarily rise).

\(^{17}\)Finicelli, Pagano, and Sbracia (2012a) focus on the theoretical conditions under which average efficiency across industries rises after opening to trade. In particular, they show that it always rises under very broad as-
Before turning to the quantification, however, let us show how the result generalizes to the case of many countries ($N \geq 2$). In Appendix A, we prove that, even in a multi-country framework, the real wage in each country $i$ has still two components, the selection effect ($SE_i$) and the reallocation effect ($RE_i$):

$$\frac{w_i}{p_i} = (SE_i + RE_i)^{1/\beta(\sigma-1)} .$$

(13)

The first term inside the brackets of the right hand side of (13) has the same expression as the corresponding term of the two-country case:

$$SE_i = \lambda_{i,o} \cdot E \left( Z_{i,o}^{\sigma-1} \right) .$$

(14)

The second term is now somewhat more cumbersome:

$$RE_i = \sum_{n \neq i} \lambda_{i,e;n} \cdot k_{ni} \cdot E \left( Z_{i,e;n}^{\sigma-1} \right) +$$

$$+ \sum_{n \neq i, h \neq i, n \neq h} \lambda_{i,e;n,h} \cdot (k_{ni} + k_{hi}) \cdot E \left( Z_{i,e;n,h}^{\sigma-1} \right) +$$

$$+ ... + \lambda_{i,e;1,...,N} \cdot (k_{1i} + ... + k_{Ni}) \cdot E \left( Z_{i,e;1,...,N}^{\sigma-1} \right) ,$$

(15)

where $\lambda_{i,e;n,h,...,k}$ is the probability that an industry of country $i$ exports in (and only) countries $n, h, ...$, and $k$; while $Z_{i,e;n,h,...,k}$ is the distribution of the efficiencies of these industries.

As shown by equations (12) and (15), in both the cases $N = 2$ and $N > 2$ the magnitude of the reallocation effect is governed by $k_{ni}$, which is a function of the exogenous variables of the model (equation (11)). In particular, $k_{ni}$ and, then, the size of the reallocation effect are larger if country $i$ is relatively more productive ($p_i/p_n$ is low, i.e. $T_i/T_n$ is high), if the destination market $n$ is near ($d_{ni}$ low), rich ($w_n/p_n$ is high relative to $w_i/p_i$) and large ($L_n$ is high relative to $L_i$).

In principle, quantifying the expressions of (14) and (15) is not an impossible task, although it may be rather daunting. Given the joint distribution of $(Z_1, ..., Z_N)$, in fact, one can always derive the distribution of any of the $Z_{i,e;n,h,...,k}$, which are just univariate conditional distributions (see Appendix A). However, in empirical applications their number might be quite large, making their computation a very challenging task. With $N$ countries, one has to compute the distributions of the efficiencies for the industries that export in each of the $N - 1$ partner countries, those for the industries that export in all the possible $N (N - 1) / 2$ couples of partner countries, etc.. For instance, in the 46-country application that we consider in the next section, one should have to compute a total of more than 35,000 billions of different distributions (that is $2^{N-1} - 1$). In the next section, instead, we show that, by considering one of the quantitative trade models of Arkolakis, Costinot, and Rodríguez-Clare (2012), the quantification of the two effects simplifies dramatically.

Assumptions about the country distributions of industry efficiencies; namely: (i) if the distributions of efficiencies are independent across countries; (ii) for many types of distributions, if their correlations are sufficiently low; (iii) regardless of cross-country correlations, if industry efficiencies belong to some families of distributions that are widely used in the literature, such as the Fréchet, Pareto and lognormal.
4 The model with Fréchet-distributed efficiencies

We now assume that, in any country $i$, industry efficiencies are Fréchet distributed, with parameters $T_i$ and $\theta$; hence, the probability that an industry of country $i$ has an efficiency lower that a positive real number $z$ is $F_i(z) = \exp \left\{ -T_i z^{-\theta} \right\}$. For the sake of simplicity, we also assume that these distributions are mutually independent across countries.

The moment of order $k$ of $Z_i$ is:

$$E \left( Z_i^k \right) = T_i^{k/\theta} \cdot \Gamma \left( \frac{\theta - k}{\theta} \right),$$

which exists if and only if $\theta > k$, where $\Gamma$ is Euler’s Gamma function. Because welfare is related to the moment of order $\sigma - 1$ of $Z_i$, we assume $\theta > \sigma - 1$. The parameter $T_i$, usually defined as the “state of technology” of country $i$, captures country $i$’s absolute advantage: an increase in $T_i$ relative to $T_n$ implies an increase in the share of goods that country $i$ produces more efficiently than country $n$. The shape parameter $\theta$, common to all countries, is inversely related to the dispersion of $Z_i$. It is related to the concept of comparative advantage because, in the Ricardian model, gains from trade depend on the heterogeneity in efficiencies. In this model, a decrease in $\theta$ (i.e. higher heterogeneity), coupled with mutual independence, generates larger gains from trade for all countries.

An important property of the model with Fréchet-distributed efficiencies is that the price distribution in country $i$ for the goods imported from country $n$ is the same for any $n$ (and equal to $P_i$). Thus, for example, source countries with a higher state of technology or lower iceberg costs exploit these advantages by selling a wider range of goods to that country but, in the equilibrium, the price distributions of the goods that the various foreign sources supply to the destination market $i$ are identical (see Eaton and Kortum, 2002, and Arkolakis, Costinot, and Rodríguez-Clare, 2012). A related key property is that, in the open economy (Finicelli, Pagano, and Sbracia, 2012a):\(^20\)

$$M_i = Z_{i,\sigma}.$$

\(^{18}\)Kortum (1997) and Eaton and Kortum (2009) show that the Fréchet distribution emerges from a dynamic model in which, at each point in time: (i) the number of ideas that arrive about how to produce a good follows a Poisson distribution; (ii) the efficiency conveyed by each idea is a random variable with a Pareto distribution; (iii) firms produce goods using always the best idea that has arrived to them. Jones (2005) shows that this set up on the flow of ideas entails two other results: the global production function is Cobb-Douglas and technical change in the long run is labor-augmenting.

\(^{19}\)The key assumption is that industry efficiencies are Fréchet distributed, while independence can easily be relaxed. In particular, Eaton and Kortum (2002) propose a multivariate Fréchet distribution for industry efficiencies that allows for correlation across countries, and Finicelli, Pagano and Sbracia (2012a) use it to compute the “productivity gains from trade” for different degrees of correlation.

\(^{20}\)In general, if the random variables $X$ and $Y$ are independent and Fréchet-distributed, then $\max (X, Y) \sim X | X \geq Y$. 

17
Hence, equation (5) becomes:

\[
\frac{w_i}{p_i} = \left[ E\left(Z_{i,o}^{\sigma-1}\right) \right]^{1/\beta(\sigma-1)}.
\]  

(17)

We now show how the analytical decomposition of welfare simplifies and how its sources can be quantified under the Fréchet assumption. Combining equation (17) with (13) and using equation (14), it turns out that:

\[
RE_i = (1 - \lambda_{i,o}) \cdot E\left(Z_{i,o}^{\sigma-1}\right),
\]

while it is still \(SE_i = \lambda_{i,o} \cdot E\left(Z_{i,o}^{\sigma-1}\right)\).

The welfare improvement induced by trade openness (equation (6)) becomes:

\[
g_i = \left[ \frac{E\left(Z_{i,o}^{\sigma-1}\right)}{E\left(Z_{i}^{\sigma-1}\right)} \right]^{1/\beta(\sigma-1)},
\]

that, in turn, can be decomposed as:

\[
g_i = \left[ \frac{\lambda_{i,o} \cdot E\left(Z_{i,o}^{\sigma-1}\right)}{E\left(Z_{i}^{\sigma-1}\right)} + (1 - \lambda_{i,o}) \cdot \frac{E\left(Z_{i,o}^{\sigma-1}\right)}{E\left(Z_{i}^{\sigma-1}\right)} \right]^{1/\beta(\sigma-1)}.
\]

In other words, given the overall gain from trade \(g_i\), a share \(\lambda_{i,o}\) of the gain is due to the selection effect, while the remaining \(1 - \lambda_{i,o}\) is due to the reallocation effect.\(^{21}\)

We can now turn to the measurement. Finicelli, Pagano and Sbracia (2012a) have shown that, in this context, \(Z_{i,o}\) is still a Fréchet, with parameters \(\Lambda_i\) and \(\theta\), where

\[
\Lambda_i = T_i + \sum_{i \neq k} T_k \left( \frac{c_{ik}d_{ik}}{c_i} \right)^{-\theta}.
\]

It follows that:\(^{22}\)

\[
\frac{E\left(Z_{i,o}^{\sigma-1}\right)}{E\left(Z_{i}^{\sigma-1}\right)} = \left( \frac{\Lambda_i}{T_i} \right)^{(\sigma-1)/\theta}.
\]

\(^{21}\)In interpreting the two shares, we can safely ignore the complication due to the exponent \(1/\beta(\sigma - 1)\). In fact, a monotone transformation of the utility function, such as the one that can be obtained by taking \(U_i\) at the \(\beta(\sigma - 1)\) power, would yield the same equilibrium quantities and relative prices. In this transformed model, then, welfare would be the same as in the original model, but at the \(\beta(\sigma - 1)\) power, making the exponent of the gain from trade equal to 1 (while leaving the base unchanged).

\(^{22}\)Note that \(\Lambda_i > T_i\). If industry efficiencies are Fréchet distributed, then, in each country surviving industries are on average more efficient than the whole set of national industries (i.e., the set comprehensive of the industries that shut down under trade openness). This feature of the "quantitative Ricardian trade model," however, does not appear to be particularly restrictive, as it holds under very broad conditions on industry efficiencies (see footnote 17) and is consistent with the available empirical evidence.
To quantify $g_i$, we borrow from Finicelli, Pagano and Sbracia (2012a) and (2012b) the result that:

$$
\Lambda_i = T_i \cdot \Omega_i \\
\text{where } \Omega_i \equiv 1 + \frac{IMP_i}{PRO_i - EXP_i},
$$

(19)

in which $IMP_i$ is the value of country $i$’s aggregate imports, $PRO_i$ is the value of its production, and $EXP_i$ is the value of aggregate exports. Thus:

$$
g_i = (\Omega_i)^{1/\beta}. 
$$

(20)

This is the result established by Arkolakis, Costinot, and Rodríguez-Clare (2012) for a large class of quantitative trade models (extended to a framework that encompasses intermediate goods).

The quantification of the selection and reallocation effect can be completed once we derive $\lambda_{i,o}$, that is the probability that an industry of country $i$ survives international competition. With computations similar to those that lead to $Z_{i,o}$, it is easy to find that:

$$
\lambda_{i,o} = \frac{T_i (c_i)^{-\theta}}{\sum_k T_k (c_k d_{ik})^{-\theta}} = \frac{1}{\Omega_i}
$$

(21)

Interestingly, note that, because welfare gains are increasing in $\Omega_i$, it follows that gains are larger when the selection effect is less important and the reallocation effect is more important. This result can be readily explained. When gains from trade are small, the selection effect matters mostly because there are few exporters in the domestic economy and, then, the possibilities of reallocating workers in these industries are fewer. On the other hand, as the export sector grows and the gains from trade increase, the importance of the reallocation effect also rises because exporting industries — which are, on average, more productive — can absorb more workers.

What does real data show about the size of these two effects? Table 1 provides a quantification of the welfare gains from trade as well as the contribution of the selection and reallocation effect for a sample of 46 countries (33 OECD economies and 13 emerging economies) in two different years, 2000 and 2005. Gains are computed using equation (20), taking the value of the main parameters from literature. In particular, we assume that the shape parameter is $\theta = 4$, as advocated by Simonovska and Waugh (2011), and the share of intermediate goods in production is $\beta = 0.33$, a conventional measure of the share of value added in total output. The share of the gain from trade pertaining to the selection and reallocation effects, respectively equal to $\lambda_{i,o}$ and $1 - \lambda_{i,o}$, are computed using equation (21).

Given that the Ricardian theory laid out in this paper best describes trade in manufactures, rather than in natural resources or primary goods, we follow the literature and consider data on the values of domestic production, exports and imports — which is all is needed to compute the gains from trade as well as the size of their sources — all relative to
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Source: authors’ calculations on OECD STAN data.

(1) Real wage relative to the autarky economy (values of $(g_i-1)\%$) and contributions of the selection and the reallocation effect (in percentage).
the manufacturing sector. In addition, given that the model assumes that trade is balanced, in the application we impose that exports are identical to imports (equal to their average).

For each year, Table 1 shows the percentage increase in welfare due to international trade and the shares (in percentage) due to the selection and the reallocation effect (the latter being simply the complement to 100 of the former). Results show that gains from trade are considerable (almost 60 and 70 percent in 2000 and 2005, for the cross-country average). As it is well known, the size of the gains is quite sensitive to the assumptions about the value of the shape parameter (trade elasticity) and the share of intermediate goods in production. For instance, by taking $\theta = 6.66$ instead of $\theta = 4$ (as Alvarez and Lucas, 2007), the gains would be about 60 percent of those reported in Table 1. By the same token, in the model without intermediate goods ($\beta = 1$), gains from trade would be about one third of those reported in the table.

Overall, the size of the selection effect is somewhat more important (close to 60 percent in the year 2000 and around 55 per cent in 2005) than the reallocation effect in our sample of countries. It is worth noting that, unlike the gains from trade, the two shares remain unchanged irrespectively of the exact value of $\theta$ and $\beta$. Unsurprisingly, the reallocation effect is more important in small open economies, such as Denmark, Estonia, Ireland, the Netherlands, Slovenia, Singapore, Thailand, and Vietnam. For these countries, the share of the welfare gains pertaining to the reallocation effect is above 70 percent in at least one year. On the other hand, for large and relatively more closed countries, it is the selection effect that is dominant. For instance, among the OECD economies, only the United States and Japan record a share of the welfare gains pertaining to the selection effect above 80 percent in at least one year. Among non-OECD economies, only the BRIC countries (Brazil, Russia, India, and China) show the same record as the United States and Japan.

5 Conclusion

This paper provides a deconstruction of the sources of the welfare gains from trade in a Ricardian model. Under general distributions of industry efficiencies, welfare gains arise from two distinct sources. The former is an effect due to the selection of industries that survive international competition. The latter is related to the reallocation of workers from the industries that shut down, as well as from those servicing only the domestic market, to the industries that start servicing the foreign market. If industry efficiencies are Fréchet distributed, so that the model becomes one of the quantitative trade models of Arkolakis, Costinot and Rodríguez-Clare (2012), these two effects can be easily measured.

Following Arkolakis, Costinot, and Rodríguez-Clare (2012), the literature appears to be taking two different directions. One explores how the measurement of the gains from trade changes when the assumptions of quantitative trade models are relaxed (an example is Arkolakis, Costinot, Donaldson, and Rodríguez-Clare, 2012). The route taken is this paper,
instead, explores the sources of the gains from trade within the class of quantitative trade models (although our results can be generalized, but not yet quantified, also outside the domain of quantitative trade models). In particular, a key insight from our analysis is that quantitative trade models seem to be useful not only in order to assess the overall welfare gains, but also to properly measure their sources — an issue that deserves to be further explored in future studies tackling other models in this class. It is apparent, then, that examining the welfare gains from trade and their sources will continue to be a promising area of theoretical and empirical research.
Appendix

A Welfare decomposition with many countries

In order to prove equation (13), let us start by generalizing the resource constraint (9) to a context with more than just two countries. As in the two-country case, we still have: \( q_i (j) = 0 \), if \( j \in O_{i,z} \) and \( q_i (j) = c_i (j) / \beta \), if \( j \in O_{i,d} \). Now consider the set of industries of country \( i \) that export in (and only) the countries \( n, h, \ldots, k \) — for any \( \{n, h, \ldots, k\} \in \{1, \ldots, N\} \setminus \{i\} \) — and denote this set by \( O_{i,e}^{n, h, \ldots, k} \).\(^{23}\) the resource constraint for these industries becomes:

\[
q_i (j) = \frac{1}{\beta} [c_i (j) + c_n (j) d_{ni} + c_h (j) d_{hi} + \ldots + c_h (j) d_{ki}] .
\]

Solving the resource constraint for the number of workers in industry \( j \), we obtain:

\[
L_i (j) = \begin{cases} 
0 & \text{if } j \in O_{i,z} \\
\frac{z_i^{\sigma-1} (j)}{\beta(1-\sigma)} \cdot \left( \frac{w_i}{p_i} \right)^{(1-\sigma)} L_i & \text{if } j \in O_{i,d} \\
\frac{z_i^{\sigma-1} (j)}{\beta(1-\sigma)} \cdot \left( \frac{w_i}{p_i} \right)^{\beta(1-\sigma)} L_i \cdot (1 + k_{ni} + k_{hi} + \ldots + k_{ki}) & \text{if } j \in O_{i,e}^{n, h, \ldots, k}
\end{cases}
\]

(22)

where the terms \( k_{li} \) are defined as in equation (11), for any destination market \( l \).

Note that the sets \( O_{i,z}, O_{i,d}, O_{i,e}^{n, h, \ldots, k} \) (for any \( \{n, h, \ldots, k\} \) as above) form a partition of the set of tradable goods. By aggregating across industries both sides of equation (22), we obtain the following:

\[
\left( \frac{w_i}{p_i} \right)^{\beta(\sigma-1)} = \lambda_i,d \cdot E \left( Z_{i,d}^{\sigma-1} \right) + \ldots + \lambda_{i,e,n,h,\ldots,k} \cdot (1 + k_{ni} + k_{hi} + \ldots + k_{ki}) \cdot E \left( Z_{i,e,n,h,\ldots,k}^{\sigma-1} \right) + \ldots
\]

(23)

where \( \lambda_{i,d} \) is the probability that an industry of country \( i \) survives international competition and serves only the domestic market (i.e. \( \lambda_{i,d} = \Pr(Z_i \in O_{i,d}) \)); \( \lambda_{i,e,n,h,\ldots,k} \) is the probability that an industry of country \( i \) exports in (and only) countries \( n, h, \ldots, k \) (i.e. \( \lambda_{i,e,n,h,\ldots,k} = \Pr(Z_i \in O_{i,e}^{n, h, \ldots, k}) \)); \( Z_{i,e,n,h,\ldots,k} \) is the distribution of the efficiencies of these industries (i.e. \( Z_{i,e,n,h,\ldots,k} = Z_i | Z_i \in O_{i,e}^{n, h, \ldots, k} \)). Considering that:

\[
\lambda_{i,o} \cdot E \left( Z_{i,o}^{\sigma-1} \right) = \lambda_i,d \cdot E \left( Z_{i,d}^{\sigma-1} \right) + \ldots + \lambda_{i,e,n,h,\ldots,k} \cdot E \left( Z_{i,e,n,h,\ldots,k}^{\sigma-1} \right) + \ldots,
\]

we can conveniently rearrange the right-hand side of equation (23) into the sum of two terms, given by equations (14) and (15). By taking the \( 1/\beta (\sigma - 1) \) power of both sides, we finally obtain equation (13).

\(^{23}\)The analytical definition of \( O_{i,e}^{n, h, \ldots, k} \) is as follows: this set includes all the industries that export in countries \( n, h, \ldots, k \), which are those for which \( z_i (j) / c_i > z_i (j) d_{li} / c_l \), for \( l = n, h, \ldots, k \); and excludes those that export in countries different from \( n, h, \ldots, k \), that are those for which \( z_i (j) / c_i < z_i (j) d_{li} / c_l \) for \( l \neq n, h, \ldots, k \).
References


