Price Competition among Oligopolistic Firms in a Spatial Economy

Preliminary Version

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Abstract

This paper presents a spatial economy in the spirit of the so called "new economic geography" literature in which strategic interactions among firms are operative. It is shown that the forces driving regional concentration of economic activity crucially depend on firms' pricing decisions. Strategic interactions affect the critical levels of trade costs at which the symmetric equilibrium is broken, core-periphery patterns become sustainable and trade occurs. In particular, when firms located in the same region collude a bell-shaped curve of spatial development may be observed. For decreasing transport costs from high to intermediate agglomeration and divergence of industrial activity take place. Further reductions encourage firms to locate in different regions and a symmetric configuration eventually arises. On the other hand, when firms do not collude a core-periphery pattern always emerges for low transportation costs as in the standard core-periphery model with monopolistic competition.

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1 Introduction

One of the key contributions of the "new economic geography" (NEG) literature (see e.g. Fujita et al. 1999, Baldwin et al. 2003 and Ottaviano and Thisse, 2004) consists in providing an analytical framework to describe the trade-off between centrifugal and centripetal forces in a spatial economy. Standard core-periphery models (CP) adopt a Dixit-Stiglitz monopolistic competition market structure allowing the analysis of aggregate implications of increasing returns to scale but neglecting strategic interactions among firms, as pointed out by Matsuyama (1995) and Neary (2000). Furthermore, as remarked by Fujita and Thisse (1996), firms tend to be worried about the choice of their close competitors, so that strategic interactions are inherent to spatial models and could drive location decisions.

The main aim of the present paper is to show how interactions among oligopolistic firms affect the spatial pattern of economic structure in a CP model with transport costs, increasing returns, product differentiation and labor mobility in a two-region, two-sector, two-factor economy. For this scope we modify the basic linear CP setup proposed by Ottaviano et al. (2002) by introducing oligopolistic pricing competition. Ottaviano et al. (2002) present an alternative model of agglomeration and trade. They depart from the Dixit-Stiglitz version of monopolistic competition using a quadratic sub-utility function and show that firms' pricing policies are affected by their geographical location and by the total number of competitors.[1]

To the best of our knowledge the only contribution studying the implications for spatial agglomeration of a different market structure along the lines of the NEG literature is given by Combes (1997). Combes builds a two-country, two-sector partial equilibrium model with Cournot competition, in which strategic interactions act as a force driving location. He shows that quantity competition may lead to the agglomeration of production in the initially developed region in presence of high scale economies or when transportation costs are low enough, consistently with the NEG literature with monopolistic competition market

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[1]In Ottaviano et al. (2002) the general insights of the basic core-periphery model still hold. In particular, a core-periphery pattern is always sustainable for low transportation costs.

structure. In this situation, the fiercer competition, which tends to reduce profits of firms located in the region where more firms operate, is counterbalanced by the reduction in imports and by larger total market shares.

In the present paper we distinguish between two possible types of strategic interaction on pricing decisions: no collusion and collusion. From our analysis it emerges clearly that interactions matter in driving location decisions and affect critical levels of trade costs at which the symmetric equilibrium is broken, core-periphery patterns become sustainable and trade takes place.

When firms collude spatial configurations emerging for different levels of trade costs are shown to critically depend on the main features of the economy, such as the degree of increasing returns, the dimension of agriculture sector, the degree of substitution between goods and the intensity of consumers' preference for differentiated goods. On the other hand, in the absence of collusion the relationship between the spatial distribution of economic activity and the level of economic integration is robust to changes in the main parameters of the model. The centrifugal role played by the dimension of the agriculture sector in driving manufacturing activity spatial allocation seems to be much stronger under collusion.

Most importantly, when oligopolistic firms do not collude a core-periphery pattern is a stable equilibrium for low transportation costs, consistently with standard CP models. By contrast, when firms located in the same region collude in both regions we observe a bell-shaped curve of spatial development: spatial inequalities in the location of production activity first arise and then fall during a process of economic integration. In these circumstances we observe that for low transportation costs a symmetric distribution of economic activity is a stable configuration.[2] In addition, in the presence of collusive firms partial agglomeration equilibria might emerge.

Finally, using this framework we are able to show that market forces yield agglomeration

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[2]The standard CP models are unable to explain why in the process of increasing regional integration a core-periphery production structure is followed by a phase involving interregional convergence (see Williamson, 1965). This limitation of the standard model has been overcome by removing some peculiar assumptions; in particular, the so-called "bell-shaped" curve of spatial development may emerge introducing urban costs or heterogenous workers in the CP models. For details, see Puga (1998), Krugman and Venables (1995), Picard and Zeng (2005), Tabuchi and Thisse (2002).

(dispersion) for values of trade costs for which it is socially optimal to keep manufacturing activities dispersed (agglomerated). In the particular case of low trade costs we observe that a core-periphery pattern is always socially desirable. Clearly, full economic integration will be welfare-improving only in the absence of any collusive behavior among firms.

The remainder of the paper is organized as follows. Section 2 presents the basic model distinguishing between non collusive and collusive pricing decisions; Section 3 analyzes how the spatial distribution of industrial activity and the implied welfare level are affected by economic integration, strategic interactions and other underlying economic features; Section 4 concludes.

2 The Model

The economy presents two regions, Home, H, and Foreign, F. It has two sectors, perfectly competitive agriculture and oligopolistic manufacturing. Each sector employs a single factor: labor for manufacturing, L, and farmers for agriculture, A. Both factors are assumed to be sector-specific and exogenously given. Farmers are evenly distributed across the two regions and are spatially immobile, while workers in manufacturing are mobile between regions.

The economy produces two types of goods: a homogenous agricultural good and a number of horizontally differentiated manufacturing goods. The agricultural good is produced under constant returns of scale and perfect competition and can be traded freely inter and intra regions, without incurring in any transportation cost. The homogenous good is chosen as numéraire and consumers are assumed to have a positive initial endowment of it.

Horizontally differentiated manufacturing goods are produced under increasing returns to scale and imperfect competition, using the mobile factor L as the only input. In the manufacturing sector each firm is assumed to have a certain market power and is aware of its influence on the market outcome. It follows that each firm takes into consideration the reactions of other firms when deciding on price.

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There is a discrete number of firms N. The existence of high economies of scales and of an exogenous total amount of manufacturing labor force limits the number of firms operating with profit in the market. Each variety of the differentiated goods is produced by a single firm. As a result each producer has a certain market power in its own variety market. The manufacturing good can be traded across regions at a positive cost of τ units of the numéraire good for each unit shipped. The cost τ accounts for all the impediments to trade and measures the level of market integration. On the other hand, intra-regional sales in manufacturing good are costless. Markets are segmented in the sense that firms can perfectly price discriminate across markets, so that same goods can have different prices in different regions.

2.1 Demand Side

Following Ottaviano et al. (2002) preferences are described by a quasi-linear utility function with a quadratic sub-utility that is assumed to be symmetric in all varieties and identical across individuals:

$$U(q) = q_0 + \alpha \sum_{i=1}^{N} q_i - \frac{\beta}{2} \sum_{i=1}^{N} q_i^2 - \frac{\gamma}{2} \sum_{i=1}^{N} \sum_{j \neq i}^{N} q_i q_j, \tag{1}$$

where q_i is the quantity of variety $i = 1..N$ and q_0 is the quantity of the numéraire good.[3] All parameters are assumed to be positive. In particular $\beta > \gamma > 0$, implying that consumers love variety. These assumptions ensure that U is strictly concave. The parameter γ measures the degree of substitution between varieties so that goods are substitutes, independent or complements according to whether $\gamma \gtreqless 0$. The larger γ the closer substitutes goods are. When $\beta = \gamma$ goods are perfect substitutes and equation (1) degenerates into a standard quadratic utility defined over a homogenous product. Finally, the parameter α indicates the intensity of consumers' preferences for differentiated goods. The utility function (1) can be better expressed after some simple manipulations as:

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[3]The use of a quasi-linear utility function leads to a partial equilibrium analysis, in that the income effect on the demand for differentiated goods is completely neglected. At the same time, the numéraire good can be seen as a composite good, formed by the rest of the goods produced in the economy, which captures all the variations in income level. See Vives (1999) and Ottaviano et al. (2002) for details.

$$U(q) = q_0 + \alpha \sum_{i=1}^{N} q_i - \frac{\beta - \gamma}{2} \sum_{i=1}^{N} q_i^2 - \frac{\gamma}{2} \left(\sum_{i=1}^{N} q_i \right)^2. \tag{2}$$

Each individual is endowed with $\overline{\mathbf{q}}_0 > 0$ units of the numéraire and one unit of labor of type L or A. Her budget constraint is defined as follows:

$$\sum_{i=1}^{N} p_i q_i + q_0 = m + \overline{\mathbf{q}}_0, \tag{3}$$

where p_i is the price of variety i, m is labor income and the price of the agricultural good is normalized to one.

The inverse demand function for each variety is obtained by maximizing the utility function (2), subject to the budget constraint (3). Given the strict concavity of the utility function the following first order conditions are also sufficient for a maximum:

$$\frac{\partial U(q)}{\partial q_i} = -p_i + \alpha - (\beta - \gamma) q_i - \gamma \sum_{j=1}^{N} q_j = 0, \tag{4}$$

where $i = 1..N$.

Solving the budget constraint (3) for the numéraire q_0, substituting it into the utility function and solving for the first order condition for each variety i yield the inverse demand function:

$$p_i = \alpha - (\beta - \gamma) q_i - \gamma \sum_{j=1}^{N} q_j. \tag{5}$$

Solving the inverse demand function of each variety for q_i and summing up all the N inverse demand functions, after some algebraic manipulations one can obtain the demand function for each variety:

$$q_i = a - b p_i + c \sum_{j=1}^{N} (p_j - p_i), \tag{6}$$

that we can expressed as:

$$q_i = a - (b + cN) p_i + c \sum_{j=1}^{N} p_j, \tag{7}$$

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where $a \equiv \frac{\alpha}{\beta+\gamma(N-1)}$, $b \equiv \frac{1}{\beta+\gamma(N-1)}$ and $c \equiv \frac{\gamma}{(\beta-\gamma)[\beta+\gamma(N-1)]}$.

Note that $(b + cN)$ denotes the own price effect and c is the cross price effect. The own price effect is larger than the cross price effect and the sum of all the cross price effects cN. According to equation (7) the demand of a certain variety falls when its own price rises not only in absolute terms but also relatively to the average price.

2.2 Supply Side

In the supply side the agricultural sector is perfectly competitive and the homogeneous good is produced under constant returns to scale. Technology in agriculture requires that one unit of output is produced using one unit of labor A. The assumption that the agricultural good can be freely traded between regions implies that in equilibrium the wage of the farmers in both regions is the same, that is $w_h^A = w_f^A = 1$.

In the manufacturing sector the differentiated good is produced in oligopoly and under increasing returns to scale. Technology in manufacturing requires that any amount of a variety is produced using ϕ units of labor L. The marginal cost of production is set equal to zero as in Ottaviano et al. (2002).

The fixed amount ϕ measures the degree of increasing returns and determines the number of oligopolistic firms operating in the market. The total number of varieties produced in the economy N (i.e. the number of firms) depends on the total labor force of the economy, L, and on the level of increasing returns:

$$N = \frac{L}{\phi}. \tag{8}$$

Let n_h and n_f be the number of firms in region H and F, respectively. Labor market clearing conditions in both regions imply:

$$n_h = \frac{\lambda L}{\phi}, \tag{9}$$

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$$n_f = \frac{(1-\lambda)L}{\phi}, \qquad (10)$$

where $N = n_h + n_f$ and $\lambda \in [0,1]$ is the share of workers L located in region H. According to (9) and (10) the region presenting the larger proportion of workers is also the region where the majority of firms operate.

Equilibrium wage in both regions is determined as the results of a bidding process among firms demanding labor. Firms interact in the oligopolistic market of the manufacturing good, but they compete in the labor market so that all gross profits are eventually absorbed by labor costs. Since we assume the existence of positive transport costs τ, each firm is able to set a price specific to the region where the product is sold. In other words, the positive transport cost allows firms to segment markets. From this time onrward we will focus on region H, given that for region F the same equations can be derived by symmetry.

From (7), the demand functions faced by a Home firm i in H and in F are, respectively:

$$q_{i,hh} = a - (b + cN)p_{i,hh} + cP_h, \qquad (11)$$

and

$$q_{i,hf} = a - (b + cN)p_{i,hf} + cP_f, \qquad (12)$$

where P_h and P_f are the price indices in region H and region F defined as $P_h = \sum_{j=1}^{n_h} p_{j,hh} + \sum_{k=1}^{n_f} p_{k,fh}$ and $P_f = \sum_{j=1}^{n_h} p_{j,hf} + \sum_{k=1}^{n_f} p_{k,ff}$.

Since we solve the model under oligopolistic pricing competition, that is when firms maximize their profits setting price, we express profits as function of prices. Profits made by the representative Home firm i in both markets are defined as follows:

$$\Pi_{i,h} = p_{i,hh}q_{i,hh}(p_{i,hh})(A/2 + \lambda L) + (p_{i,hf} - \tau)q_{i,hf}(p_{i,hf})[A/2 + (1-\lambda)L] - \phi w_h, \quad (13)$$

where w_h is the wage prevailing in region H and $q_{i,hh}(p_{i,hh})$ and $q_{i,hf}(p_{i,hf})$ are the demand functions faced by Home firm in the two regions given by (11) and (12). It should be noted that λL denotes the fraction of manufacturing workers located in H, while farmers are

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evenly spread between the two regions.

The representative Home firm i maximizes its profits (13) with respect $p_{i,hh}$ and $p_{i,hf}$, taking into account the effects that its pricing decisions will produce on the price index of each region, P_h and P_f. From this point of view firms interact strategically in both markets. In the following sections we will consider, in turn, the case in which firms interact among each others without colluding and the case in which Home (Foreign) producers collude in both markets H and F. In particular, we will consider the case in which firms of the same nationality collude among each others, while firms of different nationalities compete in both markets.[4]

2.3   No Collusion

In this section we consider the case in which producers interact among each others without colluding. In this case firms independently set the prices they charge for their output and have to supply all the demand arising at the price they charge.

The Home firm i maximizes its profits (13) with respect to $p_{i,hh}$ and $p_{i,hf}$. By solving the optimization problem we obtain the following best reaction functions:

$$p_{hh} = \frac{a + cn_f p_{fh}}{2(b + cN) - cn_h - c},$$
(14)

$$p_{hf} = \frac{a + cn_f p_{ff}}{2(b + cN) - cn_h - c} + \frac{b + cN - c}{2(b + cN) - cn_h - c}\tau.$$
(15)

where we have used the facts that *ex post* $p_{i,hh} = p_{hh}$ and $p_{i,hf} = p_{hf}$ for each $i = 1..n_h$ and that $p_{k,ff} = p_{ff}$, $p_{k,fh} = p_{fh}$ for each $k = 1..n_f$. By symmetry one can derive analogous best reaction functions for the representative Foreign firm.

Combing all the reaction functions and solving for the prices, p_{hh}, p_{ff}, p_{hf} and p_{fh} we

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[4]In the Appendix it is shown that in the case of collusion among all firms of the economy any spatial configuration is stable. In such circumstances, firms will set the same price in both markets and there will be no incentives for workers to migrate from one region to another.

obtain:

$$p_{hh} = \frac{a}{2b + cN - c} + \frac{b + cN - c}{(2b + 2cN - c)(2b + cN - c)} cn_f \tau, \qquad (16)$$

$$p_{ff} = \frac{a}{2b + cN - c} + \frac{b + cN - c}{(2b + 2cN - c)(2b + cN - c)} cn_h \tau, \qquad (17)$$

$$p_{fh} = p_{hh} + \frac{b + cN - c}{2b + 2cN - c} \tau, \qquad (18)$$

$$p_{hf} = p_{ff} + \frac{b + cN - c}{2b + 2cN - c} \tau. \qquad (19)$$

From the above results it clearly emerges that for $\tau = 0$ equilibrium prices do not depend upon the geographical allocation of firms and $p_{hh} = p_{fh} = p_{ff} = p_{hf}$.

Given the equilibrium prices (16)-(19) the level of freight absorption crucially depends on the localization of firms. In particular we have the following result.

PROPOSITION 1: *Given the equilibrium prices* (16)-(19) *the fraction of trade costs that the Home (Foreign) firm passes on to the consumers,* $p_{hf} - p_{hh}$ ($p_{fh} - p_{ff}$)*, is smaller than* $\frac{1}{2}\tau$ *if* $n_h - n_f < (>)\frac{2b + cN - c}{2(2b + 2cN - c)}$. PROOF: *see Appendix.*

The proposition asserts that there is more freight absorption for the Home firm when the foreign market is the largest one. In the attempt to penetrate the larger distant market the Home firm will bear the burden of a higher fraction of trade costs.

For high trade costs firms can set higher prices in their local market since they are more protected from international competition. By contrast, prices set in the distant markets net of transportation cost $p_{hf} - \tau$ ($p_{fh} - \tau$) are decreasing in τ, because entering foreign markets is more difficult. Let τ_{trade} denote the threshold level of transport costs ensuring trade we have the following result.

PROPOSITION 2: *Given the equilibrium prices* (16)-(19) *the Home (Foreign) firm's price*

set on Foreign (Home) market net of transport costs is positive for any workers distribution if $\tau < \tau_{trade} = \frac{a(2b+2cN-c)}{(2b+cN-c)(b+cN)}$. PROOF: *see Appendix.*

According to PROPOSITION 2 the threshold level of transportation cost depends in a complex fashion on the number of firms operating in the economy and on the preference parameters and is such that $\tau_{trade} < a/b$.

Finally, we observe that prices are lower in the largest market because competition among firms is stronger. The price index is lower in the region where more firms are located.

PROPOSITION 3: *If* $n_h > (<)n_f$ *then* $p_{hh} < (>)p_{ff}$ *and* $p_{fh} < (>)p_{hf}$ *implying that* $P_h < (>)P_f$. PROOF: *see Appendix.*

2.4 Collusion

Consider now the case in which producers located in the same region collude in both markets. Firms producing in the same region set the same price in each market, it follows that the relevant price indexes can be re-written as $P_h = n_h p_{hh} + n_f p_{fh}$ and $P_f = n_h p_{hf} + n_f p_{ff}$ *ex ante.* The representative Home firm maximizes its profits (13) with respect to p_{hh} and p_{hf}.

By solving the optimization problem we get the following best reaction functions:

$$p_{hh} = \frac{1}{2} \frac{a + c n_f p_{fh}}{b + c n_f}, \tag{20}$$

and

$$p_{hf} = \frac{1}{2} \frac{a + c n_f p_{ff} + \tau(b + c n_f)}{b + c n_f}. \tag{21}$$

Symmetric best reaction functions can be derived for the representative Foreign producer. By solving the above functions for the prices we obtain equilibrium prices:

$$p_{hh} = \frac{a c n_f + (b + c n_h)(2a + n_f c \tau)}{4b(b + cN) + 3c^2 n_h n_f}, \tag{22}$$

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$$p_{hf} = p_{hh} + \frac{(2b + cn_f)(b + cn_h)}{4b(b + cN) + 3c^2 n_h n_f} \tau, \tag{23}$$

$$p_{ff} = \frac{acn_h + (b + cn_f)(2a + n_h c\tau)}{4b(b + cN) + 3c^2 n_h n_f}, \tag{24}$$

$$p_{fh} = p_{ff} + \frac{(2b + cn_h)(b + cn_f)}{4b(b + cN) + 3c^2 n_h n_f} \tau. \tag{25}$$

Equilibrium prices depend upon demand and firms' spatial allocation across space. The dependence on the firms' distribution between regions relies on the level of transportation costs τ: the lower τ, the lower the impact of reallocation of firms on equilibrium prices. When τ is equal to zero we observe that $p_{hh} = p_{hf}$ and $p_{ff} = p_{fh}$. In other words, in the absence of transportation costs Home (Foreign) producers set the same price in both markets. However, as long as $n_h \neq n_f$ we observe that $p_{hh} \neq p_{fh}$ and $p_{ff} \neq p_{hf}$, that is equilibrium prices still depend on the spatial distribution of firms, contrary to the non collusive case. Furthermore, both p_{hh} and p_{ff} are increasing in τ, because if transportation costs are high firms tend to face a less fierce competition by the foreign firms in their local market. By contrast, net equilibrium prices $p_{hf} - \tau$ and $p_{fh} - \tau$ are decreasing in τ, because if barriers to trade are high it is more difficult for firms to sell their products in the foreign markets. Equilibrium prices increase with the parameter a which is proportional to the relative desirability of the manufacturing goods with respect to the numéraire good, α.

We observe that at the given equilibrium prices oligopolistic firms bear the burden of more than one half of transportation costs as summarized in the following proposition.

PROPOSITION 4: *Given the equilibrium prices (22)-(25) both Home and Foreign firms absorb part of the transportation cost and transfer on the consumers less than one half of it,* $(p_{hf} - p_{hh}), (p_{fh} - p_{ff}) < \frac{1}{2}\tau$. PROOF: *see Appendix.*

According to the above proposition in order to enter the Foreign market the Home firm bears the burden of a higher fraction of trade costs, no matter what the dimension of the

distant market is.

Trade occurs between the two regions provided that prices net of transportation costs are positive. In this case we have the following result.

PROPOSITION 5: *Home (Foreign) firm's price set on Foreign (Home) market net of transport cost is positive if and only if* $\tau < \tau_{trade} = \frac{2ab+acN+acn_h}{2b^2+c^2n_fn_h+2Nbc}$. PROOF: *see Appendix.*

As in the non collusive case the critical level of transportation costs depends on preferences and the number of firms operating in the economy.

Finally, we have the following results on price differentials.

PROPOSITION 6: *When the trade condition holds* $(\tau < \tau_{trade})$ *and* $\tau > 0$ *if* $n_h > (<)n_f$, *then* $p_{hh} > (<)p_{ff}$ *and* $p_{hf} > (<)p_{fh}$. *The net effect on the price index is such that if* $n_h > (<)n_f$ *then* $P_h < (>)P_f$. PROOF: *see Appendix.*

According to PROPOSITION 6 the larger the number of colluding firms of the same nationality the higher the prices that these firms will be able to set in both markets. However, the price index is lower in the region where more firms are located.

3 Firms Spatial Distribution

The agricultural labor force, as seen above, is exogenously given and, by assumption, it is evenly distributed between the two regions. Manufacturing labor force, by contrast, is mobile. Workers are assumed to move toward the region which offers a higher level of utility. The driving force of migration is the utility differential between regions, so that a spatial equilibrium arises when no worker can get a higher utility level by migrating to the other region.

By the properties of the quasi-linear utility functions, the indirect utility in regions H and F are defined as follows:

$$V_h = S_h + w_h + \overline{\mathbf{q}}_0, \tag{26}$$

$$V_f = S_f + w_f + \overline{\mathbf{q}}_0. \tag{27}$$

A spatial equilibrium is verified at $\lambda \in (0,1)$ when the utility differential is zero:

$$\Delta V(\lambda) \equiv V_h - V_f = (S_h - S_f) + (w_h - w_f) = 0, \tag{28}$$

or at $\lambda = 0$ when $\Delta V(0) \leq 0$ or at $\lambda = 1$ when $\Delta V(1) \geq 0$. Clearly, from (28) the symmetric configuration, $\lambda = 1/2$, is always an equilibrium. According to the standard literature on migration and agglomeration of economic activity mobile workers are assumed to be attracted by the region displaying a utility higher than the average utility. In general, the distribution of manufacturing workers between regions changes over time to the extent that utility differs across regions. The equation of motion describing the migration dynamics with respect to time t is the following:

$$\overset{\bullet}{\lambda} \equiv \frac{d\lambda}{dt} = \lambda[V_h - \lambda V_h - (1 - \lambda)V_f] = \lambda(1 - \lambda)\Delta V(\lambda). \tag{29}$$

In spatial equilibrium no migration occurs, so that $\overset{\bullet}{\lambda} = 0$. Workers move from region F to H when $\Delta V(\lambda) > 0$, and from region H to F when $\Delta V(\lambda) < 0$. According to the differential equation (29) a spatial equilibrium is stable if for any marginal deviation from the equilibrium the economy is driven back to its original state by the equation of motion. In other words, a spatial equilibrium is locally stable if for any marginal deviation of the population distribution from the equilibrium the equation of motion (29) brings the distribution of workers back to the original one. When some workers move from one region to the other we assume that local labor markets adjust instantaneously. More precisely, the number of firms in each region must be such that the labor market clearing conditions (9) and (10) remain valid for the new distribution of workers. At the same time utility

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differential depends on the distribution of manufacturing activity in the economy.

The symmetric equilibrium is stable (unstable) if $\left.\frac{d\Delta V(\lambda)}{d\lambda}\right|_{\lambda=\frac{1}{2}} < (>)0$. Conversely, the core-periphery pattern is stable (unstable) at $\lambda = 1$ if the utility differential is positive (negative), $\Delta V(1) > (<)0$. By symmetry, the core-periphery equilibrium is stable (unstable) at $\lambda = 0$ if the utility differential is negative (positive), $\Delta V(0) < (>)0$.

From this point of view there exists a value of τ where $d\Delta V(1/2)/d\lambda$ changes sign and an other one where $\Delta V(\lambda)$ changes sign for $\lambda = 0, 1$. Consistently with the literature, let define the first critical value of transportation cost the "break point", τ_b, and the second one the "sustain point", τ_s. The break point is the critical level of trade costs at which the diversified equilibrium is on the brink of instability and symmetry is broken. The sustain point is the threshold level of trade costs at which agglomeration of manufacturing starts to be a stable equilibrium.

3.1 No collusion

In the absence of collusion it can be shown that $\left.\frac{d\Delta V(\lambda)}{d\lambda}\right|_{\lambda=\frac{1}{2}}$ is a quadratic function of τ of the form (see Appendix C):

$$\left.\frac{d\Delta V(\lambda)}{d\lambda}\right|_{\lambda=\frac{1}{2}} = \tau B_1 - \tau^2 B_2^2, \tag{30}$$

where B_1, B_2 are positive coefficients which depend in a very complex fashion on all the other parameters of the model. The above equation simply implies that there exists one break point at which the symmetric equilibrium ceases to be stable, $\tau_b = \frac{B_1}{B_2}$.

Similarly, it can be shown that

$$\Delta V(1) = \tau S_1 - \tau^2 S_2^2, \tag{31}$$

where $S_1, S_2 > 0$. From (31) it clearly emerges that the sustain point at which the CP becomes sustainable is $\tau_s = \frac{S_1}{S_2}$.

Since the model is not analytically tractable in order to analyze how spatial equilibrium

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changes with trade costs we will rely on several numerical examples, assigning different values to the parameters of the model. We will also check for robustness of our results carrying out a sensitivity analysis. The baseline calibration is reported in Table 1.

Figure 1 plots the utility differential between regions, $\Delta V(\lambda)$, against the share of manufacturing workers in the Home region, λ, for different values of trade costs in the interval $(0, \tau_{trade})$. Figure 1 represents a variant of the so-called "wiggle diagram" used in the standard CP literature (see e.g. Baldwin et al., 2005 and Fujita et al. 1999). Curves, in fact, wiggle as trade costs change. All four curves are calculated at baseline calibration reported in Table 1 with the agriculture labor force A being equal to 4. As it will be shown below, the dimension of the agriculture sector should be sufficiently large for trade to occur under all possible spatial allocations of manufacturing activity.[5] For low levels of τ ($\tau = 0.2, 0.5$) we observe that only core-periphery configurations are stable. By contrast, for high trade costs ($\tau = 1.06$) the symmetric equilibrium is the unique stable pattern. Finally, it can be noted that both the core-periphery pattern and the symmetric equilibrium are stable for intermediate levels of trade costs ($\tau = 0.96$).

In order to better illustrate the above results we can consider the bifurcation diagram reported in Figure 2 plotting λ against trade costs, τ. Solid and dotted lines denote stable and unstable equilibria, respectively. A core periphery pattern is sustainable for $\tau \leq \tau_s$, while the stability of the symmetric equilibrium is broken at $\tau = \tau_b$. Since $\tau_s > \tau_b$ all configurations are stable for $\tau_b < \tau < \tau_s$.

These results are consistent with the main predictions of the standard NEG literature, according to which agglomeration of manufacturing activity is more likely when transport costs are low. Moreover, the existence of an interval of trade costs for which all configurations are stable suggests that, at least under the baseline calibration, the bifurcation diagram, describing the stability of the equilibria is given by the standard "tomahawk" diagram.[6]

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[5]In the following section we will show that the centrifugal force of the agriculture sector is much stronger under collusion. For high levels of agricultural labor force, in fact, trade occurs only when manufacturing firms are identically split between the two regions.

[6]In Ottaviano et al. (2002) since equation (28) is linear in λ the "break point" corresponds to the "sustain

In what follows we analyze how the critical values of τ change in function of the degree of increasing returns, the dimension of agriculture sector, the degree of substitution between goods and the intensity of consumers' preference for differentiated goods.

Degree of increasing returns, ϕ

Figure 3a reports the sustain and the break points (τ_s, τ_b) as function of the degree of increasing returns, ϕ. Let the solid line indicate τ_s and the dotted line τ_b. The critical level of trade costs τ_{trade} is represented by the lighter dotted line. We observe that τ_s is always larger than τ_b though the difference is negligible. However, this tiny gap between τ_b and τ_s opens up the possibilities of multiple equilibria.

The interval of trade cost for which the core-periphery pattern is stable tends to be larger for a high degree of increasing returns (i.e. low number of firms). Conversely, for $\phi \to 0$ the symmetric equilibrium is the only possible configuration. The dynamics of the economy described in the bifurcation diagram of Figure 2 is thus robust to changes in the degree of increasing returns.

Dimension of agriculture sector, A

Figure 3b illustrates the sustain and the break points as function of the dimension of the agriculture of the economy A. As expected, both critical levels of τ are decreasing in A implying that a core-periphery configuration is not sustainable for very large levels of the agriculture sector. The dimension of the agricultural sector works as a centrifugal force. It should be also noted that for low levels of A trade occurs only in the symmetric equilibrium.

Degree of substitution, γ

Figure 3c plots the sustain and the break points as function of the degree of substitution among goods, γ. Higher γ implies that consumers consider different varieties as closer substitutes. The interval of trade costs under which the core-periphery configuration is

point".

sustainable tends to be larger for low degrees of substitution. On the other hand, when goods tend to be perfect substitutes, the manufacturing activity is allocated evenly across regions.

Intensity of consumers' preference for differentiated goods, α

Figure 3d shows the critical levels of trade costs as function of the intensity of consumers' preference for differentiated goods with respect to the homogenous one. As expected for high α full agglomeration of manufacturing activity is sustainable for a larger interval of τ. Intensity of consumers' preference for differentiated goods works as a centripetal force since the higher α the larger the share of income consumers wish to spend for differentiated products.

3.1.1 Welfare analysis

Let Ω be the world social welfare defined as the sum of the indirect utilities of all the individuals of the economy:

$$\Omega = (S_h + w_h)\,\lambda L + (S_f + w_f)\,(1 - \lambda)L + \frac{A}{2}\,(S_h + 1) + \frac{A}{2}\,(S_f + 1). \qquad (32)$$

It can be shown that the above expression has always an interior extremum at $\lambda = 1/2$. The symmetric configuration is the optimum only if trade costs are above a certain critical value.

The grey areas in the set of Figures 4 represent the combinations of trade costs and critical parameters for which the symmetric configuration is never desirable. We observe that for very low values of trade costs a symmetric equilibrium is not socially optimal. By using the results in the set of Figures 3 we notice that in most cases market forces yield a dispersed (agglomerated) configuration for a whole range of trade cost values for which it is socially desirable to have an agglomerated (dispersed) pattern of activities.

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3.2 Collusion

In the case of collusion the function $\frac{d\Delta V(\lambda)}{d\lambda}\big|_{\lambda=\frac{1}{2}}$ is of the form (see Appendix C):

$$\frac{d\Delta V(\lambda)}{d\lambda}\bigg|_{\lambda=\frac{1}{2}} = -\overline{B}_0 + \tau\overline{B}_1 - \tau^2\overline{B}_2^2, \tag{33}$$

where $\overline{B}_0, \overline{B}_1, \overline{B}_2 > 0$.

Similarly, the indirect utility gap computed at $\lambda = 1$ is a quadratic function of τ:

$$\Delta V(1) = -\overline{S}_0 + \tau\overline{S}_1 - \tau^2\overline{S}_2^2, \tag{34}$$

where $\overline{S}_0, \overline{S}_1, \overline{S}_2 > 0$ (see Appendix C for details).

From the results it emerges that there may exist four critical values of τ for which the stability conditions change.[7] In particular, there may be two break points, τ_{b1} and τ_{b2} (with $\tau_{b1} < \tau_{b2}$, for convenience) at which $d\Delta V(1/2)/d\lambda$ changes sign, and two sustain points, τ_{s1} and τ_{s2} (with $\tau_{s1} < \tau_{s2}$), where $\Delta V(\lambda)$ changes sign for $\lambda = 0, 1$.

As done in the previous section we will use several numerical examples to study how spatial equilibrium changes with trade costs for different parameter values.

Figure 5 plots the utility differential between regions, $\Delta V(\lambda)$, against λ for different levels of trade costs in the interval $(0, \tau_{trade})$. All five curves are calculated at baseline calibration under the assumption that the agricultural labor force A is equal to 0.5. As anticipated in the previous section, in fact, the agriculture sector centrifugal force is much stronger when firms collude.

For $\tau = 2.8$ the utility differential is always positive (negative) if $\lambda < (>)1/2$ implying that the symmetric configuration is stable. When $\tau = 2.6$ a more complex dynamics emerges since the utility differential crosses the zero-axe in three points. We notice that $\Delta V(0) < 0$, $\Delta V(1) > 0$ and $d\Delta V(1/2)/d\lambda < 0$ suggesting that there are three stable configurations: one diversified and two agglomerated. On the other hand, for $\tau = 2$ we observe that the

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[7]Clearly, (33) and (34) may not have real roots.

utility differential is always negative if $\lambda < 1/2$ and positive if $\lambda > 1/2$, implying that manufacturing may agglomerate in either region. For $\tau = 0.5$ we observe that $\Delta V(0) > 0$, $\Delta V(1) < 0$ and $d\Delta V(1/2)/d\lambda > 0$, implying that both the core-periphery pattern and the dispersed equilibrium are unstable. In this case partial agglomeration equilibria may emerge. Finally, for $\tau = 0.2$ the core-periphery configuration cannot be sustained and there exist only a symmetric diversified equilibrium. If transport costs are close to zero the region where more than half of the manufacturing workers are located is less attractive than the other region in utility terms. This result runs counter the common wisdom of the core-periphery analysis, in which, for transport costs close to zero a core-periphery pattern always arises.

The results of Figure 5 can be better illustrated in the bifurcation diagram plotted in Figure 6a. It can be noted that the break and the sustain points are such that $\tau_{b1} < \tau_{s1} < \tau_{b2} < \tau_{s2}$. For $\tau < \tau_{b1}$ manufacturing firms are evenly distributed between the two regions and agglomeration is no sustainable. For $\tau_{b1} < \tau < \tau_{s1}$ the symmetric equilibrium is broken and the economy exhibits partial agglomeration. In the interval $\tau_{s1} < \tau < \tau_{b2}$ the world economy moves toward an equilibrium with full agglomeration. For trade costs such that $\tau_{b2} < \tau < \tau_{s2}$ both the core-periphery pattern and the dispersed equilibrium are stable. Finally, for high trade costs $\tau > \tau_{s2}$ agglomeration ceases to be sustainable and diversification is the only stable equilibrium.

We will show below that the critical values of transportation costs change for different values of the parameters of the model and other possible configurations may emerge. In particular, we will observe other three cases. Figure 6b shows the case in which there are four critical points, such that $\tau_{b1} < \tau_{s1} < \tau_{s2} < \tau_{b2}$. For $\tau < \tau_{b1}$ there is symmetric equilibrium. For trade costs such that $\tau_{b1} < \tau < \tau_{s1}$ we observe partial agglomeration of economic activity. For $\tau_{s1} < \tau < \tau_{s2}$ all economic activity is concentrated in a single region. For $\tau_{s2} < \tau < \tau_{b2}$ partial agglomeration equilibria take place. For higher trade cost such that $\tau > \tau_{b2}$ firms are equally located in both regions. Figure 6c reports the

case in which there exist two break points but there are no sustain points.[8] Specifically, for $\tau < \tau_{b1}$ economic activity is evenly distributed across space. For trade costs in the interval $\tau_{b1} < \tau < \tau_{b2}$ symmetry is broken and equilibria with partial agglomeration arise. For $\tau > \tau_{b2}$ the symmetric equilibrium becomes the only stable pattern. Finally, Figure 6d illustrates the case in which neither sustain points nor break points exist. In this case the only possible stable equilibrium is the symmetric one.[9]

We observe that for low values of τ the symmetric equilibrium always prevails. By contrast, the core-periphery configuration is sustainable for values of trade costs that are neither too high nor too low. As in the standard case, for high trade costs firms are locally dispersed. Collusion among firms could explain the observed bell-shaped curve of spatial distribution of economic activity. We deduce that collusion among firms operating in the same country may act as a centrifugal force at very low levels of transport costs. The economic intuition for this result is the following.

Manufacturing workers as consumers are always more content to locate in a single region (the core) since if located in the distant market (the periphery) they would bear a larger fraction of trade costs. By contrast, as workers employed in the collusive sector, their wage crucially depends on the firms' profits and on the level of trade costs. For high trade costs the existence of an immobile labor force in agriculture is the reason why it is always convenient for firms to be geographically dispersed. At intermediate levels of trade costs it is more beneficial for firms to be located in the same region, serving a largest local market and reducing trade costs that they must sustain in the distant market. In the case of full agglomeration, colluding firms act as a monopolist in both markets. At very low trade costs agglomeration is not sustainable anymore, because the level of firms' freight absorbtion becomes negligible. This effect encourages stability of the diversified equilibrium, since it becomes convenient for firms to move from the core to the periphery in order to serve the distant market at a lower price.

The four possible dynamics illustrated in the bifurcation diagrams plotted in Figures

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[8]In this case the polynomial (34) has not real roots.

[9]In this case both polynomials (33) and (34) have not real roots.

5a-5d are summarized in Table 2. In the remainder of this section we will refer to this Table to discuss how the critical values of τ vary with the degree of increasing returns, the dimension of agriculture sector, the degree of substitution between goods and the intensity of consumers' preference for differentiated goods.

Degree of increasing returns, ϕ

Consider Figure 7a reporting the critical values of trade costs in the case of collusion as function of the degree of increasing returns. When the degree of increasing returns is high (i.e. low number of firms) the possible spatial allocation of economic activity is as in CASE 1 (figure 6a). Conversely, for a low level of increasing returns the spatial allocation is as in CASE 3 (figure 6c). For large number of firms we observe in fact either symmetric equilibrium or partial agglomeration. In other words, full agglomeration is not sustainable when the number of firms is sufficiently high. In the limiting case $\phi \to 0$ the symmetric equilibrium is the only possible configuration.

Dimension of agriculture sector, A

Figure 7b shows how break and sustain points are affected by the dimension of agriculture sector. When the dimension of the agriculture sector is sufficiently small the allocation of the differentiated sector across region in function of trade costs is as in CASE 1 (figure 6a). Increasing A the configuration is first as in CASE 2 (figure 6b), then as in CASE 3 (figure 6c), finally as in CASE 4 (figure 6d). The dimension of agricultural sector works as a centrifugal force.

Degree of substitution, γ

Figure 7c illustrates how critical points change in function of the degree of substitution among differentiated products. For a low degree of substitution the geographical distribution of the manufacturing sector is as in CASE 1 (figure 6a). For a higher level, instead, the distribution is as in CASE 3 (figure 6c). A core-periphery equilibrium is in fact sustainable

only for a low degree of substitution among differentiated product. In the limiting case $\gamma \to 1$ the symmetric equilibrium is the only possible configuration.

Intensity of consumers' preference for differentiated goods, α

Finally, Figure 7d plots the relationship between critical trade costs and the degree of consumers' preference for manufacturing goods. For any value of the intensity of consumers preference for differentiated goods the possible allocation of manufacturing activity for different values of transportation cost is as in CASE 1 (figure 6a). We observe that in general α works as a centripetal force. Agglomeration is sustainable for a larger interval of τ when households preference for differentiated goods is high.

3.2.1 Welfare analysis

Turning to the social desirability of the equilibrium it can be shown that total welfare (32) has always an interior extremum at $\lambda = 1/2$. The grey areas in the set of Figures 8 represent the combinations of trade costs and critical parameters for which the dispersed equilibrium is not socially desirable. As in the absence of collusion the symmetric configuration is the optimum only if trade costs are sufficiently high. By using the results in the set of Figures 7 we observe that in most cases market forces yield a dispersed (agglomerated) configuration for a whole range of trade cost values for which it is socially desirable to have an agglomerated (dispersed) pattern of activities. Clearly, the symmetric equilibrium that emerges for fully integrated market under collusion is not optimal.

4 Conclusions

In this paper we have studied how strategic interactions among oligopolistic firms may influence the location and the industrialization process. We have modified the basic CP model setup proposed by Ottaviano et al. (2002) by introducing oligopolistic pricing competition. In the standard CP models, in fact, the assumption of a large number of firms leads to

leaving aside any form of strategic interaction.

The analysis carried out in this paper has shown that strategic interactions play a crucial role in driving spatial distribution of economic activity and that the robustness of the results obtained in the standard CP models crucially depends on the type of pricing competition. From the sensitivity analysis it stems out that, in the absence of collusion, spatial configurations emerging for different levels of trade costs are robust to changes in the parameters of the model. On the other hand, under collusion a more complex picture emerges, since potential spatial allocations of manufacturing activity depend on market fundamentals. The dimension of the agriculture sector is shown to be a much stronger centrifugal force in the presence of colluding firms. Most importantly, when oligopolistic firms do not collude a core-periphery pattern is always sustainable for low transportation costs, consistently with the basic CP models. On the other hand, when oligopolistic firms located in the same region collude in both countries, we obtain a bell-shaped curve of spatial development and for low transportation costs diversification occurs. In other words, the collusive behavior of firms could be an additional reason why we observe re-dispersion of economic activity as trade costs decrease.

Turning to the welfare properties of the model we have shown that market forces alone are able to yield agglomeration (dispersion) for levels of economic integration for which it would be socially desirable to keep industrial activities geographically dispersed (agglomerated). In the particular case of zero trade costs we have observed that a core-periphery pattern is always optimal, that is why we deduce that full economic integration will be welfare improving only in the absence of any collusive behavior among firms. The explicit consideration of a spatial dimension in an oligopolistic market provides supplementary reasons to enforce competition among firms in an increasingly integrated world economy.

References

Baldwin, R., R. Forslid, P. Martin, G. Ottaviano and F. Robert-Nicoud (2005), *Economic Geography and Public Policy*, Princeton: Princeton University Press.

Combes, P.P. (1997), Industrial Agglomeration under Cournot Competition", *Annales d'Economie et de Statistique*, 45, pp. 161-182.

Combes, P.P. and H.G. Overman (2004), The Spatial Distribution of Economic Activities in the European Union, in J.V. Henderson and J.F. Thisse (eds), *Handbook of Regional and Urban Economics*, Amsterdam: North Holland, vol. 4, chapter 64, pp. 2845-2909.

Dixit, A. K. and J. E. Stiglitz (1977), Monopolistic Competition and Optimum Product Diversity, *American Economic Review*, 67, pp. 297-308.

Fujita, M., and J-F. Thisse (1996), Economics of Agglomeration, *Journal of the Japanese and International Economies*, 10, pp. 339-78.

Fujita, M., and J-F. Thisse (1997), Economie géographique. Problèmes anciens et nouvelles perspectives, *Annales D'Economie et de Statistique*, 45, pp. 7-85.

Fujita, M., P. Krugman and A. J. Venables (1999), *The Spatial Economy*, Cambridge Massachusetts: The MIIT Press.

Krugman, P. R. (1991a), Increasing Returns and Economic Geography, *Journal of Political Economy*, 99(3), pp. 483-99.

Krugman, P. R. (1991b), *Geography and Trade*, Cambridge Massachusetts: The MIT Press.

Krugman, P.R. and A.J. Venables (1995), Globalization and the inequality of nations, *Quaterly Journal of Economics,* 60, pp. 857-880.

Matsuyama, K. (1995), Complementarities and Cumulative Processes in Models of Monopolistic Competitions, *Journal of Economic Literature*, 33, pp. 701-29.

Neary, J. P. (2001), Of Hype and Hyperbolas: Introducing the New Economic Geography, *Journal of Economic Literature*, 39, pp. 536-561.

Ottaviano, G., and D. Puga (1998), Agglomeration in the Global Economy: A Survey

of the New Economic Geography, *World Economy*; 21(6), pp. 707-31.

Ottaviano, G., T. Tabuchi and J.F. Thisse (1999), Agglomeration and Trade Revisited, *International Economic Review, 43*, pp. 409-435.

Ottaviano, G. and J.F. Thisse (2004), Agglomeration and Economic Geography, in J.V Henderson and J.F. Thisse (eds), *Handbook of Regional and Urban Economics*, Amsterdam: North Holland, vol. 4, chapter 58, pp. 2563-2608.

Picard P. and D.Z. Zeng (2005), Agricultural sector and industrial agglomeration, Journal of Development Economics, 77, pp. 75-106

Puga D. (1999), The rise and fall of regional inequalities, *European Economic Review*, 43, 303-334.

Tabuchi T. and J.F. Thisse (2002) Taste heterogeneity, labor mobility and economic geography, *Journal of Development Economics,* 69, 155-177.

Thisse, J-F. and X. Vives (1988), On the Strategic Choice of Spatial Price Policy, *American Economic Review*, 78, pp. 122-37.

Vives, X. (1999), *Oligopoly Pricing: Old Ideas and New Tools*, Cambridge Massachusetts: The MIT Press.

Williamson J. (1965) Regional inequality and the process of national development, *Economic Development and Cultural Change,* 14, pp. 3-45.

Appendix A

Proof of Proposition 1

The result immediately follows from equations (16)-(19) by noting that

$$p_{hf} - p_{hh} = \frac{b + cN - c}{2b + 2cN - c}\tau \left[1 + \frac{c\left(n_h - n_f\right)}{2b + cN - c}\right], \tag{A1}$$

$$p_{fh} - p_{ff} = \frac{b + cN - c}{2b + 2cN - c}\tau \left[1 + \frac{c\left(n_f - n_h\right)}{2b + cN - c}\right]. \tag{A2}$$

From the above equations it follows that $p_{hf} - p_{hh}(p_{fh} - p_{ff}) < \frac{1}{2}\tau$ if $n_h - n_f < (>$
$)\frac{2b+cN-c}{2(2b+2cN-c)}$ ∎

Proof of Proposition 2

From equations (19) given (17) we have

$$p_{hf} - \tau = \frac{a}{2b + cN - c} + \tag{A3}$$
$$+ \left[\frac{b + cN - c}{(2b + 2cN - c)(2b + cN - c)} cn_h + \frac{b + cN - c}{2b + 2cN - c} - 1 \right] \tau.$$

The price p_{hf} net of transportation cost τ is larger than zero provided that

$$\tau < \frac{a(2b + 2cN - c)}{(2b + cN - c)(b + cN) - (b + cN - c)cn_h}. \tag{A4}$$

For any n_h in the interval $0 \le n_h \le N$ the RHS of the above inequality has a minimum at $n_h = 0$. It follows that for any n_h such that $0 \le n_h \le N$, $p_{hf} - \tau > 0$ if $\tau < \frac{a(2b+2cN-c)}{(2b+cN-c)(b+cN)}$. The result for $p_{fh} - \tau$ follows by symmetry ∎

Proof of Proposition 3

The result follows from a close inspection of the price gaps, $p_{hh} - p_{ff}$ and $p_{fh} - p_{hf}$ using (16)-(19):

$$p_{hh} - p_{ff} = \frac{b + cN - c}{(2b + 2cN - c)(2b + cN - c)} (n_f - n_h) c\tau, \tag{A5}$$
$$p_{fh} - p_{hf} = \frac{b + cN - c}{(2b + 2cN - c)(2b + cN - c)} (n_f - n_h) c\tau. \tag{A6}$$

It follows that when $n_h > (<)n_f$ then $p_{hh} < (>)p_{ff}$ and $p_{fh} < (>)p_{hf}$.

By the definitions of price indexes:

$$P_h - P_f = n_h p_{hh} + n_f p_{fh} - n_f p_{ff} - n_h p_{hf}, \tag{A7}$$

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using the price equations one obtains:

$$P_h - P_f = -\left[\frac{c(n_h + n_f)}{2b + cN - c} + 1\right]\frac{b + cN - c}{2b + 2cN - c}\tau(n_h - n_f). \qquad \text{(A8)}$$

From the above equation it emerges that $P_h < (>)P_f$ if $n_h > (<)n_f$ ∎

Proof of Proposition 4

From (22)-(25) the price gaps are:

$$p_{hf} - p_{hh} = \frac{(2b + cn_f)(b + cn_h)}{4b(b + cN) + 3c^2 n_h n_f}\tau, \qquad \text{(A9)}$$

$$p_{fh} - p_{ff} = \frac{(2b + cn_h)(b + cn_f)}{4b(b + cN) + 3c^2 n_h n_f}. \qquad \text{(A10)}$$

Assume *ad absurdum* that the RHS of equation is larger than $\frac{1}{2}\tau$:

$$\frac{(2b + cn_f)(b + cn_h)}{4b(b + cN) + 3c^2 n_h n_f}\tau > \frac{1}{2}\tau. \qquad \text{(A11)}$$

Re-arranging the above inequality gives:

$$\frac{c}{4b^2 + 4Nbc + 3n_f n_h c^2}(2bn_h - 2bN - cn_f n_h) > 0, \qquad \text{(A12)}$$

which is impossible since $n_h \leq N$. It follows that:

$$p_{hf} - p_{hh} = \frac{(2b + cn_f)(b + cn_h)}{4b(b + cN) + 3c^2 n_h n_f}\tau < \frac{1}{2}\tau. \qquad \text{(A13)}$$

∎

Proof of Proposition 5

From equations (23) given (22) we have

$$
\begin{aligned}
p_{hf} - \tau \;=\;\; & \frac{acn_f + 2a(b + cn_h)}{4b(b + cN) + 3c^2 n_h n_f} + \\
& + \left[\frac{(b + cn_h)n_f c}{4b(b + cN) + 3c^2 n_h n_f} + \frac{(2b + cn_f)(b + cn_h)}{4b(b + cN) + 3c^2 n_h n_f} - 1 \right] \tau .
\end{aligned}
\tag{A14}
$$

The price p_{hf} net of transportation cost τ is larger than zero provided that

$$
\tau < \frac{2ab + acN + acn_h}{2b^2 + c^2 n_f n_h + 2Nbc}.
\tag{A15}
$$

The result for $p_{fh} - \tau$ follows by symmetry ∎

Proof of Proposition 6

The result can be derived form the price gaps, $p_{hh} - p_{ff}$ and $p_{fh} - p_{hf}$ using (22)-(25):

$$
p_{hh} - p_{ff} \;=\; c\frac{a - b\tau}{4b(b + cN) + 3c^2 n_h n_f}\left(n_h - n_f\right),
\tag{A16}
$$

$$
p_{fh} - p_{hf} \;=\; -c\frac{a}{4b(b + cN) + 3c^2 n_h n_f}\left(n_h - n_f\right).
\tag{A17}
$$

It follows that when $n_h > (<)n_f$, then $p_{hh} > (<)p_{ff}$ if $\tau < a/b$, while $p_{fh} < (>)p_{hf}$.

By definitions of price indexes:

$$
P_h - P_f = n_h p_{hh} + n_f p_{fh} - n_f p_{ff} - n_h p_{hf},
\tag{A18}
$$

Substituting the price equations (22)-(25) in (A18) yields:

$$
P_h - P_f = -\frac{\left(n_h - n_f\right)\left(2b^2 + c^2 n_f n_h + 2bcn_f + 2bcn_h\right)}{4b(b + cN) + 3c^2 n_h n_f}\tau.
\tag{A19}
$$

From the above equation it emerges that $P_h < (>)P_f$ if $n_h > (<)n_f$ ∎

Appendix B

Collusion among All Firms of the Economy

In the case of collusion among all firms of the economy a unique price will prevail in all markets, whether the good is produced locally or imported from abroad.

The optimal price level will be determined by maximizing of the total amount of profits of the world economy Π_{world} defined as

$$\Pi_{world} = \sum_{j=1}^{n_h} \Pi_{j,h}(p) + \sum_{k=1}^{n_f} \Pi_{k,f}(p). \tag{B1}$$

where $p = p_{hh} = p_{ff} = p_{hh} = p_{fh}$. By solving the optimization problem we obtain that the optimal price:

$$p = \frac{1}{2}\left(\frac{a}{b} + \frac{1}{2}\tau\right). \tag{B2}$$

Trade will take place provided that $p - \tau > 0$, that is when $\tau < \frac{2}{3}\frac{a}{b}$. It can be easily shown that in this case:

$$V_h - V_f = 0 \tag{B3}$$

for any τ and λ, implying that $\overset{\bullet}{\lambda} = 0$ at all times. It follows that any spatial allocation of firms is stable.

Appendix C

Break and Sustain Points in the Non-Collusive Case

With the help of a symbolic software[10] it can be shown that in the absence of collusion we have that:

$$\left.\frac{d\Delta V(\lambda)}{d\lambda}\right|_{\lambda=\frac{1}{2}} = -\frac{N}{(4\,b^2 + 6\,cNb - 4\,bc + 2\,c^2N^2 - 3\,c^2N + c^2)^2\phi} \times$$
$$[\,(12\,\phi\,b^5 - 10\,c^2b^3A + 4\,cAb^4 + 48\,Ncb^4\phi + 2\,N^5c^5\phi + c^4\phi\,b + 6\,N^3c^5\phi - 44\,N^3c^4\phi\,b) +$$
$$+ (54\,N^3c^3\phi\,b^2 + 23\,c^2\phi\,b^3 - 28\,cb^4\phi - 2\,Nc^5A - 2\,N^2c^5\phi + 35\,N^2c^4\phi\,b + 2\,N^4c^5A) +$$

---

[10]Maple files are available from the authors upon request.

$$+ \left(-6\,N^3 c^5 A + 10\,N^3 c^4 b A + 6\,N^2 c^5 A - 98\,N^2 c^3 \phi\, b^2 + 74\,N^2 c^2 \phi\, b^3\right) +$$

$$+ \left(-10\,N c^4 \phi\, b + 52\,N c^3 \phi\, b^2 - 88\,N c^2 \phi\, b^3 - 8\,c^3 \phi\, b^2 - 6\,N^4 c^5 \phi + 18\,N^4 c^4 \phi\, b\right) +$$

$$+ \left(-22\,N^2 c^4 b A + 18\,N^2 c^3 b^2 A + 14\,N c^4 b A - 26\,N c^3 b^2 A + 14\,N c^2 b^3 A - 2\,c^4 b A + 8\,c^3 b^2 A\right)]\tau^2 +$$

$$+ 2\,\frac{\left(4\,c^3 N^3 - 9\,N^2 c^3 + 14\,N^2 bc^2 + 6\,Nc^3 - 20\,Nbc^2 + 16\,Nb^2 c - c^3 + 6\,bc^2 - 11\,b^2 c + 6\,b^3\right)Na}{(-c + cN + 2\,b)(4\,b^2 + 6\,cNb - 4\,bc + 2\,c^2 N^2 - 3\,c^2 N + c^2)}\tau$$

$$\Delta V(1) = -\frac{1/2}{(4\,b^2 + 6\,cNb - 4\,bc + 2\,c^2 N^2 - 3\,c^2 N + c^2)^2 \phi}\,N \times$$

$$[(35\,N^2 c^4 \phi\, b - 98\,N^2 c^3 \phi\, b^2 + 74\,N^2 c^2 \phi\, b^3 - 10\,N c^4 \phi\, b + 52\,N c^3 \phi\, b^2 - 88\,N c^2 \phi\, b^3) +$$

$$+ \left(48\,N c b^4 \phi - 22\,N^2 c^4 b A + 18\,N^2 c^3 b^2 A + 14\,N c^4 b A - 26\,N c^3 b^2 A\right) +$$

$$+ \left(14\,N c^2 b^3 A - 2\,c^4 b A + 12\,\phi\, b^5 - 10\,c^2 b^3 A + 4\,c A b^4 + 18\,N^4 c^4 \phi\, b\right) +$$

$$+ \left(-6\,N^4 c^5 \phi + 2\,N^5 c^5 \phi - 6\,N^3 c^5 A + 10\,N^3 c^4 b A - 2\,N c^5 A + 2\,N^4 c^5 A\right) +$$

$$+ (c^4 \phi\, b - 8\,c^3 \phi\, b^2 + 23\,c^2 \phi\, b^3 - 28\,c b^4 \phi + 8\,c^3 b^2 A + 6\,N^2 c^5 A^2) +$$

$$+ \left(6\,N^3 c^5 \phi - 44\,N^3 c^4 \phi\, b + 54\,N^3 c^3 \phi\, b^2 - 2\,N^2 c^5 \phi\right)]\tau^2 +$$

$$+ \frac{\left(4\,c^3 N^3 - 9\,N^2 c^3 + 14\,N^2 bc^2 + 6\,Nc^3 - 20\,Nbc^2 + 16\,Nb^2 c - c^3 + 6\,bc^2 - 11\,b^2 c + 6\,b^3\right)Na}{(-c + cN + 2\,b)(4\,b^2 + 6\,cNb - 4\,bc + 2\,c^2 N^2 - 3\,c^2 N + c^2)}\tau$$

It can be shown that by assigning values to N the signs of the coefficients can be determined univocally. Consider the case of $N = 10$, then we have the following results:

$$\left.\frac{d\Delta V(\lambda)}{d\lambda}\right|_{\lambda=\frac{1}{2}} = \frac{\left(15390\,c^3 \phi + 4779\,c^3 L + 6750\,c^2 b\phi + 1737\,c^2 Lb + 920\,cb^2 \phi + 206\,cb^2 L + 8\,b^3 L + 40\,b^3 \phi\right)a}{(9\,c + 2\,b)(4\,b^2 + 56\,cb + 171\,c^2)\phi}\tau +$$

$$- \frac{\left(12\,b^5 L + 6543\,c^2 b^3 L + 145800\,c^5 L + 40\,cb^4 A + 15480\,c^3 A b^2 + 44712\,c^3 b^2 L\right)}{(4\,b^2 + 56\,cb + 171\,c^2)^2 \phi}\tau^2 +$$

$$- \frac{\left(139401\,c^4 Lb + 79380\,c^4 A b + 452\,cb^4 L + 145800\,c^5 A + 1300\,c^2 b^3 A\right)}{(4\,b^2 + 56\,cb + 171\,c^2)^2 \phi}\tau^2$$

$$\Delta V(1) = +10\,\frac{\left(3159\,c^3 + 1206\,bc^2 + 149\,b^2 c + 6\,b^3\right)a}{(9\,c + 2\,b)(4\,b^2 + 56\,bc + 171\,c^2)}\tau +$$

$$- 5\,\frac{\left(14580\,c^5 A + 139401\,c^4 \phi\, b + 7938\,c^4 b A + 44712\,c^3 \phi\, b^2 + 145800\,c^5 \phi\right)}{(4\,b^2 + 56\,bc + 171\,c^2)^2 \phi}\tau^2 +$$

$$- 5\,\frac{\left(1548\,c^3 b^2 A + 4\,c A b^4 + 452\,cb^4 \phi + 12\,\phi\, b^5 + 6543\,c^2 \phi\, b^3 + 130\,c^2 b^3 A\right)}{(4\,b^2 + 56\,bc + 171\,c^2)^2 \phi}\tau^2$$

Break and Sustain Points in the Collusive Case

Under collusion we have the following results:

$$\left.\frac{d\Delta V(\lambda)}{d\lambda}\right|_{\lambda=\frac{1}{2}} = -8\,\frac{(L+A)a^2 c^2 L^2}{(4\,b\phi + cL)\left(16\,b^2 \phi^2 + 16\,b\phi\, cL + 3\,c^2 L^2\right)} +$$

$$+2\,\frac{La\left(192\,b^4\phi^4+448\,b^3\phi^3Lc+380\,c^2L^2\phi^2b^2+132\,c^3L^3\phi b+13\,c^4L^4+12\,c^3L^2\phi bA+16\,c^2L\phi^2b^2A\right)}{\phi\left(16\,b^2\phi^2+16\,b\phi cL+3\,c^2L^2\right)^2}\tau+$$

$$-\frac{L\left(192\,b^5\phi^5+512\,b^4\phi^4Lc+508\,b^3\phi^3L^2c^2+220\,b^2\phi^2L^3c^3+37\,b\phi L^4c^4\right)}{\phi^2\left(16\,b^2\phi^2+16\,b\phi cL+3\,c^2L^2\right)^2}\tau^2+$$

$$-\frac{L\left(2\,c^5L^5+100\,b^2\phi^2L^2c^3A+64\,b^4\phi^4cA+24\,b\phi L^3c^4A+144\,b^3\phi^3Lc^2A+2\,c^5L^4A\right)}{\phi^2\left(16\,b^2\phi^2+16\,b\phi cL+3\,c^2L^2\right)^2}\tau^2$$

$$\Delta V(1)=-1/16\,\frac{(L+A)L^2c^2a^2}{\phi^2(b\phi+cL)b^2}+$$
$$1/16\,\frac{\left(4\,c^2L^2+c^2AL+16\,b\phi cL+12\,b^2\phi^2\right)aL}{\phi^2b(b\phi+cL)}\tau+$$
$$-1/32\,\frac{\left(8\,c^2L^2+5\,c^2AL+20\,b\phi cL+4\,cb\phi A+12\,b^2\phi^2\right)L}{\phi^2(b\phi+cL)}\tau^2$$

In this case the signs of the coefficients can be immediately determined, thus by Descartes' sign rule we know that both polynomials have two positive roots.

Table 1

Baseline calibration

$L = 1$

$q_0 = 1$

$\alpha = 3$

$\beta = 1$

$\gamma = 0.5$

$\phi = 0.25$

Implied parameter values

$N = 4$

$a = 1.2$

$b = 0.4$

$c = 0.4$

Table 2

					Stable Equilibria at		
	Fig.	No. τ_b	No. τ_s		$\lambda = 1/2$	$\lambda = 0, 1$	$0 < \lambda < 1$
Case 1	6a	2	2	$\tau_{b1} < \tau_{s1} < \tau_{b2} < \tau_{s2}$	*yes*	*yes*	*yes*
Case 2	6b	2	2	$\tau_{b1} < \tau_{s1} < \tau_{s2} < \tau_{b2}$	*yes*	*yes*	*yes*
Case 3	6c	2	0	$\tau_{b1} < \tau_{b2}$	*yes*	*no*	*yes*
Case 4	6d	0	0		*yes*	*no*	*no*

Figure 1: Wiggle Diagram under no Collusion
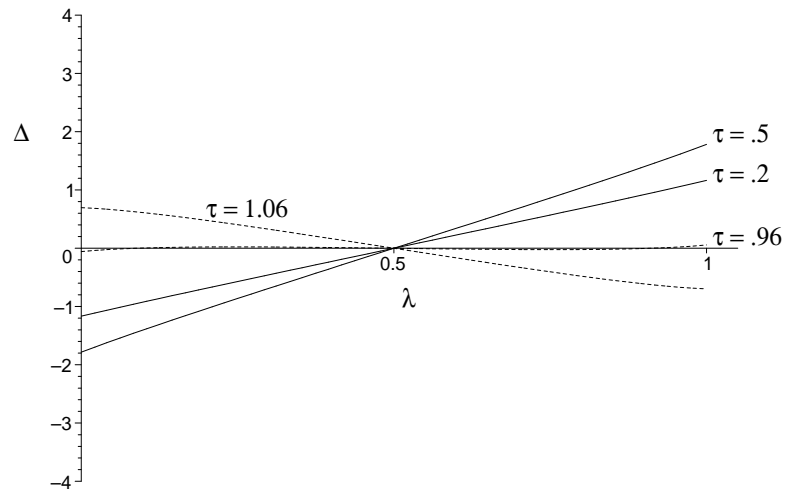
Figure 2: Bifurcation Diagram under no Collusion
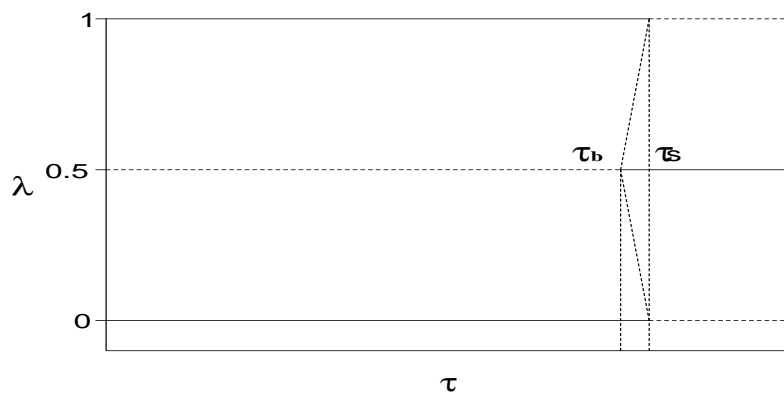
Critical Values of τ under no Collusion, Sensitivity Analysis

Figure 3a
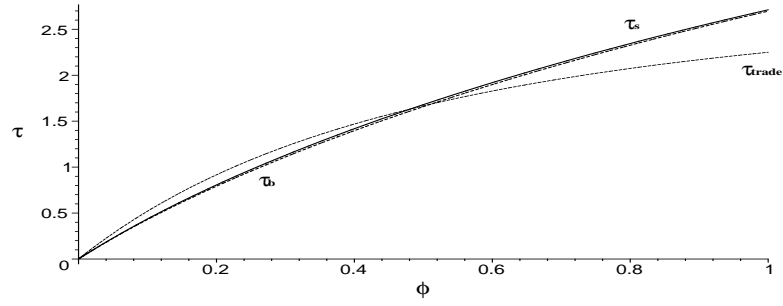
Figure 3b
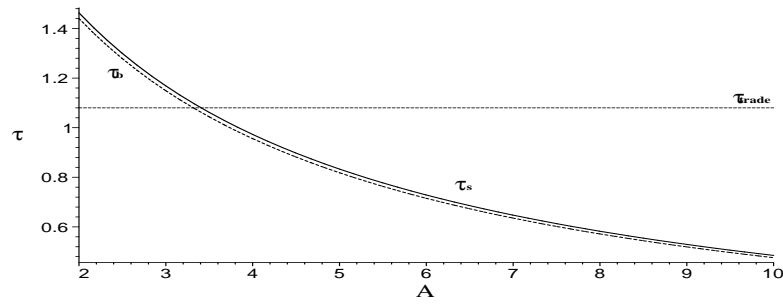
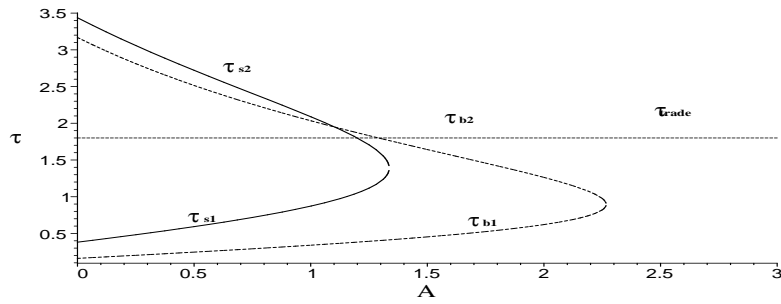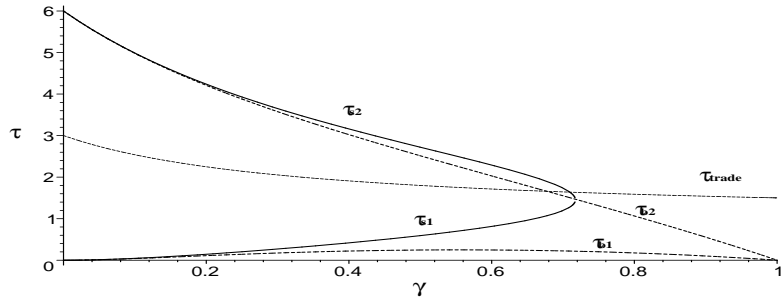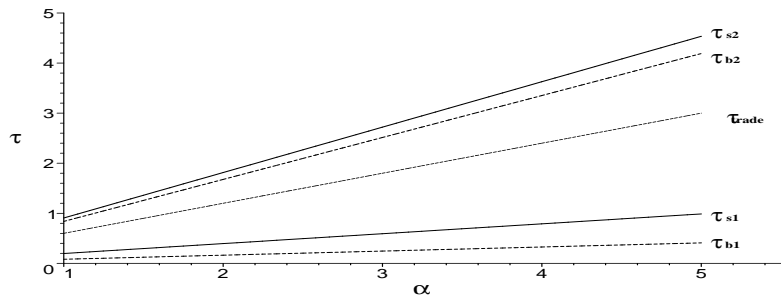## Figure 3c
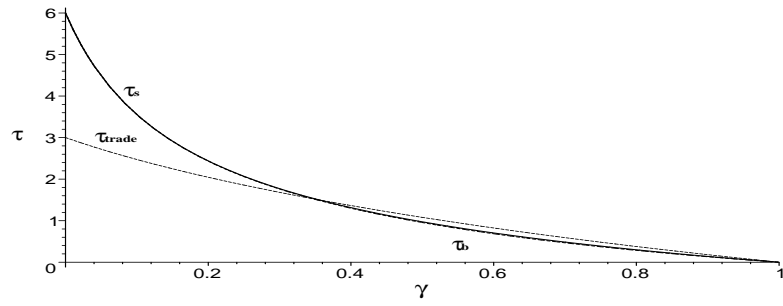
Figure 3d
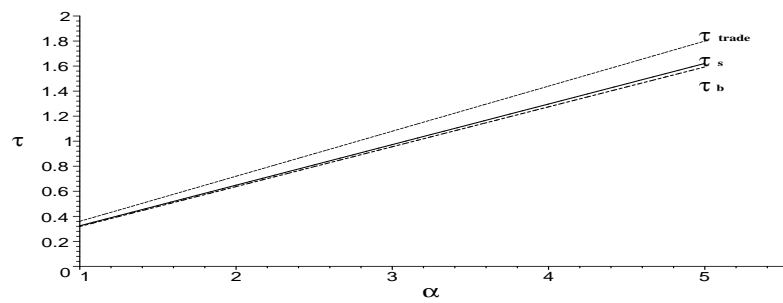
Social Desirability of the Symmetric Equilibrium under no Collusion, Sensitivity Analysis

Figure 4a
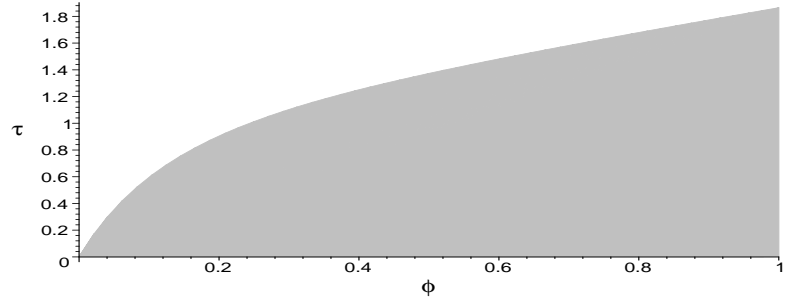
Figure 4b
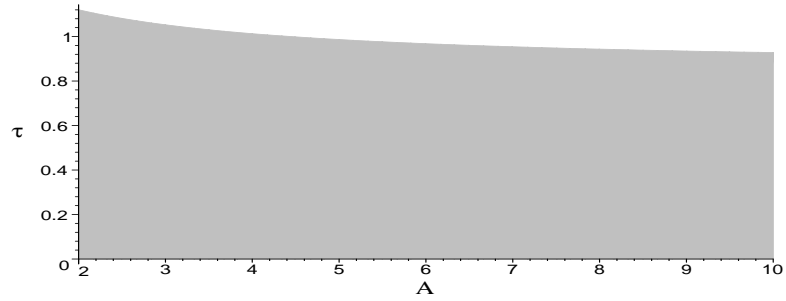
Figure 4c
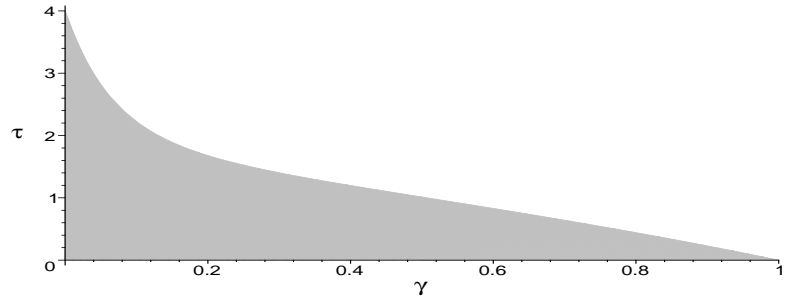
Figure 4d
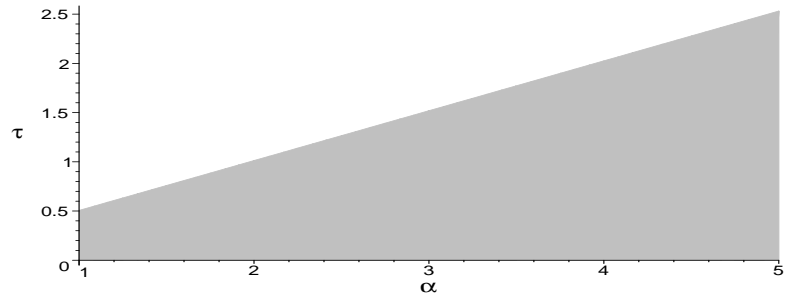
Figure 5: Wiggle Diagram under Collusion
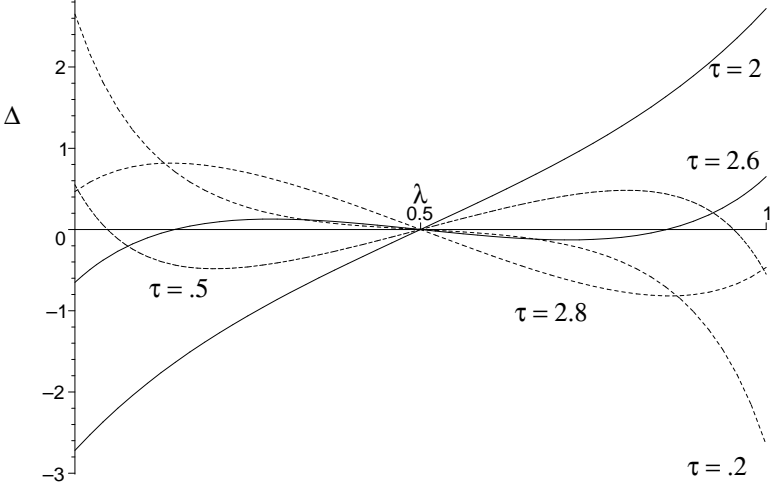
Bifurcation Diagrams under Collusion

Figure 6a
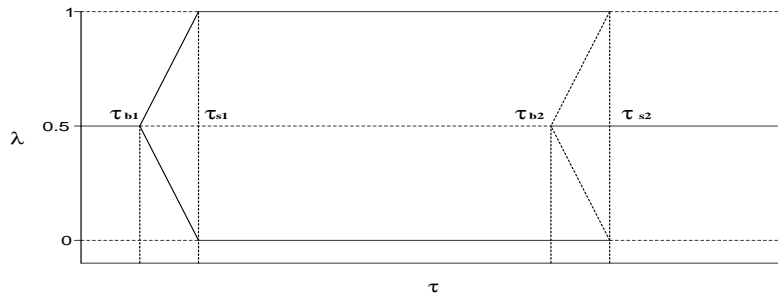
Figure 6b
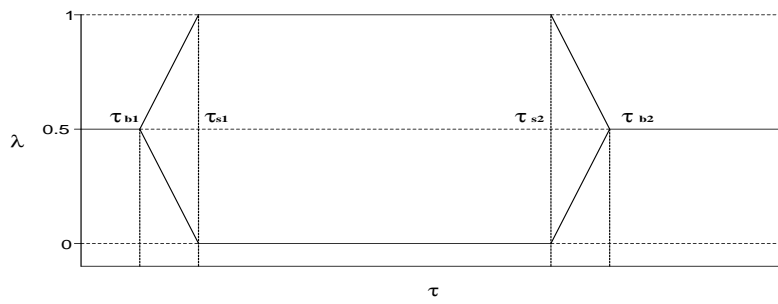
Figure 6c
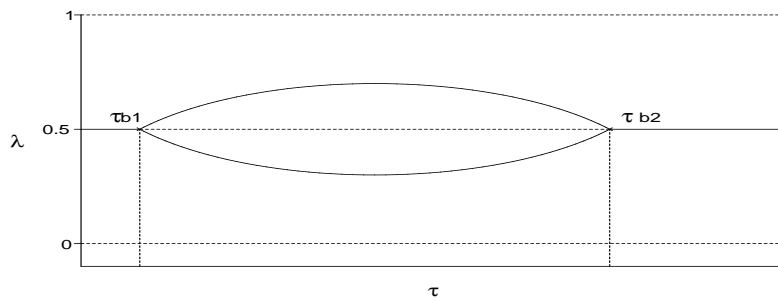
Figure6d
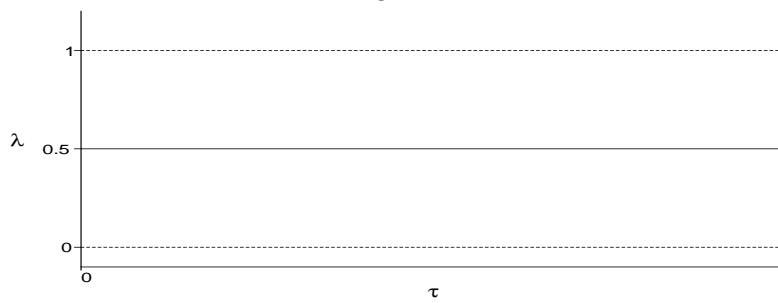
Critical Values of τ under Collusion, Sensitivity Analysis

Figure 7a
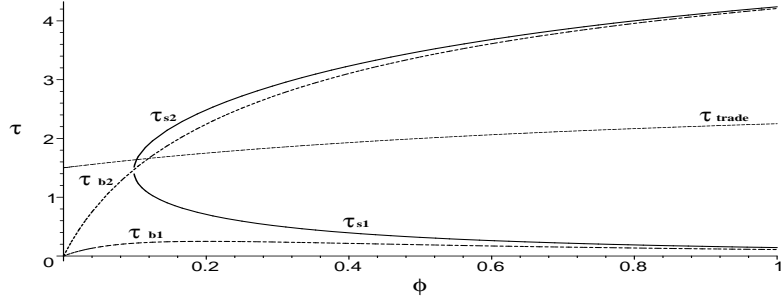
Figure 7b

Figure 7c

Figure 7d

Social Desirability of the Symmetric Equilibrium under Collusion, Sensitivity Analysis

Figure 8a
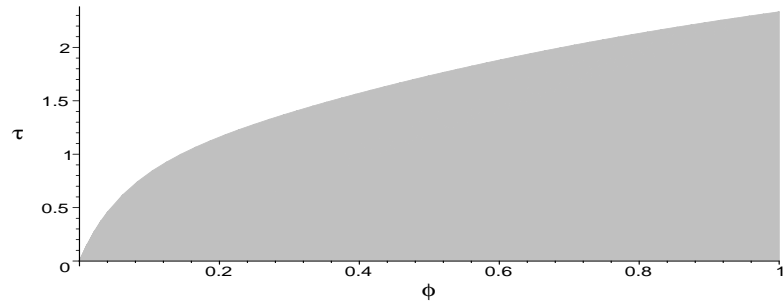
Figure 8b

Figure 8c

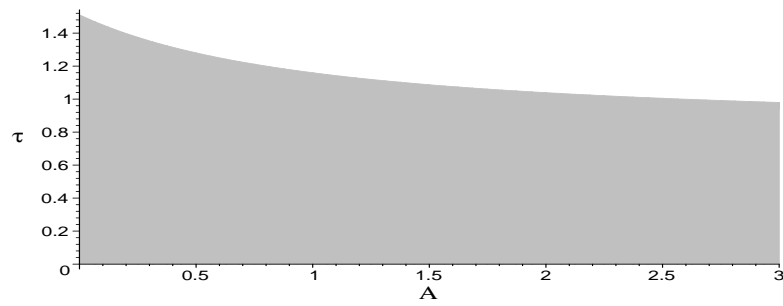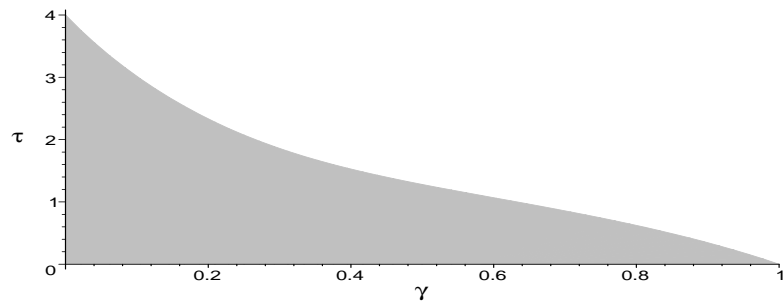### Figure 8d